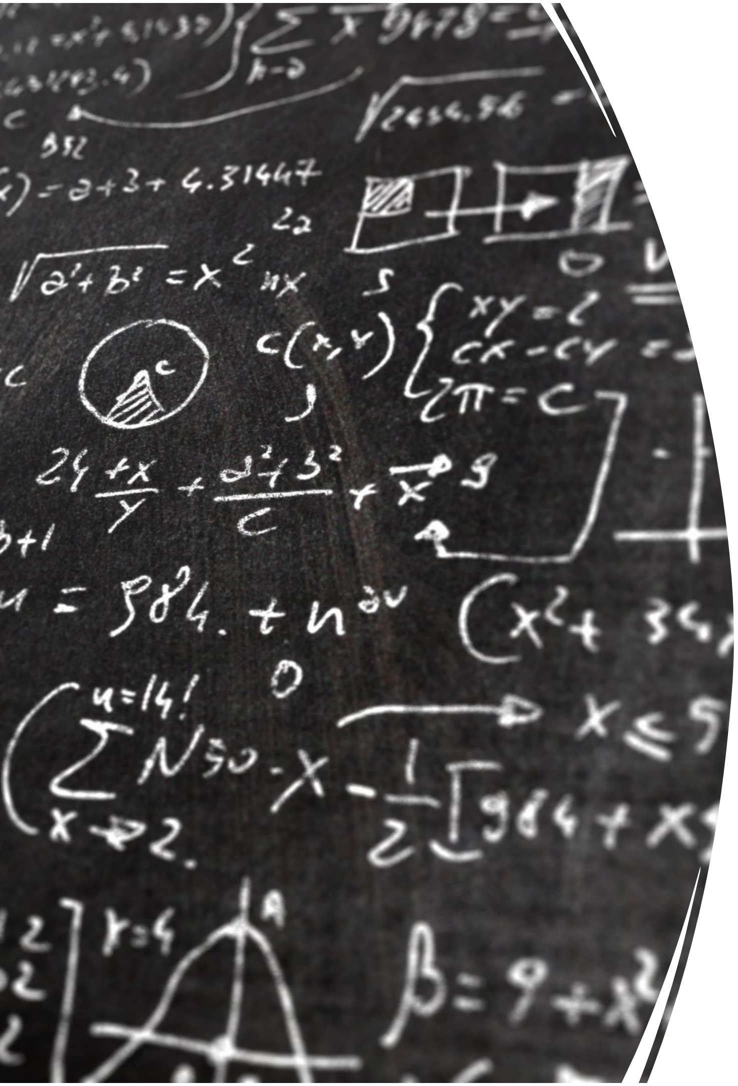


CS 383 – Computational Text Analysis

Lecture 21
Hypothesis Testing II

Adam Poliak

04/05/2023

Announcements

Final Projects:

Originally 13 project ideations submitted

Now 12 submitted

Proposal: due this Friday

I'll be offline till Saturday night so you can submit them until Saturday night

Today's lecture:

- https://inferentialthinking.com/
- Chapter 9.4 – 14 (inclusive)

Midterm

- Allowed 1 page (double sided) cheatsheet

- List of detailed topics covered on Midterm:
  - https://docs.google.com/document/d/195oRBEfG41DBYBnSklSu8Ff3h0JidqZNCZOFyigf4j4/edit?usp=sharing

Outline

- Review
- Stat Sig
- A/B Testing (difference of means)
  - Causality
- Estimation
- Boostrap
- Confidence Intervals
- Normal Distribution

Probability vs Statistics

Probability:

- Coming up with a view of the world then seeing if the data matches

Statistics:

- Creating a view of the world by looking at data

Probability vs Empirical Distribution

"Probability Distribution":

- All the possible values of a quantity
- The probability of each of the values

"Empirical" – based on observations

"Empirical Distribution":

- All observed values
  - The proportion of times each value appears

Inference

- **Statistical Inference:**
  - Making conclusions based on data in random samples

- **Example:**

fixed

  - Use the data to guess the value of an unknown number

Depends on the random sample

  - Create an **estimate** of an unknown quantity

Parameter vs Statistic

- **Parameter**
  - Numerical quantity associated with the population
- **Statistic**
  - A number calculated from the sample
- A statistic can be used as an **estimator** of a parameter

Models

A model is a set of assumptions about the data

Generative model:
- Narrative of how the data came to be

Discriminative model
- Modeling a decision based on observed data

$P(x|y)$ vs $P(y|x)$ Zoom poll:

Approach to Assessing Models

- If we can simulate data according to the assumptions of the model, we can learn what the model predicts

- We can compare the model's predictions to the observed data

- If the data and the model's predictions are not consistent, that is evidence against the model

Steps in Assessing a Model

- Choose a statistic that will help you decide whether the data support the model or an alternative view of the world
- Simulate statistic under the assumptions of the model
- Draw a histogram of the simulated values
    - This is the model's prediction for how the statistic should come out
- Compute the statistic from the sample in the study
    - If the two are not consistent => evidence against the model
    - If the two are consistent => data supports the model ***so far***

Null and Alternative

The method only works if we can simulate data under one of the hypotheses.

- **Null hypothesis**
  - A well defined chance model about how the data were generated
  - We can simulate data under the assumptions of this model
    - "Under the null hypothesis"
- **Alternative hypothesis:**
  - A different view about the origin of the data

Prediction Under the Null Hypothesis

- Simulate the test statistic under the null hypothesis
  - Draw the histogram of simulated values
  - **The empirical distribution of the statistic under the null hypothesis**
- It is a prediction about the statistic, made by the null hypothesis
  - It shows all the likely values of the statistic
  - Also how likely they are (**if the null hypothesis is true)**
- The probabilities are approximate, because we can't generate all the possible random samples

Conclusion of the Test

Resolve choice between null and alternative hypotheses

- Compare the **observed test statistic** and its empirical distribution under the null hypothesis
- If the observed value is not **consistent** with the the empirical distribution
  - The test favors the alternative
  - “data is more consistent with the alternative”

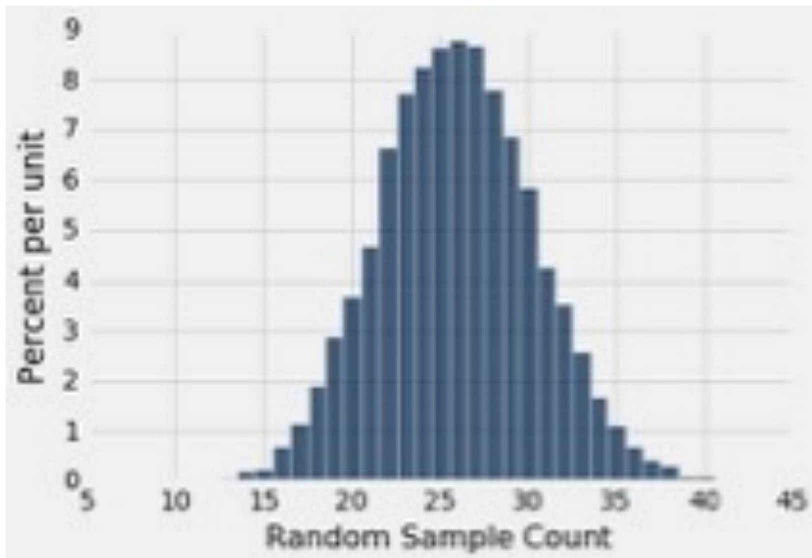
Whether a value is consistent with a distribution:

- A visualization may be sufficient
- If not, there are conventions about “consistency”

Statistical Significance

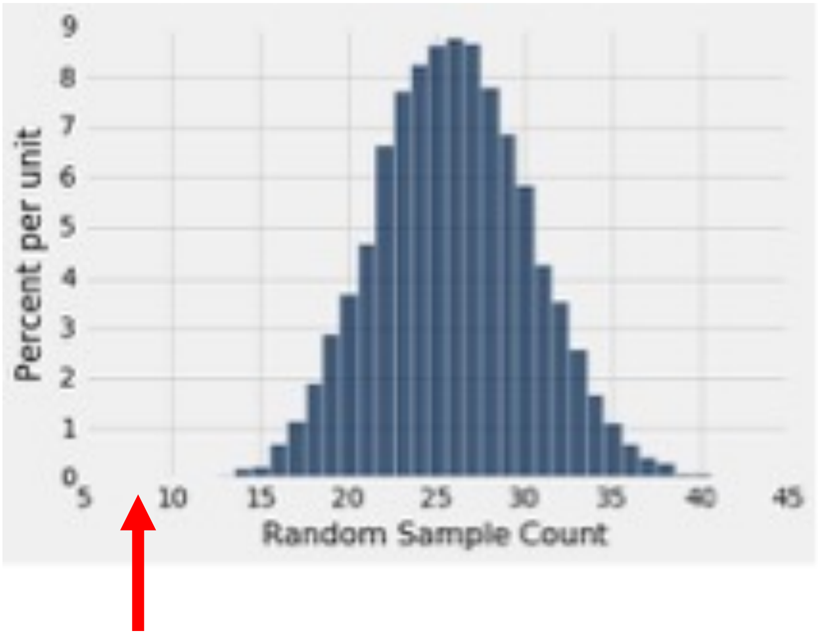
Tail Areas

Alabama Jury



Tail Areas

Alabama Jury



Observed Number (8)

Definition of the P-value

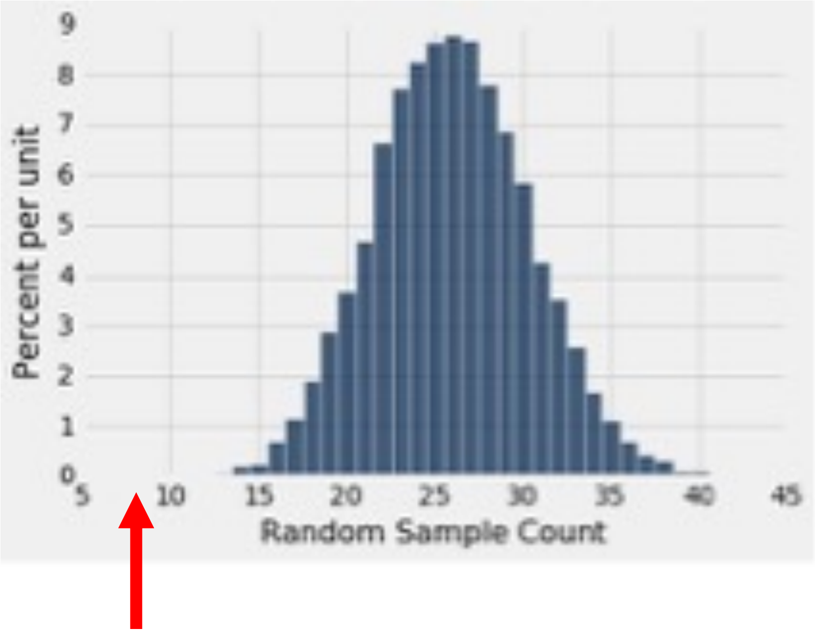
Formal name: **observed significance level**

The *P*-value is the chance,

- Under the null hypothesis,
- That the test statistic
- Is equal to the value that was observed in the data
- Or is even further in the direction of the tail

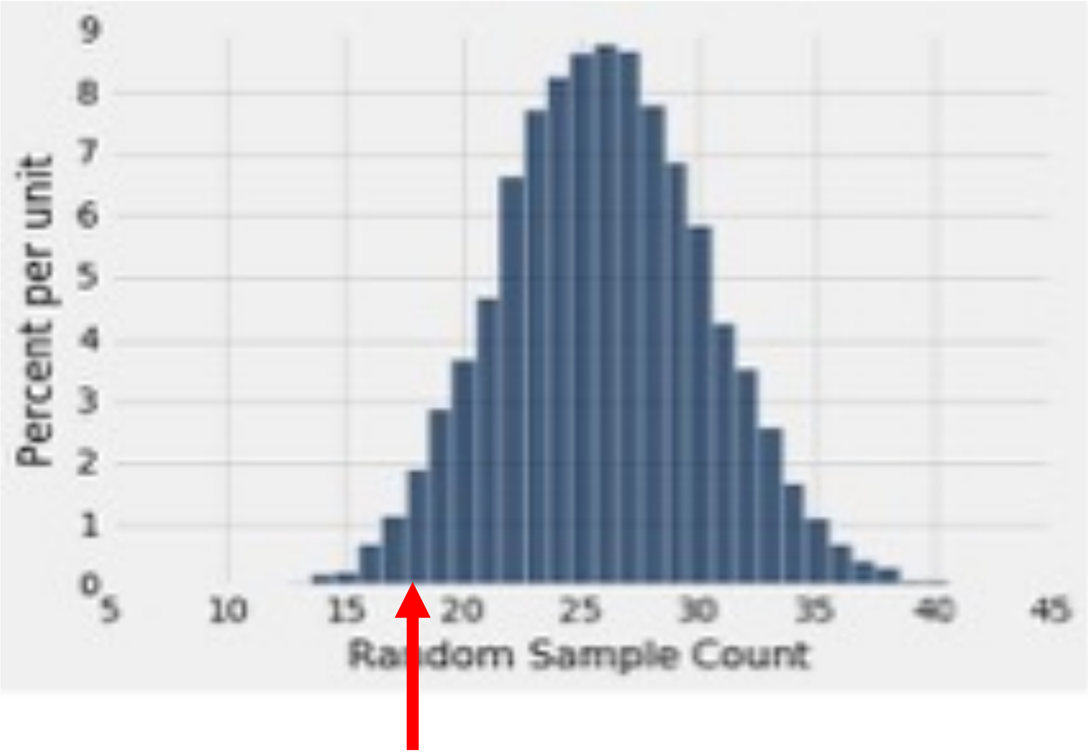
What's the p-value here

Alabama Jury



Observed Number (8)

Does this empirical distribution support the null hypothesis or not?

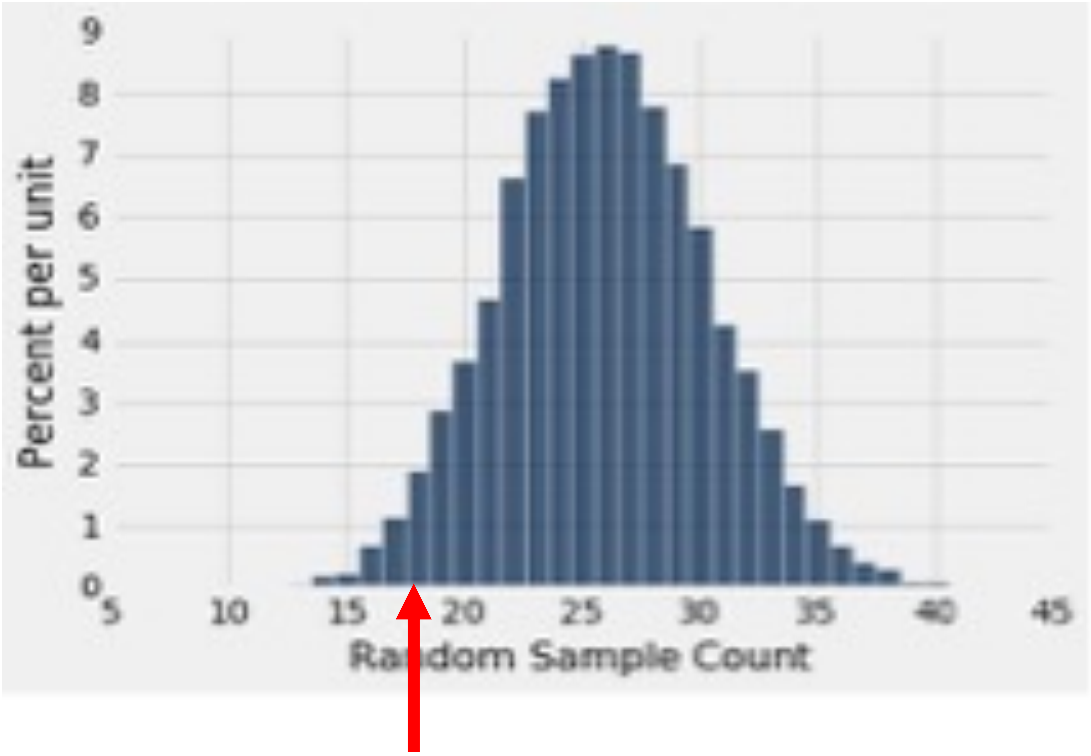


Observed Number (18)

Conventions About Inconsistency

- **"Inconsistent with the null"**: The test statistic is in the tail of the empirical distribution under the null hypothesis

Not so clear example



Conventions About Inconsistency

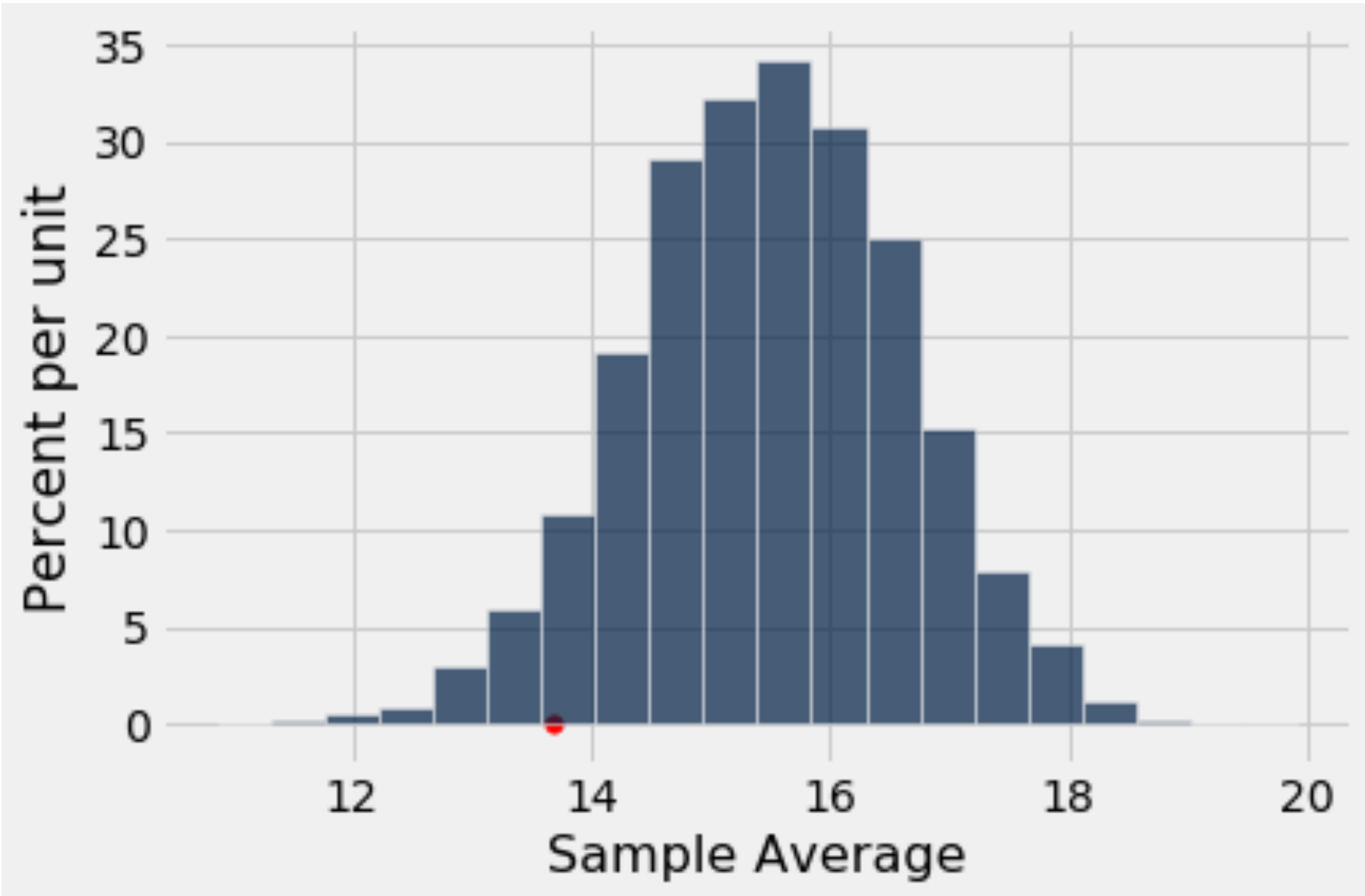
- **“Inconsistent with the null”**: The test statistic is in the tail of the empirical distribution under the null hypothesis

- **“In the tail,” first convention:**
  - The area in the tail is less than 5%
  - The result is “statistically significant”

- **”In the tail,” second convention:**
  - The area in the tail is less than 1%
  - The result is “highly statistically significant”

Histogram of simulated values & observed statistic

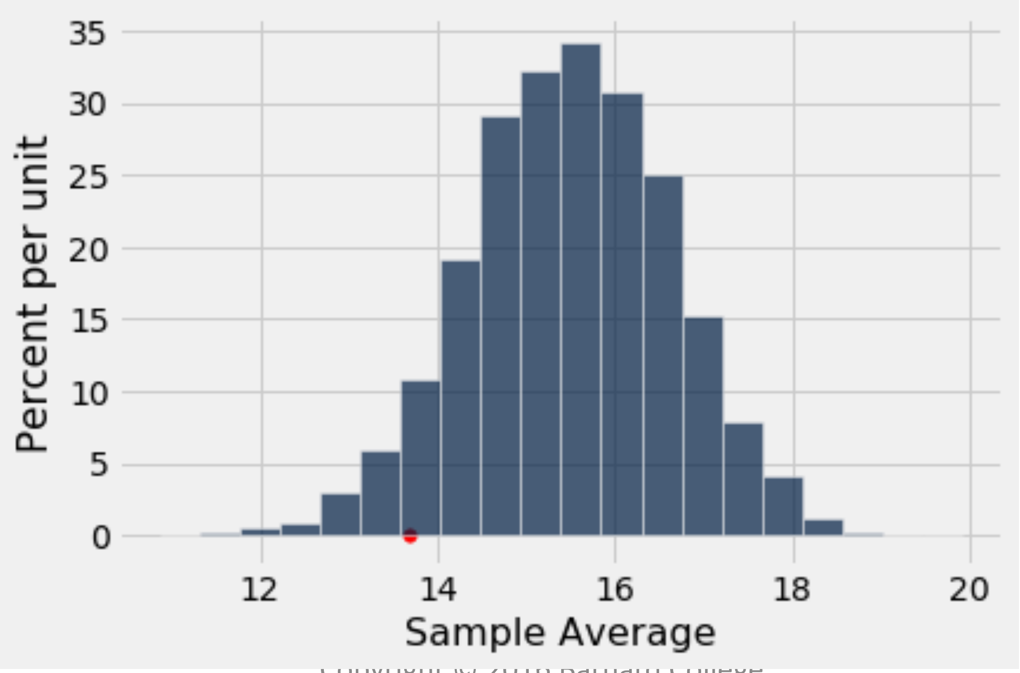
Is the observed statistic consistent with the histogram?



Compute the p-value

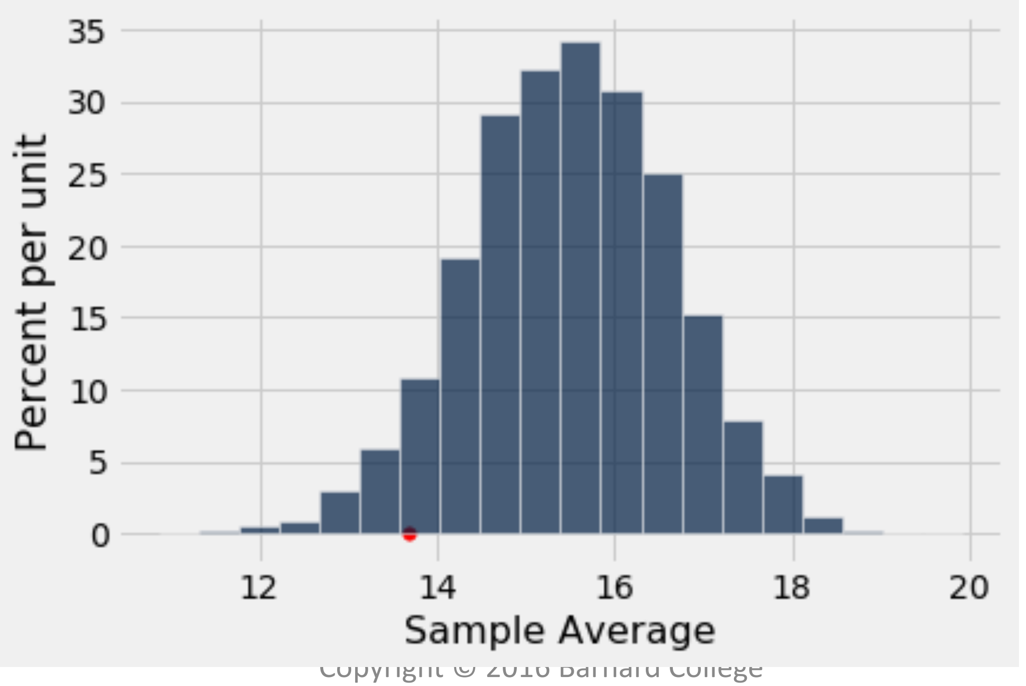
The *P*-value is the chance,

- Under the null hypothesis, that the test statistic, is equal to the value that was observed in the data, or is even further in the direction of the tail



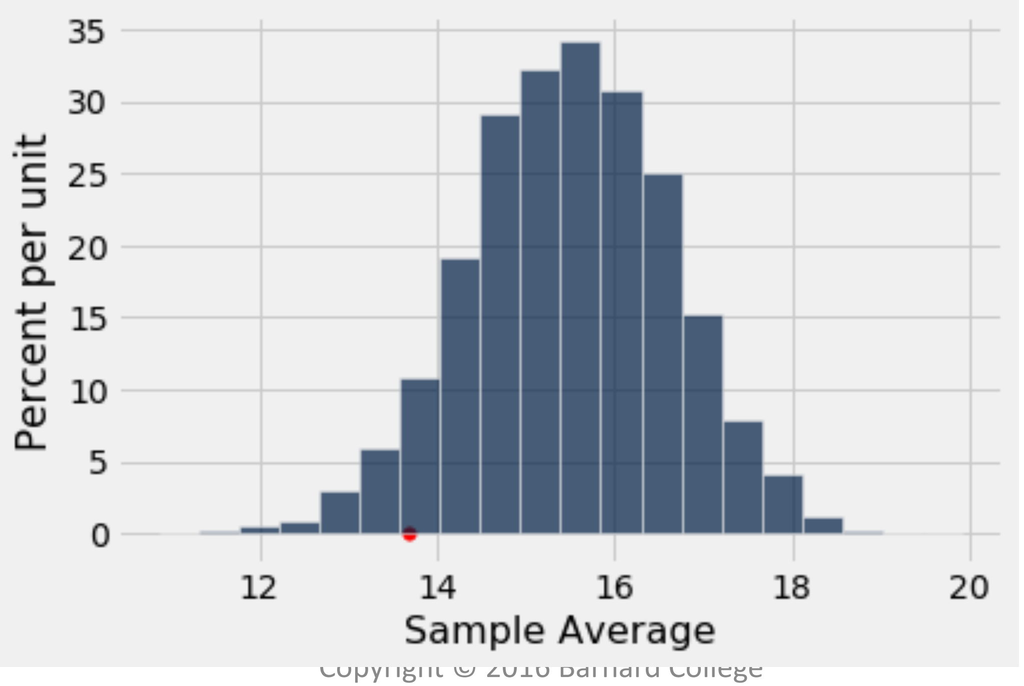
Compute the p-value

$$\text{Probability (A)} = \frac{\textit{number of outcomes that make A happen}}{\textit{total number of outcomes}}$$



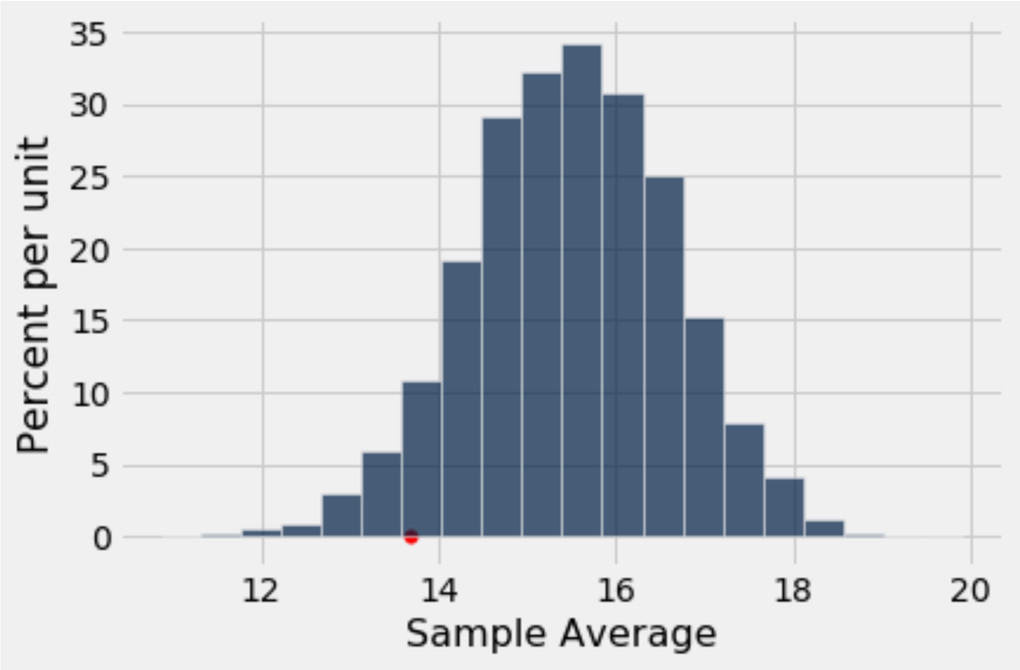
Compute the p-value

A = the sampled statistic was less than or equal to the observed statistic



Compute the p-value

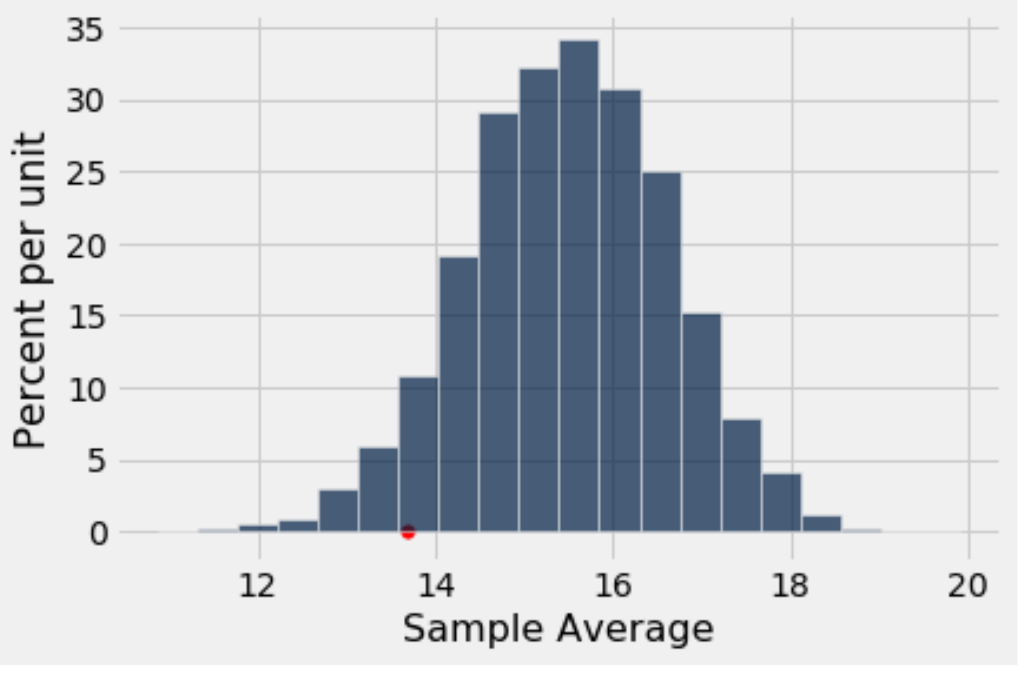
P(A) = (the number of times the sampled statistic was less than the observed statistic) divided by the number of samples



Compute the p-value

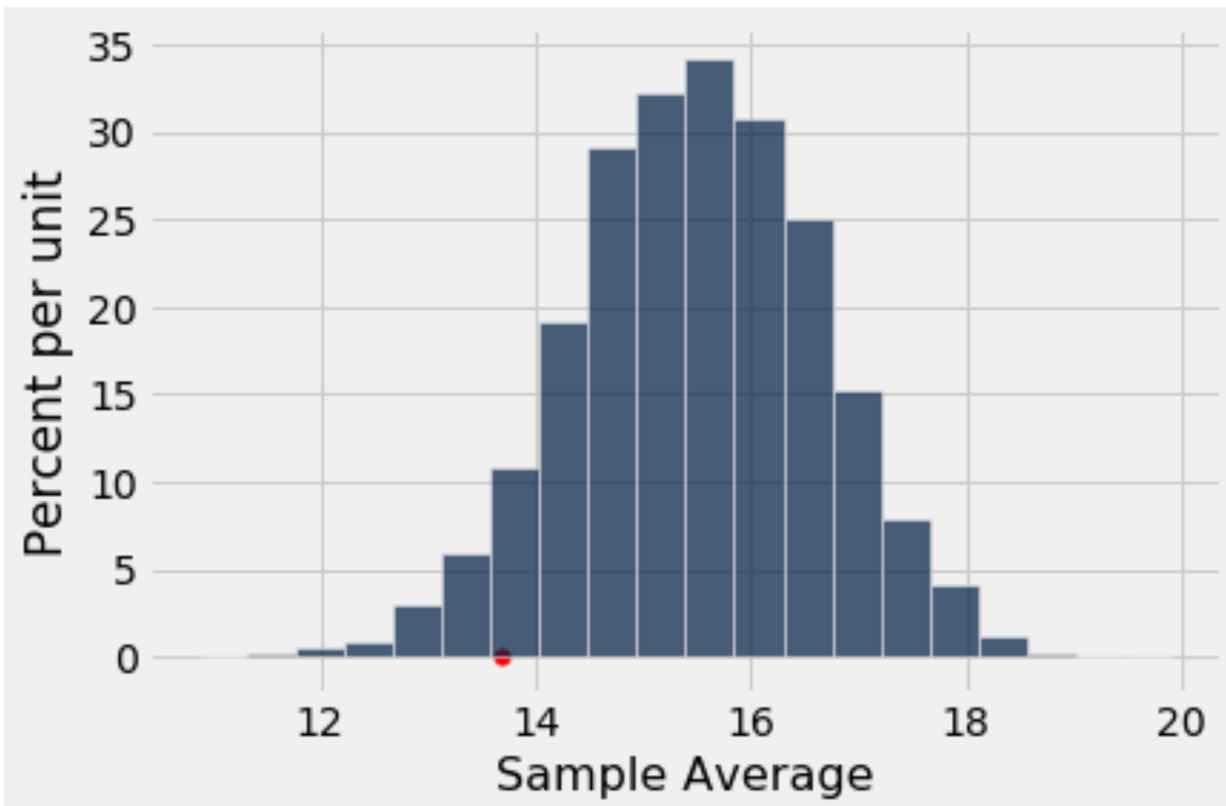
P(A) =

$$\frac{sum(sample\ averages \leq observed\ averages)}{50K}$$

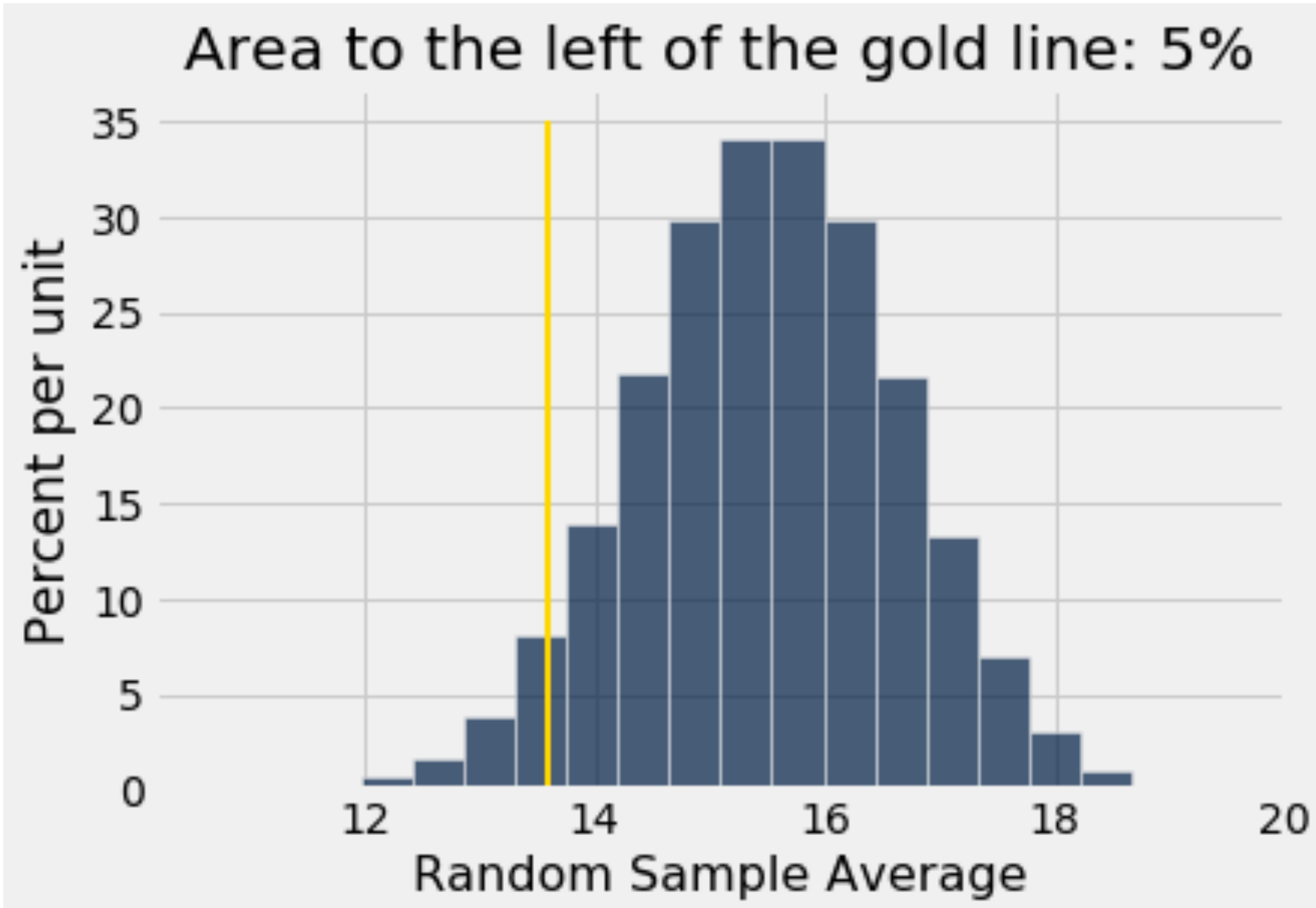


Compute the p-value

P(A) = 0.05682 ≈ 5%



Compute the p-value



Comparing Two Samples
A/B Testing

Terminology

- Compare values of sampled *individuals* in **Group A** with values of sampled *individuals* in **Group B**.
- Question: Do the two sets of values come from the same underlying distribution?
- Answering this question by performing a statistical test is called **A/B testing**.

The Groups and the Questions

- Random sample of mothers of newborns. Compare:
  A. Birth weights of babies of mothers who smoked during pregnancy
  B. Birth weights of babies of mothers who didn't smoke

- Question: Could the difference be due to chance alone?

Hypotheses

Null Hypothesis:

- In the population, the distributions of the birth weights of the babies in the two groups are the same. (They are different in the sample just due to chance.)

Alternative Hypothesis:

- In the population, the babies of the mothers who smoked weigh less, on average, than the babies of the non-smokers

Test Statistic

Group A: non-smokers

Group B: smokers

Statistic:

- Difference between average weights:
  - Group B average - Group A average

Negative values of this statistic favor the alternative

Simulating Under the Null

If the null is true, all rearrangements of labels are equally likely

Permutation Test:

- Shuffle all birth weights
- Assign some to Group A and the rest to Group B
  - Key: keep the sizes of Group A and Group B that same from before
- Find the difference between the two shuffled groups
- Repeat

Random Permutations

- Sample randomly with replacement

- With replacement:
  - Randomly choose a value from a set, then put it back into the set
  - Can result in duplicates

A-B Testing for CTA

Difference in stress before vs during COVID

Observed Statistic:

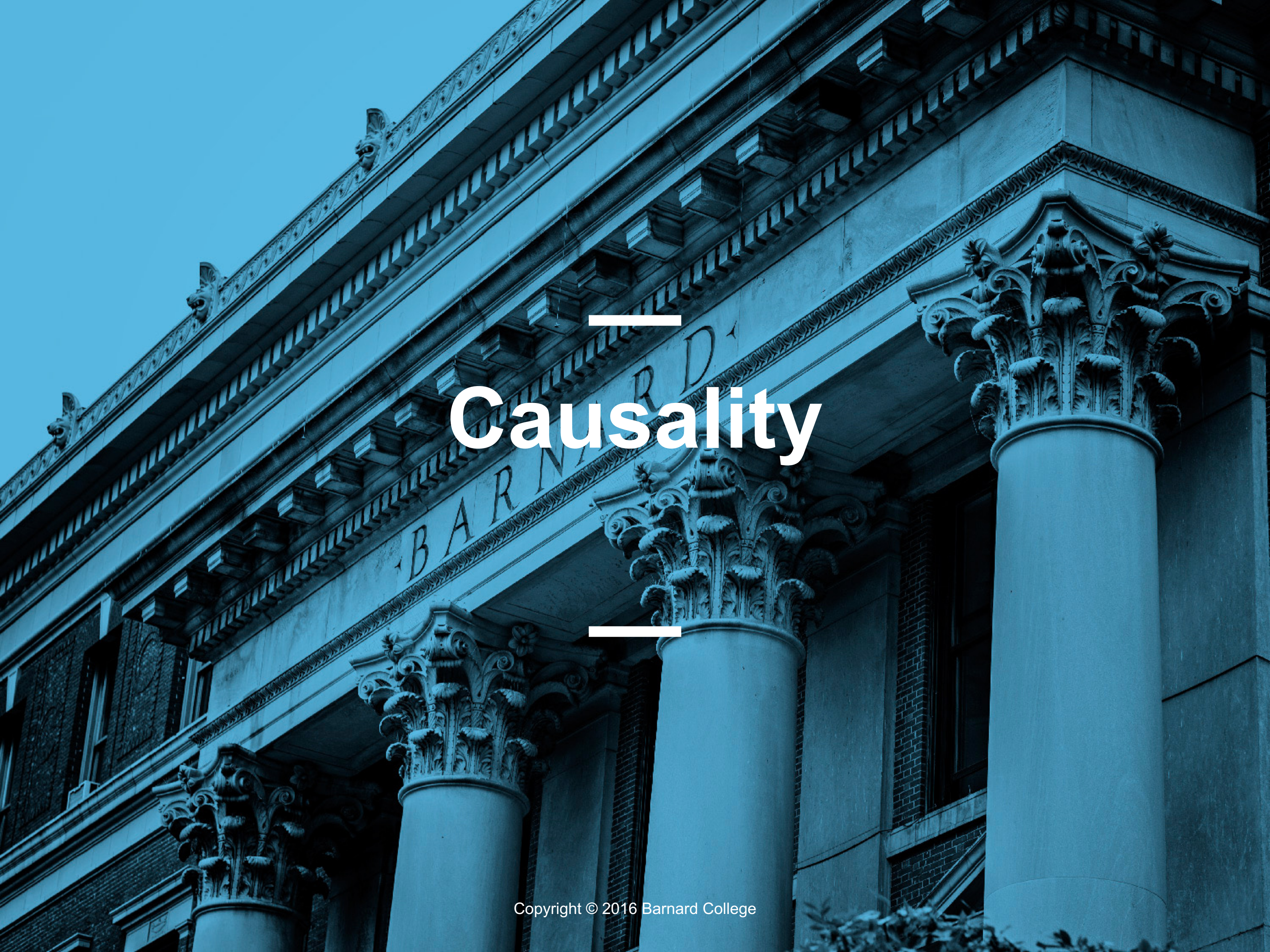
- Difference in avg LIWC score in *n* posts before COVID vs *m* posts during from a similar subreddit

Empirical distribution:

- Randomly assign *n* posts to before and *m* posts to during
- Compute difference between the two new groups

P-value

- Percent of simulated statistic that was like, or more extreme than observed statistic



Causality

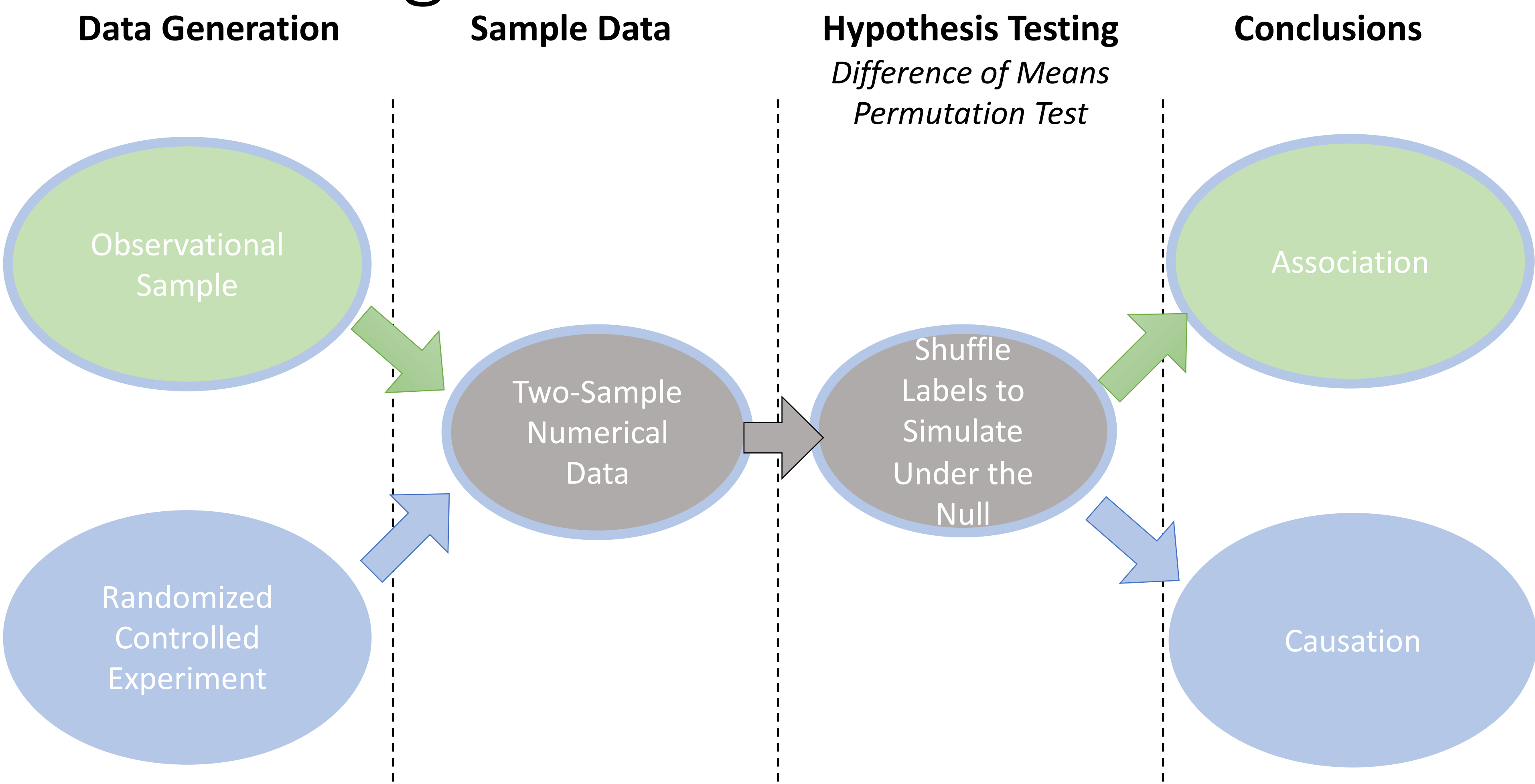
Randomized Controlled Experiment

- Sample A: **control group**
- Sample B: **treatment group**

- **if the treatment and control groups are selected at random, then you can make causal conclusions.**

- Any difference in outcomes between the two groups could be due to
  - chance
  - the treatment

Randomized Assignment & Shuffling



Estimation

Inference: Estimation

- How do we calculate the value of an unknown parameter?
- If you have a census (that is, the whole population):
  - Just calculate the parameter and you’re done
- If you don’t have a census:
  - Take a random sample from the population
  - Use a statistic as an **estimate** of the parameter

Estimation Variability

Variability of the Estimate

- One sample → One estimate
- But the random sample could have come out differently
- And so the estimate could have been different
- Big question:
  - How different would it be if we estimated again?

Quantifying Uncertainty

- The estimate is usually not exactly right.
- Variability of the estimate tells us something about how accurate the estimate is:

Estimate = Parameter + Error

- How accurate is the estimate, usually?
- How big is a typical error?
- When we have a census, we can do this by simulation

Where to Get Another Sample?

- We want to understand errors of our estimate
- Given the **population**, we could simulate
  - ...but we only have the **sample**!
- To get many values of the estimate, we needed many random samples
- Can't go back and sample again from the population:
  - No time, no money
- Stuck?

The Bootstrap

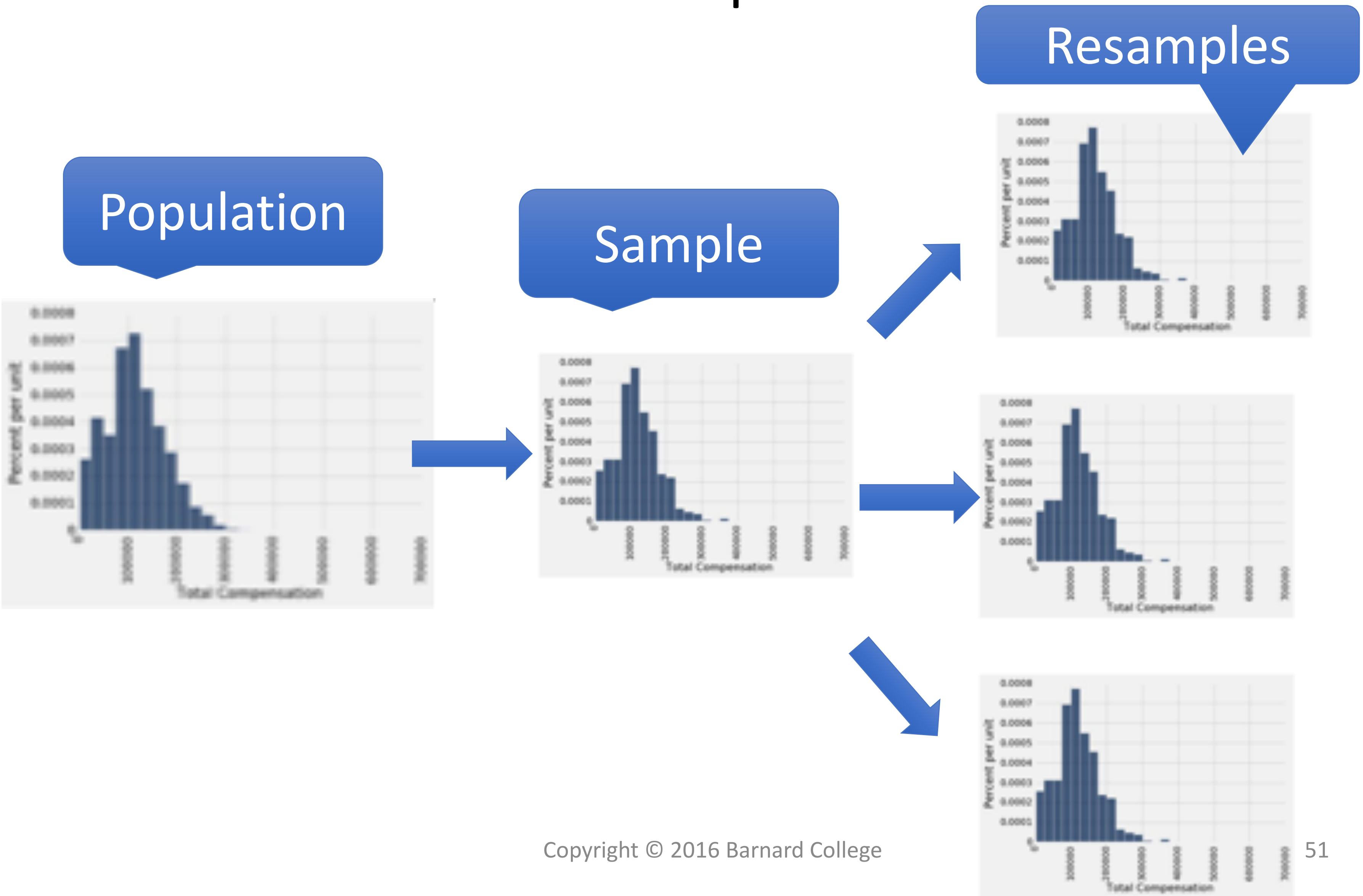
The Bootstrap

- A technique for simulating repeated random sampling

- All that we have is the original sample
  - ... which is large and random
  - Therefore, it probably resembles the population

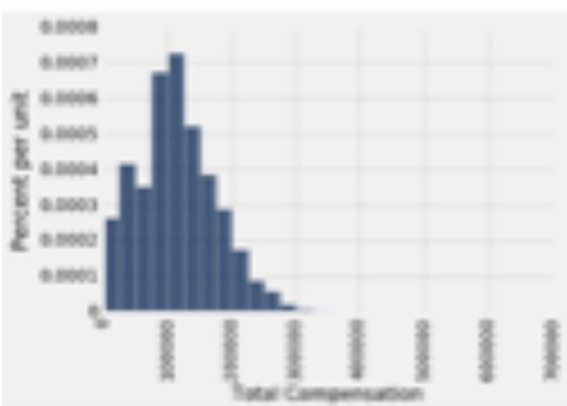
- So we sample at random from the original sample!

How the Bootstrap works



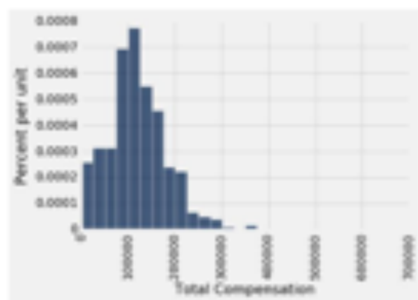
Why the Bootstrap works

Population



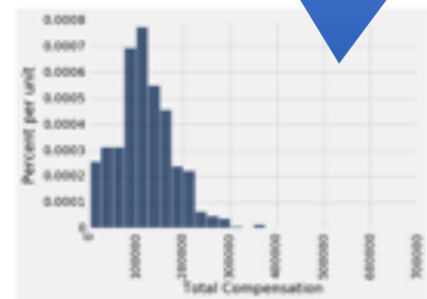
What we wish we could get

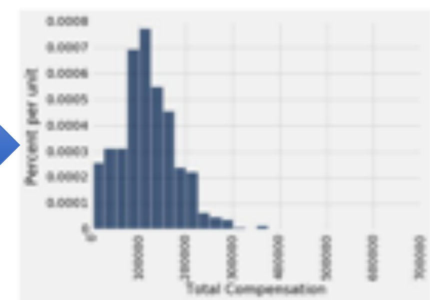
Sample

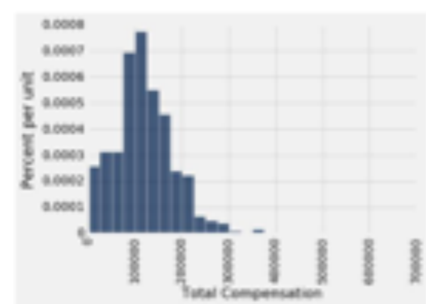


What we actually can get

Resamples







Real World vs Bootstrap World

Real World

- True probability distribution (population)
  - Random sample 1
    - Estimate 1
  - Random sample 2
    - Estimate 2
  - …
  - Random sample 1000
    - Estimate 1000

Bootstrap World

- Empirical distribution of original sample ("population")
  - Bootstrap sample 1
    - Estimate 1
  - Bootstrap sample 2
    - Estimate 2
  - …
  - Bootstrap sample 1000
    - Estimate 1000

Hope: these two scenarios are analogous

The Bootstrap Principle

- The bootstrap principle:
  - **Bootstrap-world** sampling ≈ **Real-world** sampling

- Not always true!
  - ... but reasonable if sample is large enough

- We hope that:
  a) Variability of bootstrap estimate
  b) Distribution of bootstrap errors

  ...are similar to what they are in the real world

Key to Resampling

- From the original sample,
    - draw at random
    - with replacement
    - as many values as the original sample contained

- The size of the new sample has to be the same as the original one, so that the two estimates are comparable

Variability

Our results might be different based on the original sample

How can we quantify this variability?

Confidence Intervals

95% Confidence Interval

- Interval of **estimates of a parameter**
- Based on random sampling
- 95% is called the confidence level
  - Could be any percent between 0 and 100
  - Higher level means wider intervals

- The **confidence is in the process** that gives the interval:
  - It generates a “good” interval about 95% of the time

Use Methods Appropriately

Can You Use a CI Like This?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- About 95% of the mothers in the population were between 26.9 years and 27.6 years old.

Answer:

- **False.** We're estimating that their **average age** is in this interval.

Is This What a CI Means?

An approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

There is a 0.95 probability that the average age of mothers in the population is in the range 26.9 to 27.6 years.

Answer:

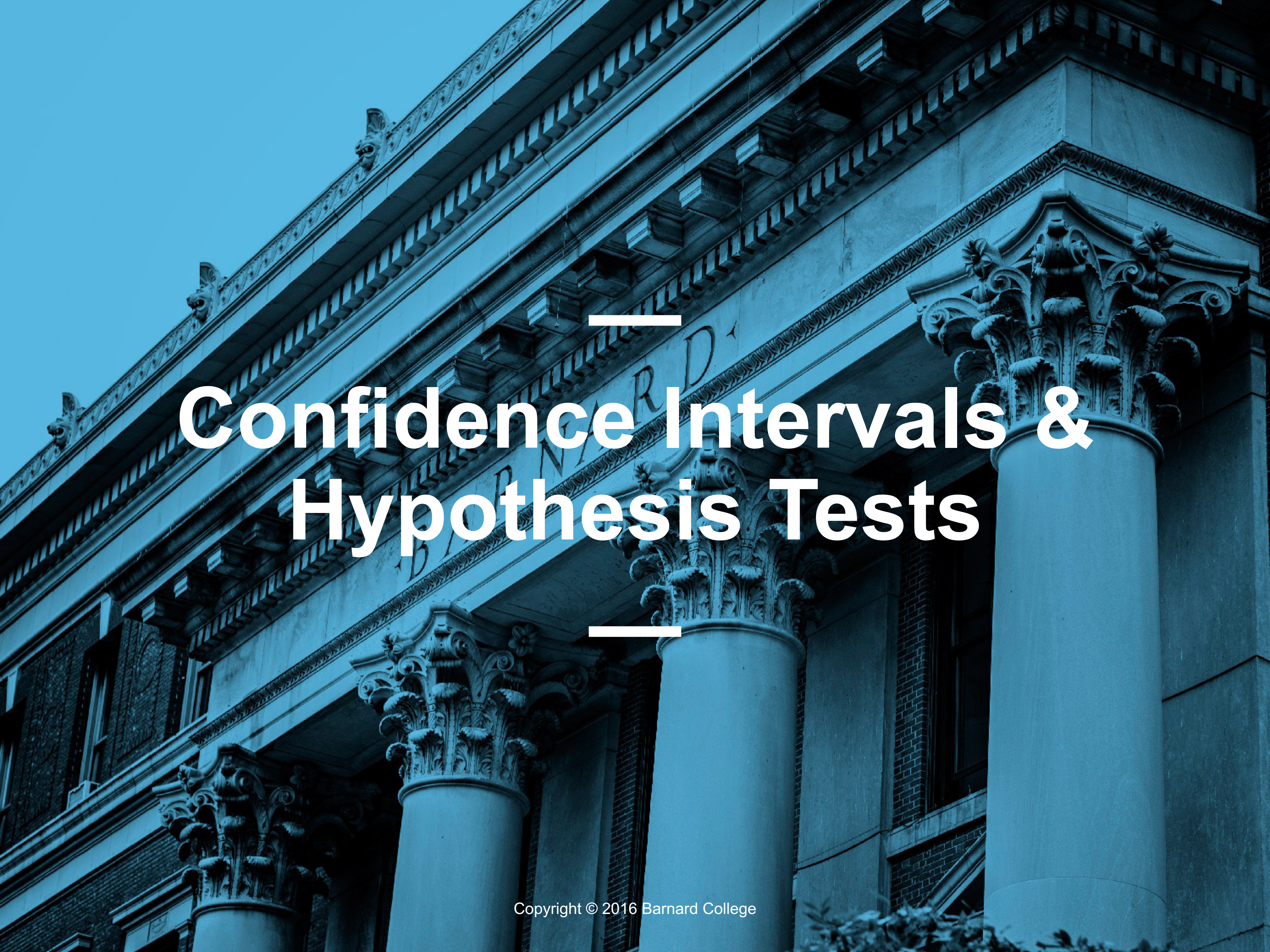
False. The average age of the mothers in the population is unknown but it's a constant. It's not random. No chances involved

When *NOT* to use the Bootstrap

- if you're trying to estimate very high or very low percentiles, or min and max
- If you're trying to estimate any parameter that's greatly affected by rare elements of the population
- If the probability distribution of your statistic is not roughly bell shaped (the shape of the empirical distribution will be a clue)
- If the original sample is very small

Using a CI for Testing

- Null hypothesis: **Population average = *x***
- Alternative hypothesis: **Population average ≠ *x***
- Cutoff for P-value: *p*%
- Method:
  - Construct a (100-*p*)% confidence interval for the population average
  - If *x* is not in the interval, reject the null
  - If *x* is in the interval, can't reject the null



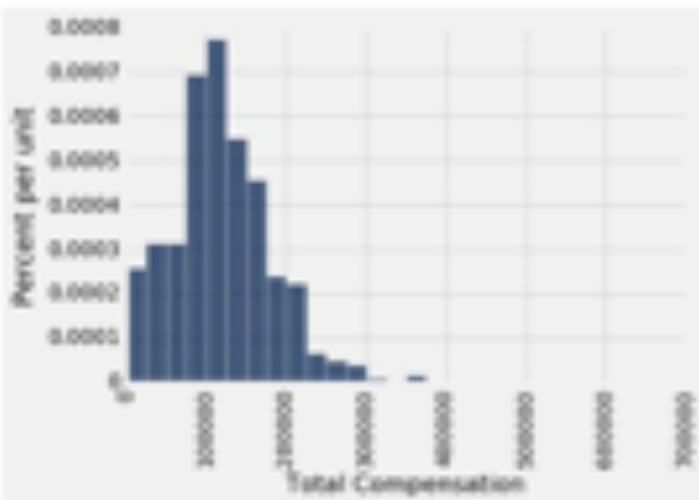
Confidence Intervals & Hypothesis Tests

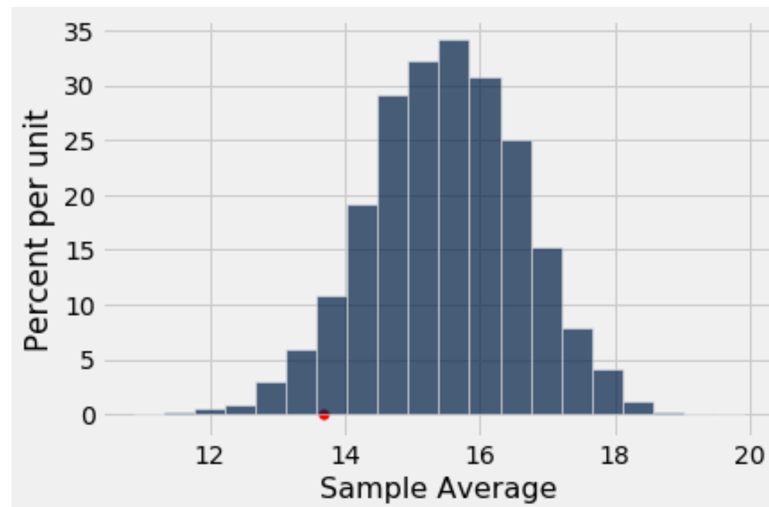
Using a CI for Testing

- Null hypothesis: **Population average = *x***
- Alternative hypothesis: **Population average ≠ *x***
- Cutoff for P-value: *p*%
- Method:
    - Construct a (100-*p*)% confidence interval for the population average
    - If *x* is not in the interval, reject the null
    - If *x* is in the interval, can't reject the null

Empirical Distribution

When we simulate the statistic under the null hypothesis, we often see a distribution like:





Why?

Center Limit Theorem

Center & Spread

Questions/Goals

- How can we quantify natural concepts like “center” and “variability”?

- Why do many of the empirical distributions that we generate come out bell shaped?

- How is sample size related to the accuracy of an estimate?

Average and the Histogram

The average (mean)

Data: 2, 3, 3, 9

Average = (2+3+3+9)/4 = 4.25

- Need not be a value in the collection
- Need not be an integer even if the data are integers
- Somewhere between min and max, but not necessarily halfway in between
- Same units as the data
- Smoothing operator: collect all the contributions in one big pot, then split evenly

Relation to the histogram

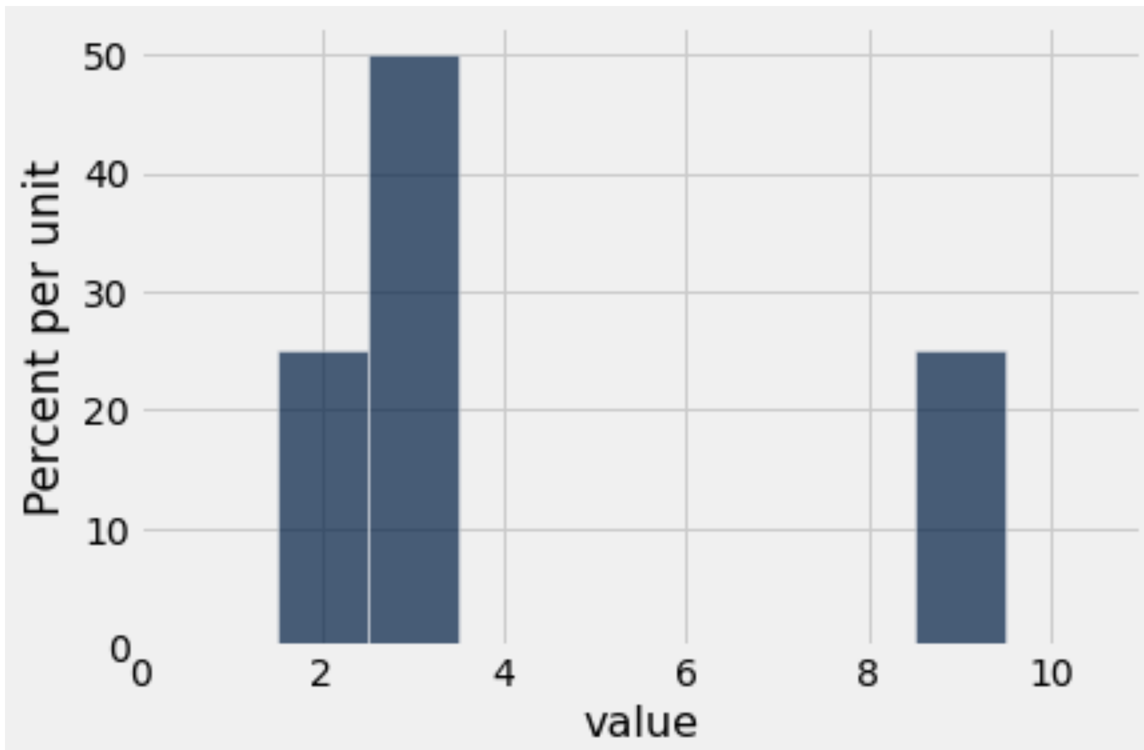
- The average depends only on the **proportions** in which the distinct values appears

- The average is the **center of gravity** of the histogram

- It is the point on the horizontal axis where the histogram balances

Average as balance point

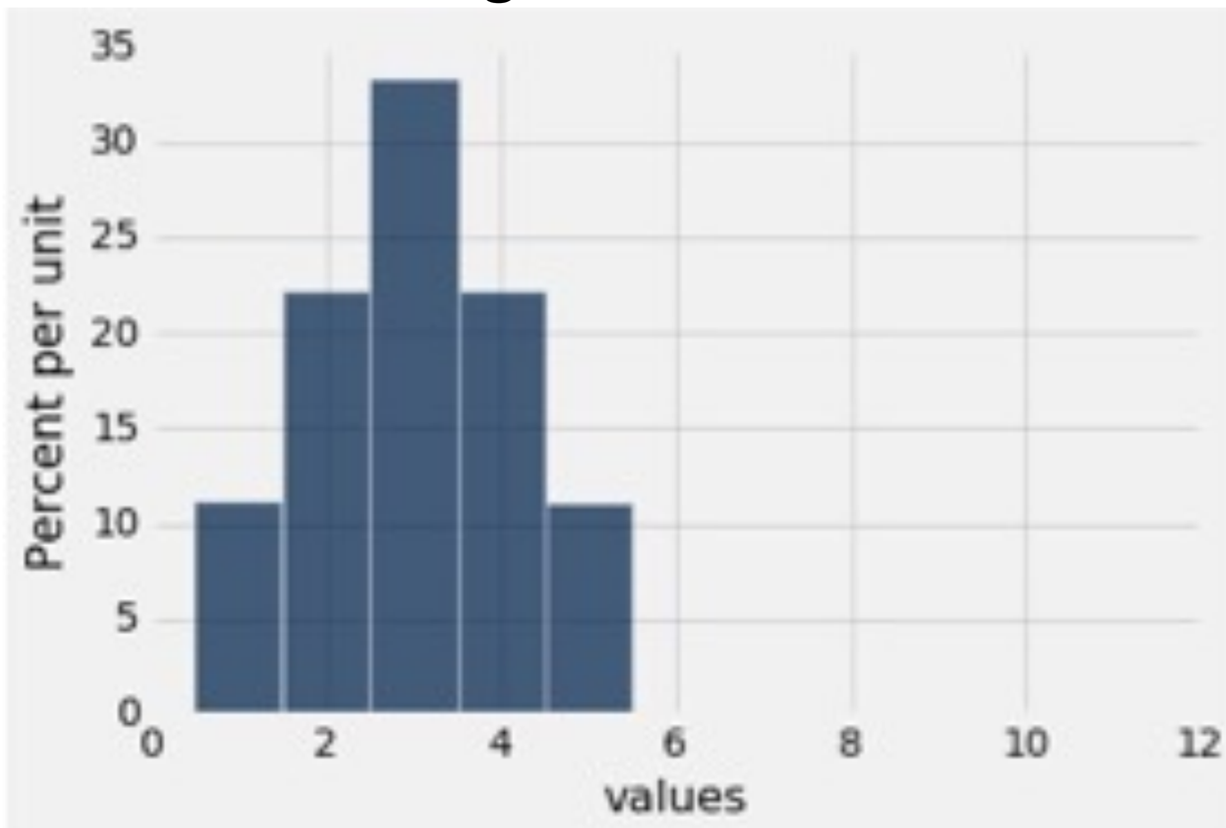
- Average is 4.25



Average and Median

Question

- What list produces this histogram?



Question

- What list produces this histogram?

1, 2, 2, 3, 3

3, 4, 4, 5

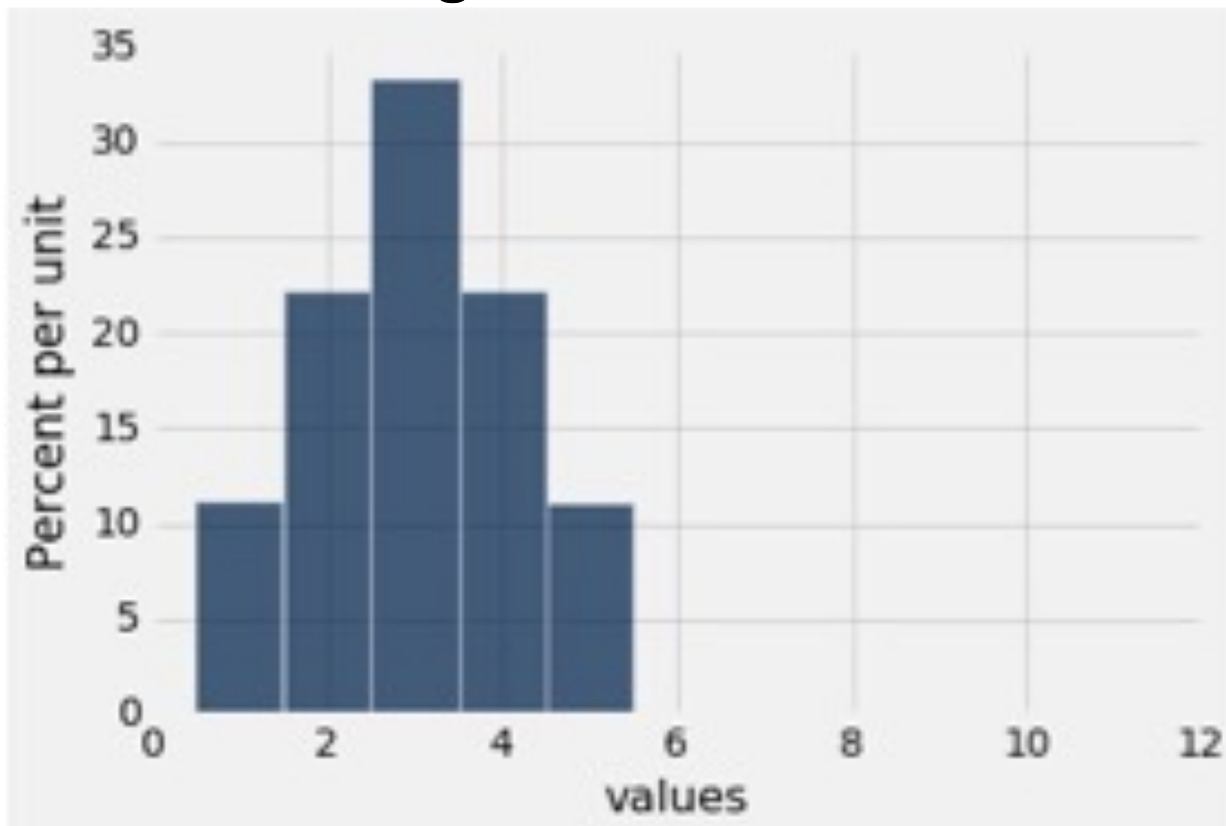
Question

- What list produces this histogram?

1, 2, 2, 3, 3

3, 4, 4, 5

- Average?

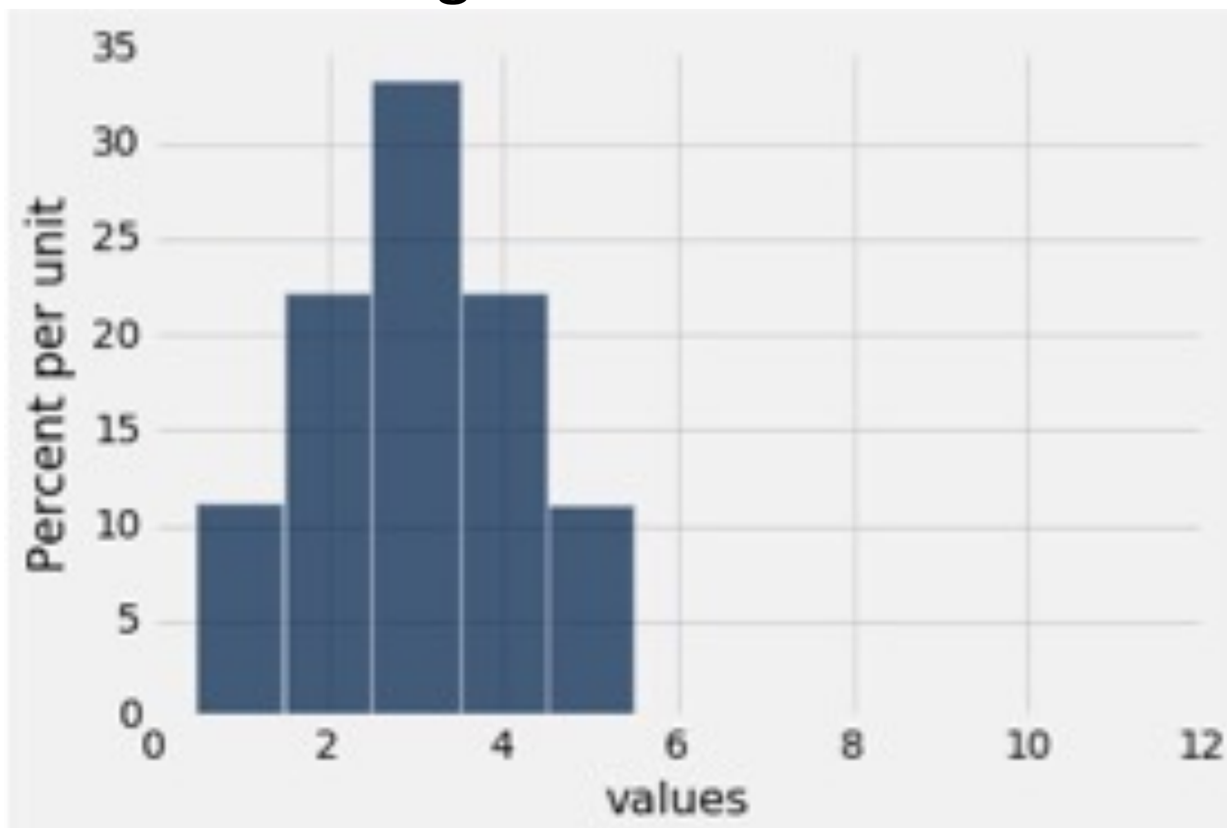


Question

- What list produces this histogram?

1, 2, 2, 3, 3

3, 4, 4, 5



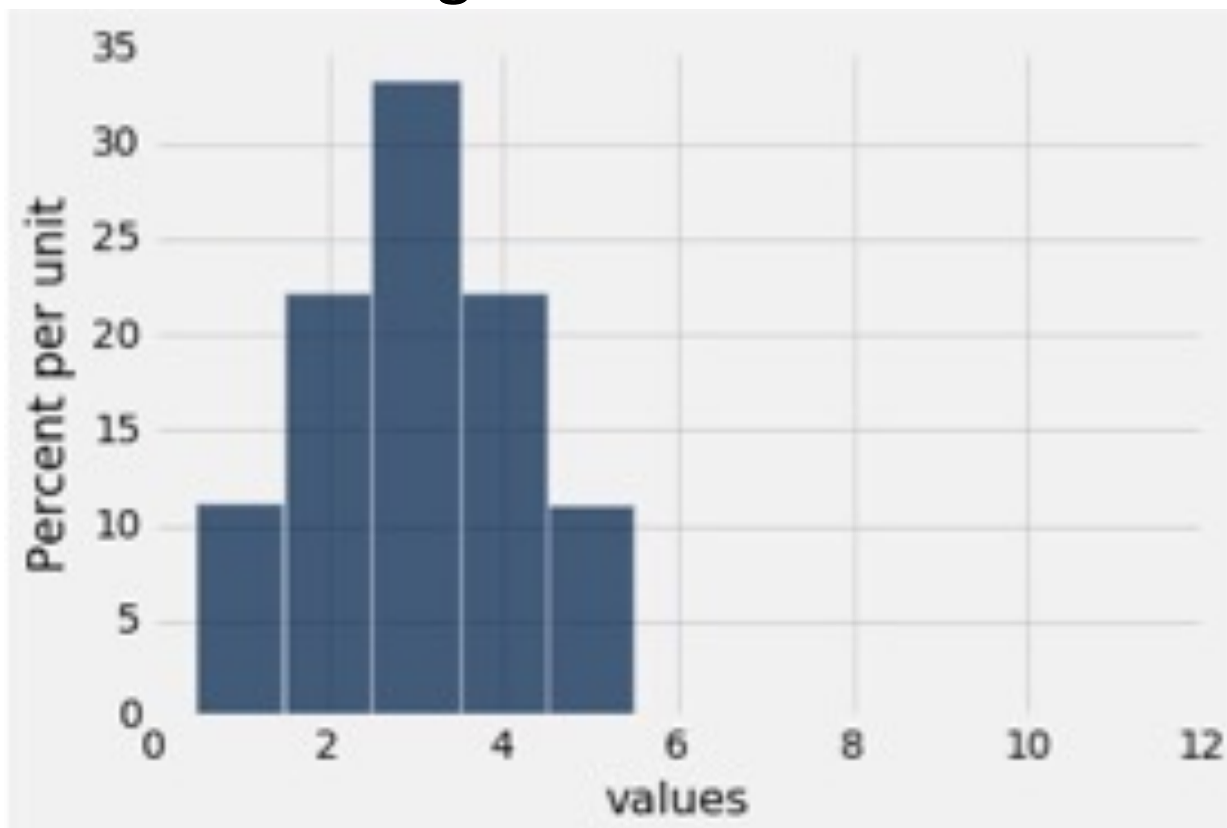
- Average?
  - 3

Question

- What list produces this histogram?

1, 2, 2, 3, 3

3, 4, 4, 5



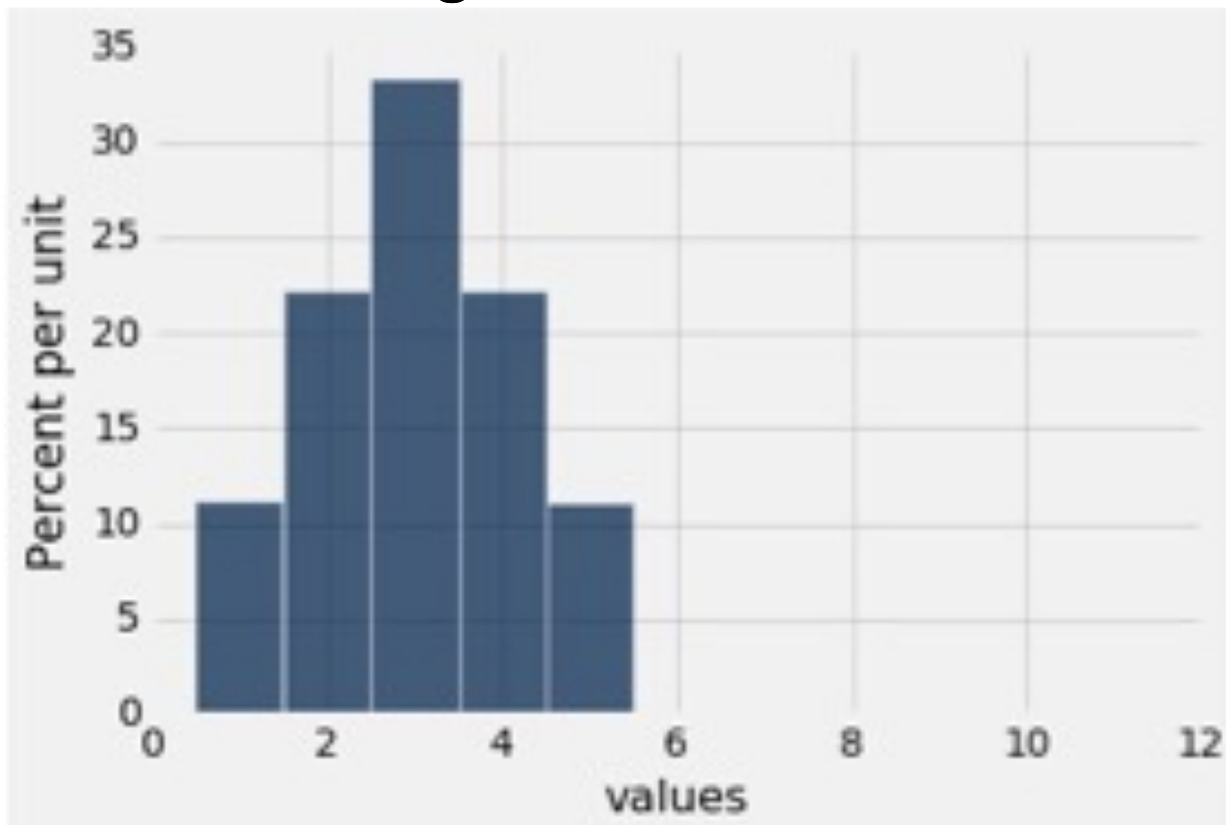
- Average?
  - 3
- Median?

Question

- What list produces this histogram?

1, 2, 2, 3, 3

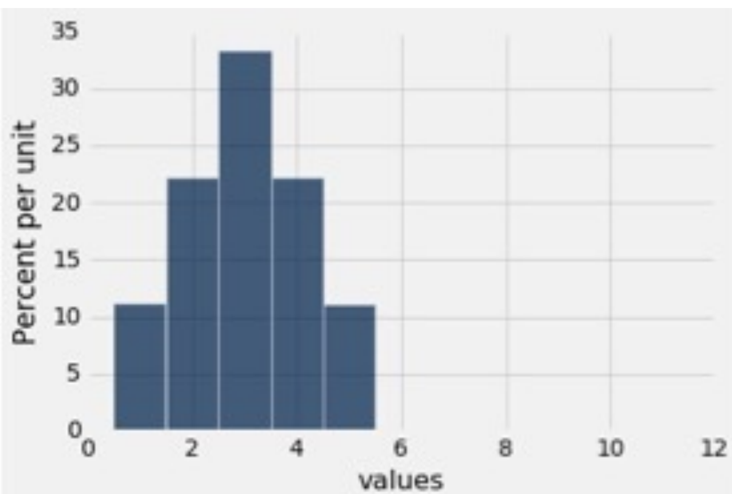
3, 4, 4, 5

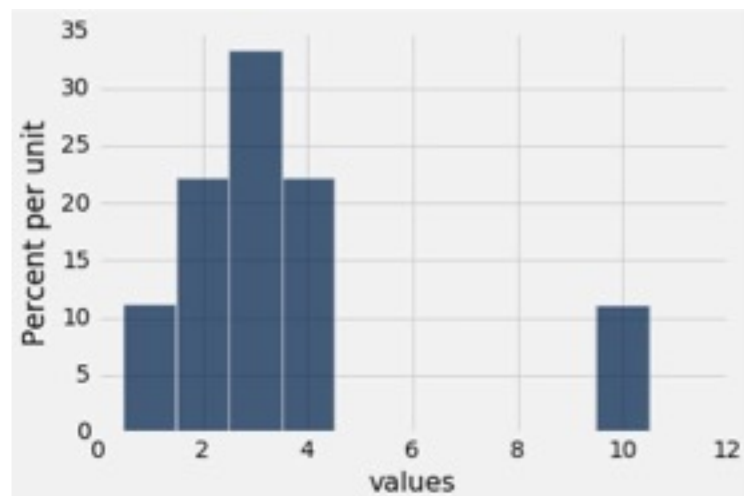


- Average?
  - 3
- Median?
  - 3

Question 2

- Are the medians of these two distributions the same or different? Are the means the same or different? If you say “different,” then say which one is bigger





Answer 2

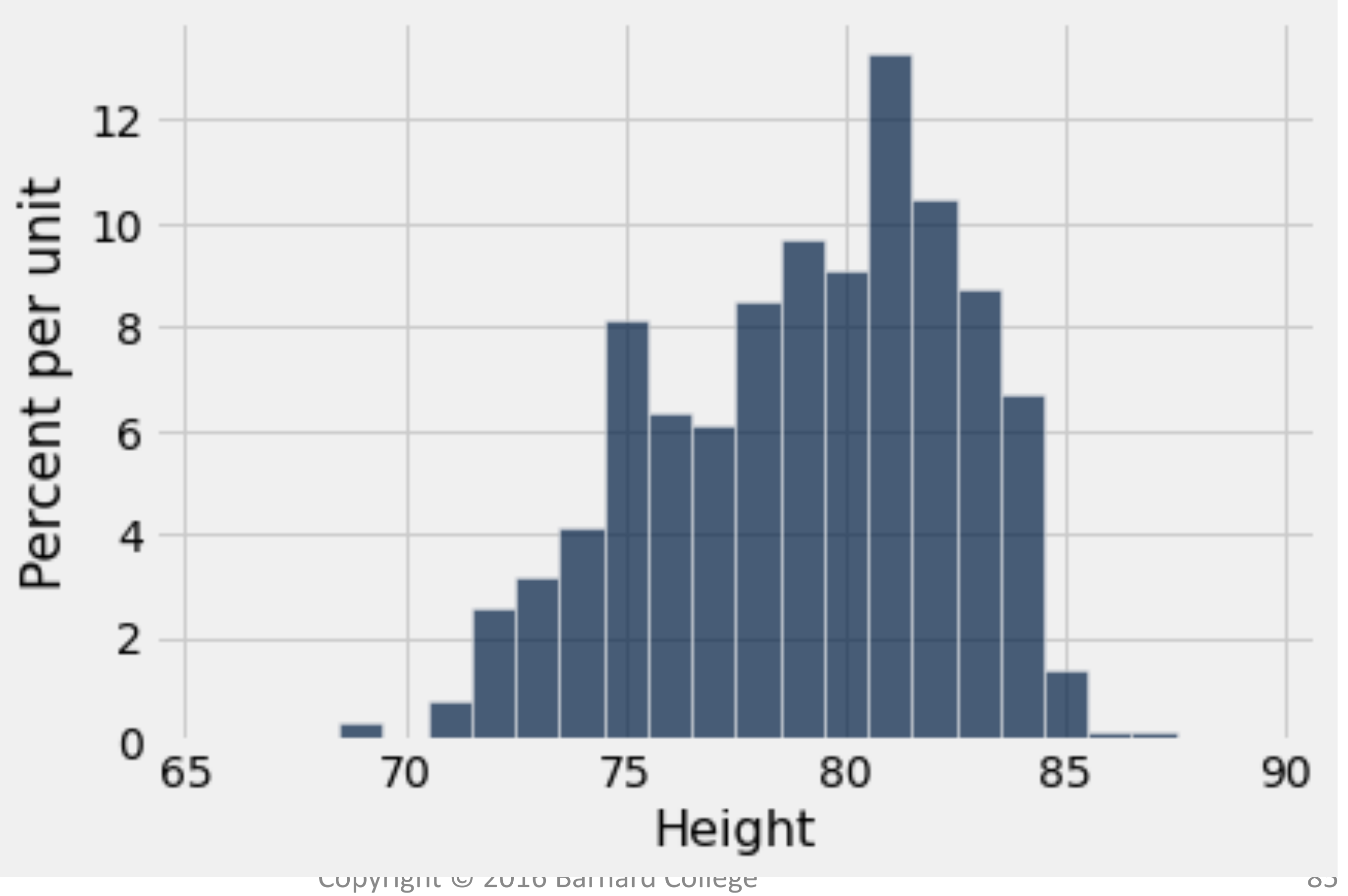
- List 1
  - 1, 2, 2, 3, 3, 3, 4, 4, 5
- List 2
  - 1, 2, 2, 3, 3, 3, 4, 4, 10
- Medians = 3
- Mean(List1) = 3
- Mean (List 2) = 3.55556

Comparing Mean and Median

- **Mean:** Balance point of the histogram
- **Median:** Half-way point of data; half the area of histogram is on either side of median
- If the distribution is symmetric about a value, then that value is both the average and the median.
- If the histogram is skewed, then the mean is pulled away from the median in the direction of the tail.

Question

- Which is bigger, median or mean?



Standard Deviation

Defining Variability

- **Plan A:** “biggest value - smallest value”
  - Doesn’t tell us much about the shape of the distribution

- **Plan B**:
  - Measure variability around the mean
  - Need to figure out a way to quantify this

How far from the average?

- Standard deviation (SD) measures roughly how far the data are from their average

- SD = root mean square of deviations from average

Steps: 5 4 3 2 1

- SD has the same units as the data

Why use Standard Deviation

- There are two main reasons.

- **The first reason:**
  - No matter what the shape of the distribution, the bulk of the data are in the range "average plus or minus a few SDs"

- **The second reason:**
  - Relation with the bellshaped curve
  - Discuss this later in the lecture

Chebyshev’s Inequality

How big are most values?

No matter what the shape of the distribution,
the bulk of the data are in the range "average ± a few SDs"

Chebyshev's Inequality

No matter what the shape of the distribution,
the proportion of values in the range "average ± *z* SDs" is

at least 1 - 1/*z*2

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/*z*2

Range	Proportion

Chebyshev's Bounds

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Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/z2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/z2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)

Chebyshev's Bounds

the proportion of values in the range "average ± *z* SDs" is at least 1 - 1/z2

Range	Proportion
average ± 2 SDs	at least 1 - 1/4 (75%)
average ± 3 SDs	at least 1 - 1/9 (88.888...%)
average ± 4 SDs	at least 1 - 1/16 (93.75%)
average ± 5 SDs	at least 1 - 1/25 (96%)

True no matter what the distribution looks like

Understanding HW Results

Statistics:

Minimum: 7.5

Maximum: 29.0

Mean: 24.55

Median: 25.0

Standard Deviation: 3.96

- At least 50% of the class had scores between 20.59 and 28.51

- At least 75% of the class had scores between 16.62 and 32.47

Standard Units

Standard Units

- How many SDs above average?
- ***z* = (value - average)/SD**
  - Negative z: value below average
  - Positive z: value above average
  - z = 0: value equal to average
- When values are in standard units:
  average = 0, SD = 1
- Chebyshev: At least 96% of the values of *z* are between -5 and 5

Question

What whole numbers are closest to

(1) Average age

(2) The SD of ages

Age in Years	Age in Standard Units
27	-0.0392546
33	0.992496
28	0.132704
23	-0.727088
25	-0.383171
33	0.992496
23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

Answers

(1) Average age is close to 27 (standard unit here is close to 0)

(2) The SD is about 6 years (standard unit at 33 is close to 1. 33 – 27 = 6)

Age in Years	Age in Standard Units
27	-0.0392546
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23	-0.727088
25	-0.383171
30	0.476621
27	-0.0392546

The SD and the Histogram

- Usually, it's not easy to estimate the SD by looking at a histogram.

- But if the histogram has a bell shape, then you can

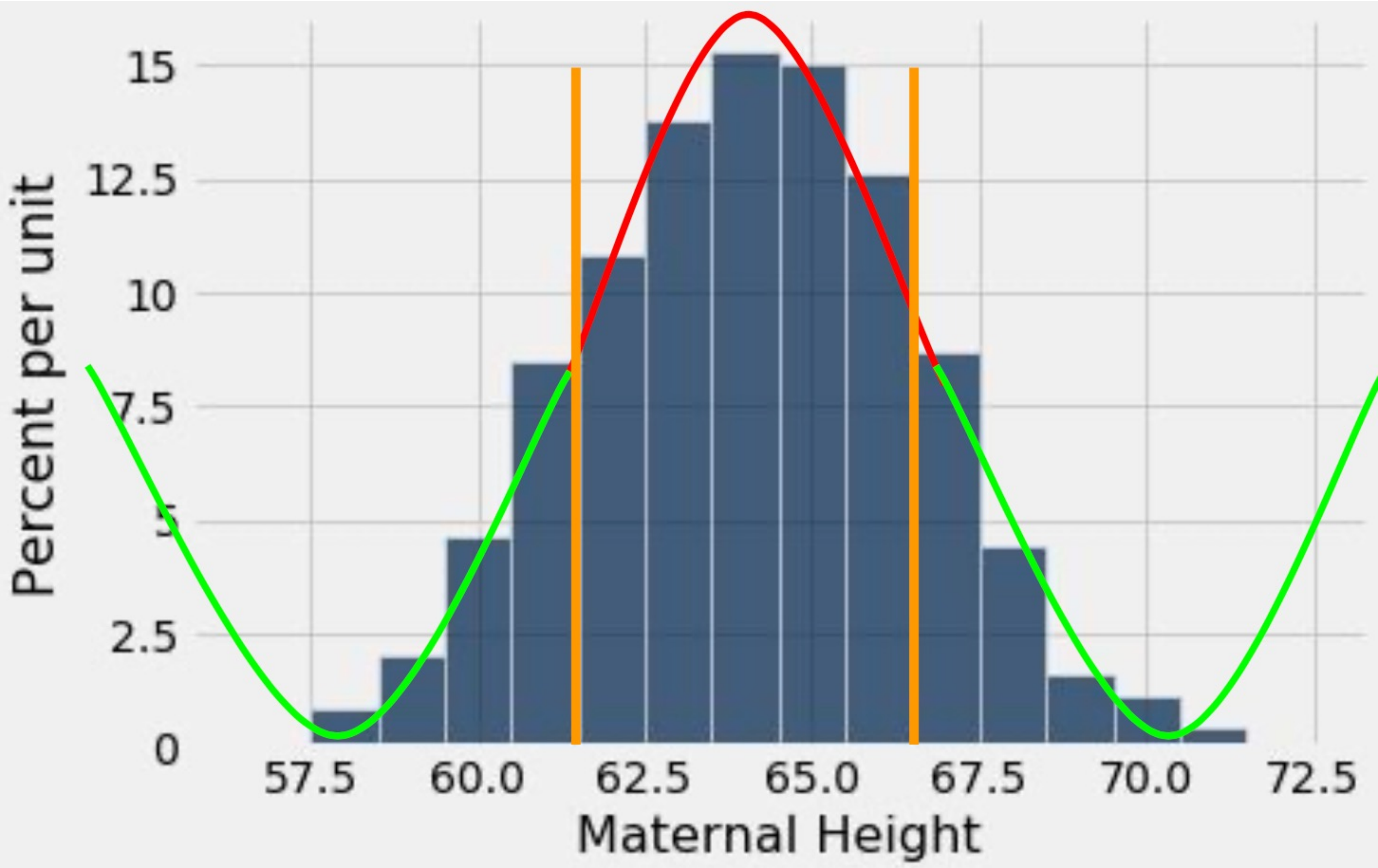
The SD and Bell Shaped Curves

If a histogram is bell-shaped, then

- the average is at the center

- the SD is the distance between the average and the points of inflection on either side

Points of Inflection



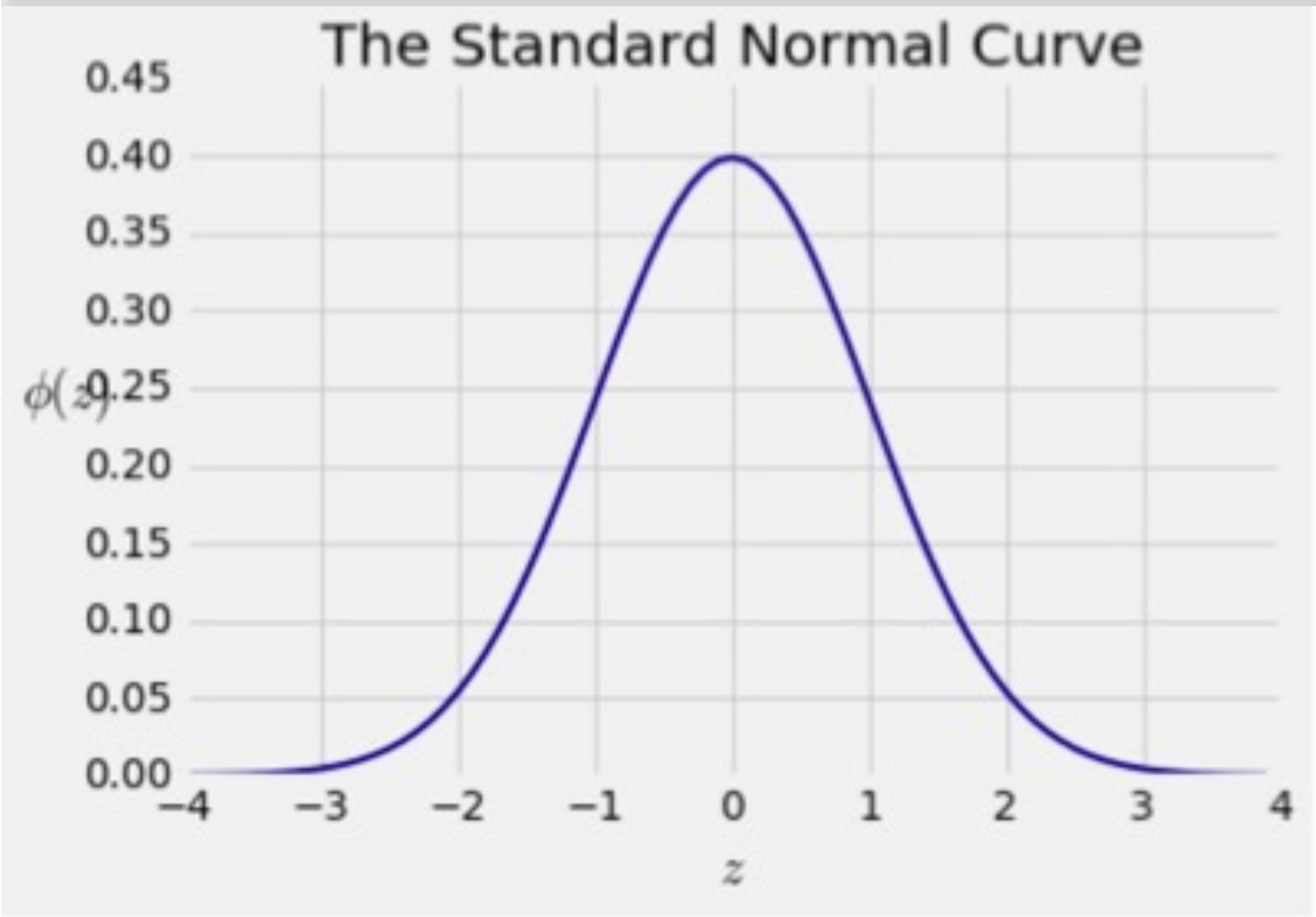
Normal Distribution

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Equation for the normal curve

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}, \qquad -\infty < z < \infty$$

Bell Curve



The Standard Normal Curve
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$\phi(z)$
−4
−3
−2
−1
0
1
2
3
4
z

How Big are Most of the Values

No matter what the shape of the distribution,

the bulk of the data are in the range “average ± a few SDs”

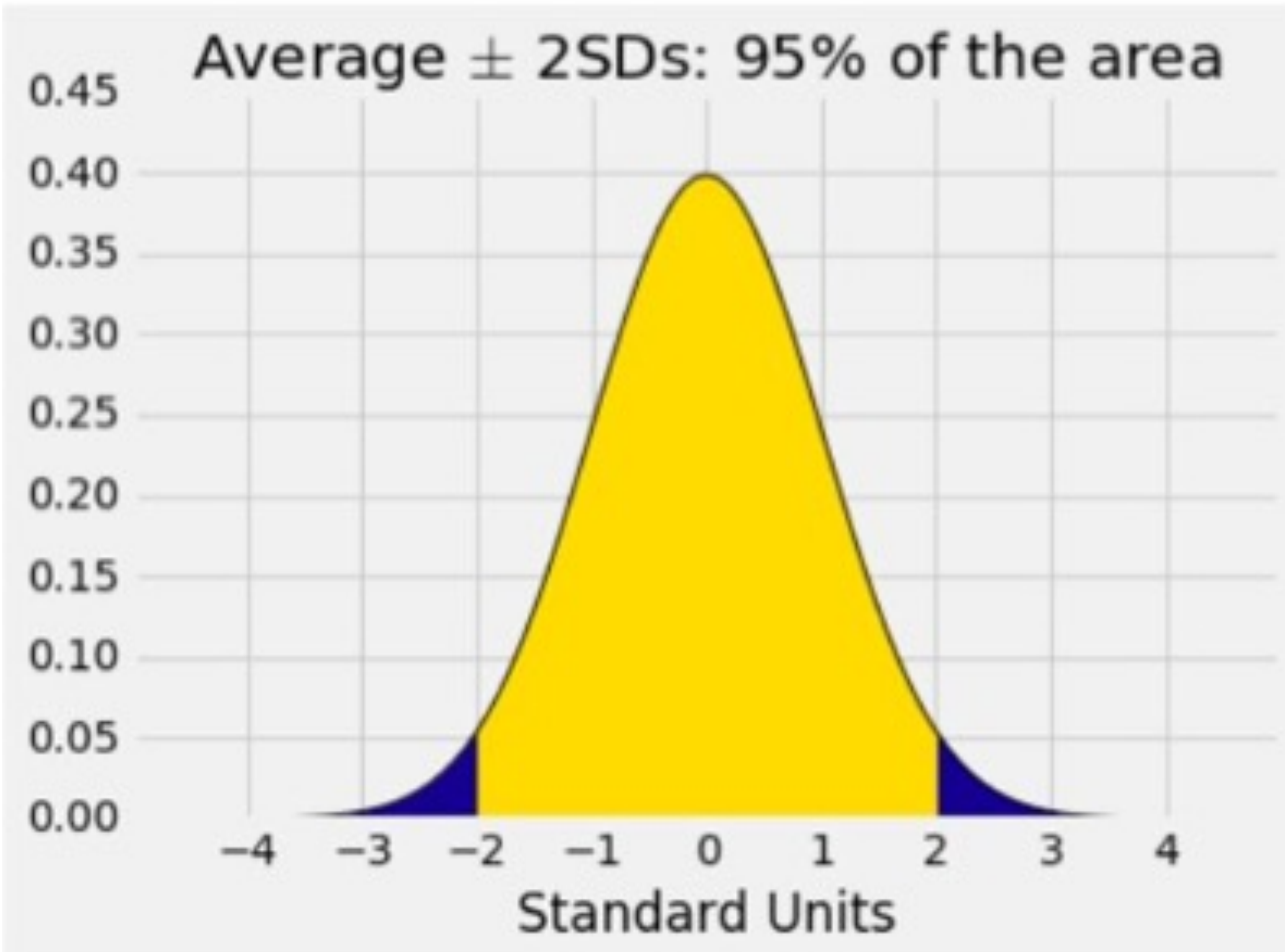
If a histogram is bell-shaped, then

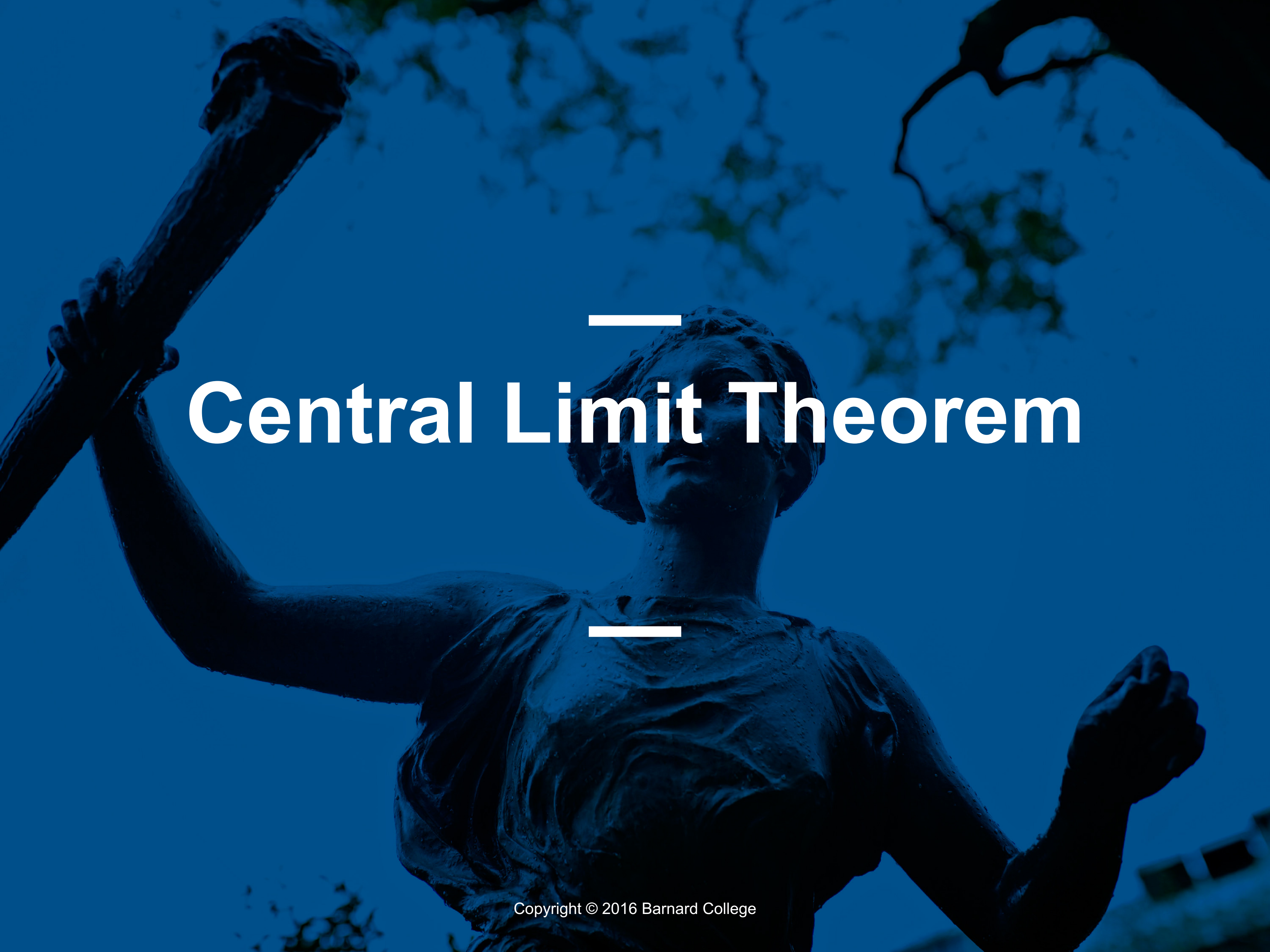
- Almost all of the data are in the range “average ± 3 SDs

Bounds and Approximations

Percent in Range	All Distributions	Normal Distributions
Average +- 1 SD	At least 0%	About 68%
Average +- 2 SDs	At least 75%	About 95%
Average +- 3 SDs	At least 88.888...%	About 99.73%

A “Central” Area





Central Limit Theorem

Central Limit Theorem

If the sample is

- large, and
- drawn at random with replacement,

Then, *regardless of the distribution of the population,*

the probability distribution of the sample sum (or the sample average) is roughly normal

Sample Average

- We often only have a sample
- We care about sample averages because they estimate population averages.
- The Central Limit Theorem describes how the normal distribution (a bell-shaped curve) is connected to random sample averages.
- CLT allows us to make inferences based on averages of random samples