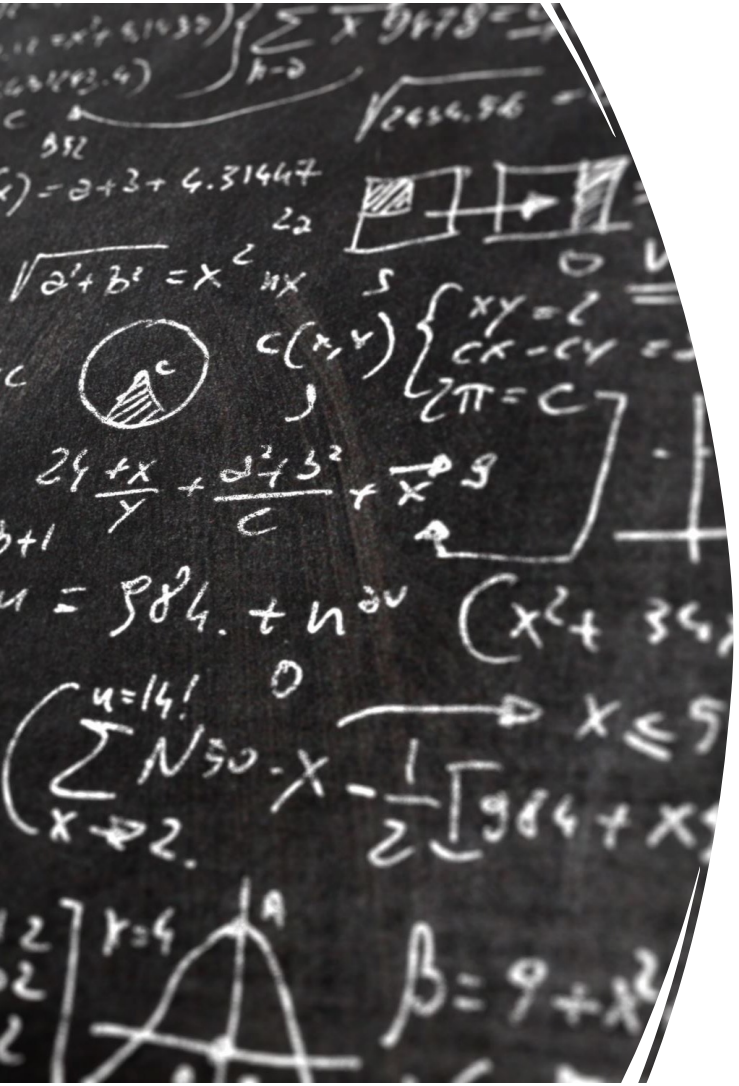


CS 383 – Computational Text Analysis

Lecture 10
Feed Forward Networks

Adam Poliak

02/20/2023

Slides adapted Dan Jurafsky, Jordan Boyd-Graber, Daniel Khashabi

Announcements

- HW04
  - Due next Wednesday 03/01
- Reading 05
  - Due Monday 02/27 - CTA/TADA/CSS papers using Word Embeddings,
  - lets move it to next Wednesday if I release today
- Office hours:
  - 3-4 pm today
  - 4-5 Thursday (tomorrow)

Final Project

Deliverables:

- Ideation
  250 write up – what idea do you have, who are you working with
  Due before Spring break

- Proposal
  Due after spring break

- Presentation
  Maybe last day of class

- Writeup, code, data
  End of finals?

Outline

Updating weights in LR

Computation Graph

Backpropagation

Issues when training NNs

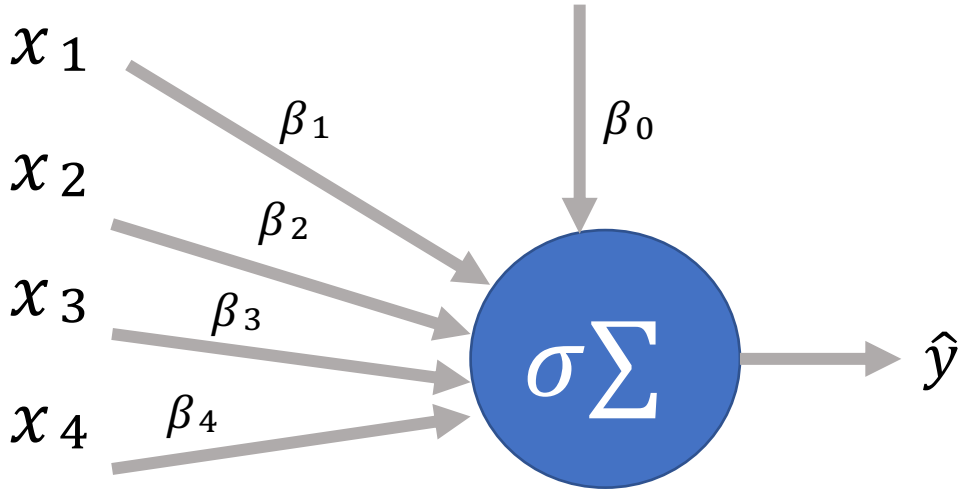
Pytorch

Deep Averaging Neural Network

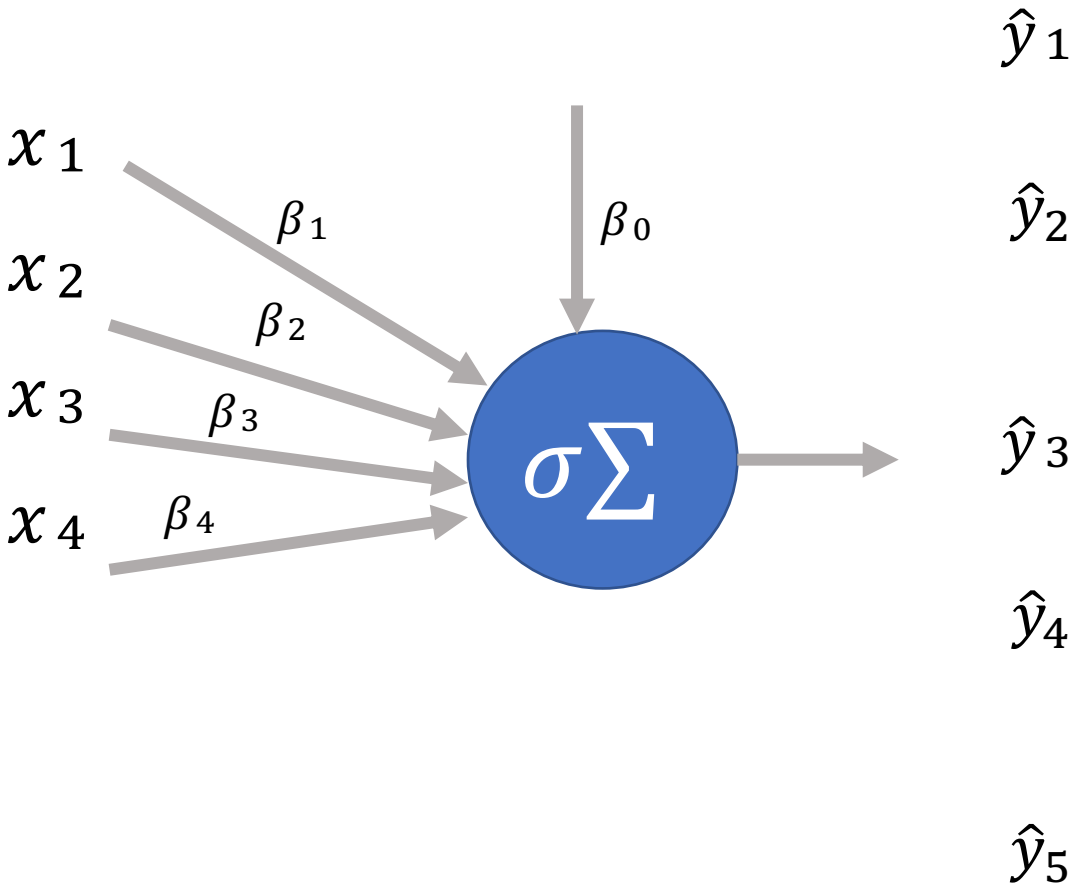
Logistic Regression

We make a prediction by applying sigmoid to the doc product of the features (covariates) and weights (coefficients)

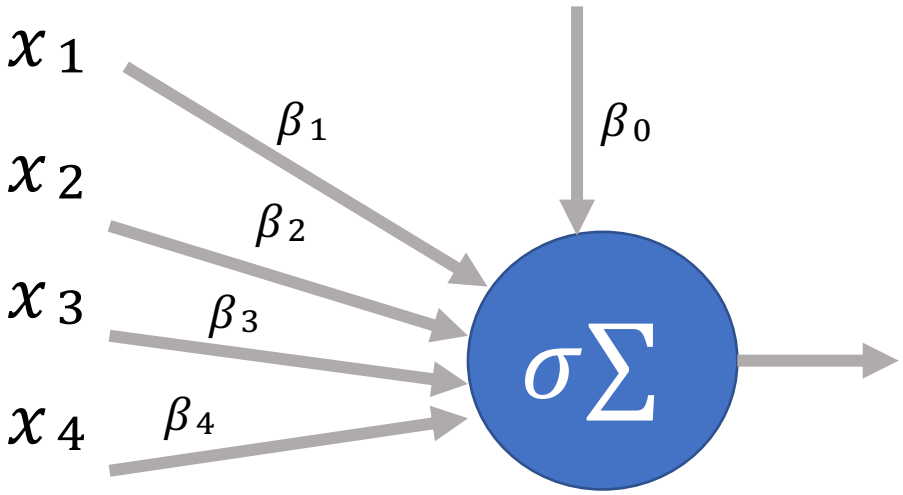
$$\sigma(\boldsymbol{\beta} \cdot \boldsymbol{x}) = \sum_i^j \sigma(\beta_i \cdot x_i)$$



Softmax


x_1
x_2
x_3
x_4
β_1
β_2
β_3
β_4
β_0
$\sigma\sum$
$\hat{y}_1$
$\hat{y}_2$
$\hat{y}_3$
$\hat{y}_4$
$\hat{y}_5$

Softmax



$\hat{y}_1$ 0.1

$\hat{y}_2$ 0.32

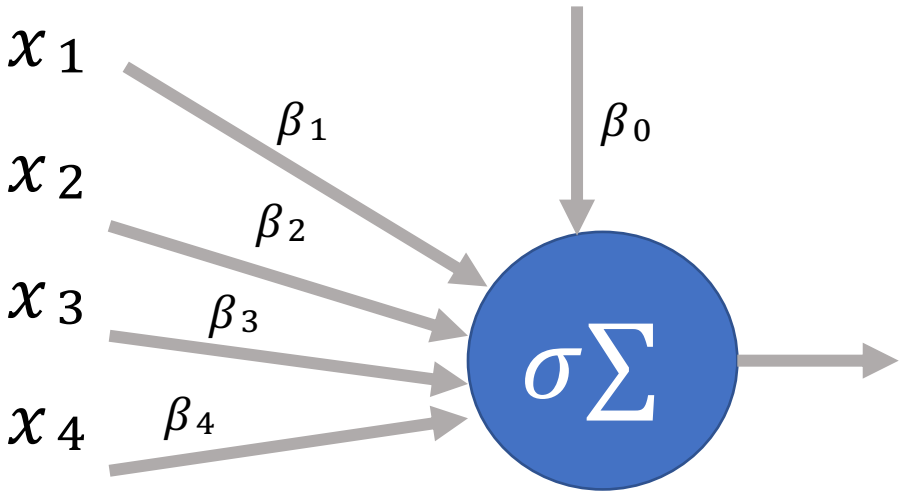
$\hat{y}_3$ 0.21

$\hat{y}_4$ 0.16

$\hat{y}_5$ 0.21

$$softmax(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{k} \exp(z_j)} \qquad 1 \leq i \leq k$$

Softmax



$\hat{y}_1$ 0.1

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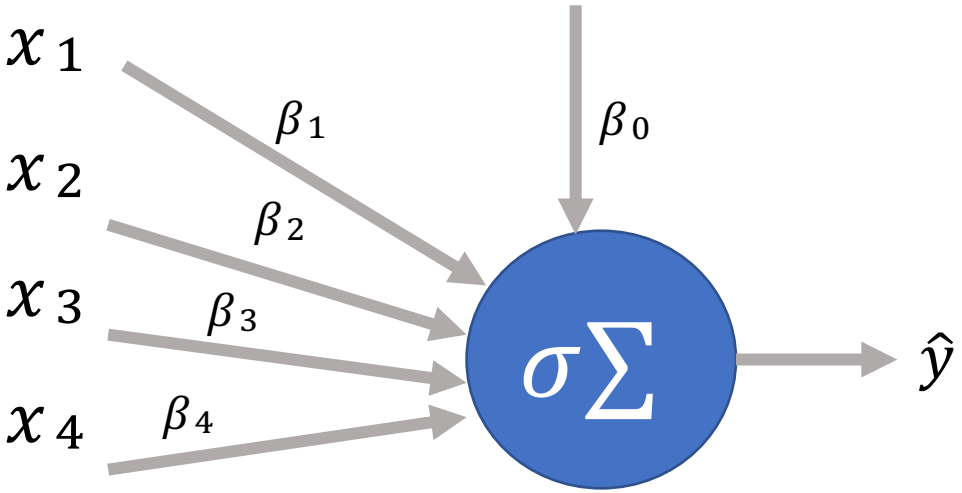
A neuron



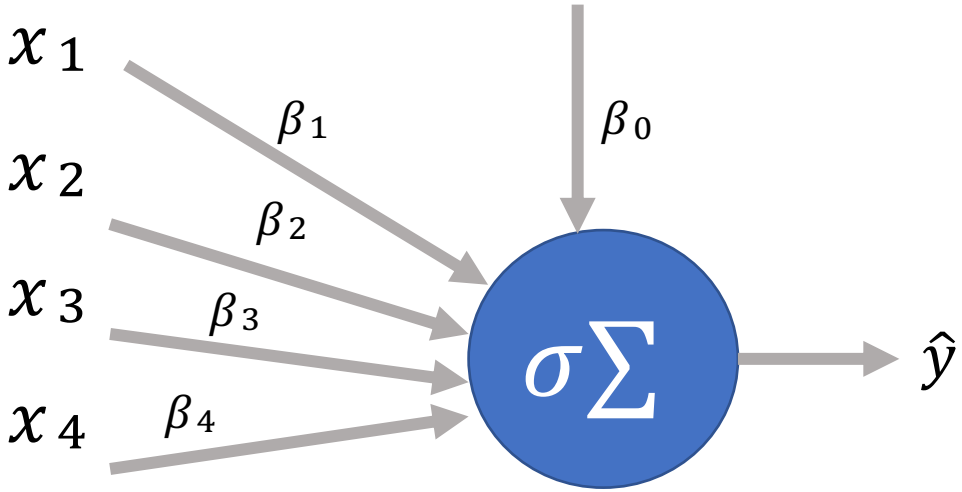
Logistic Regression as NN

A single layer neural network

Input layer: features

Output layer: prediction

We can pass the output of the neuron to another neuron

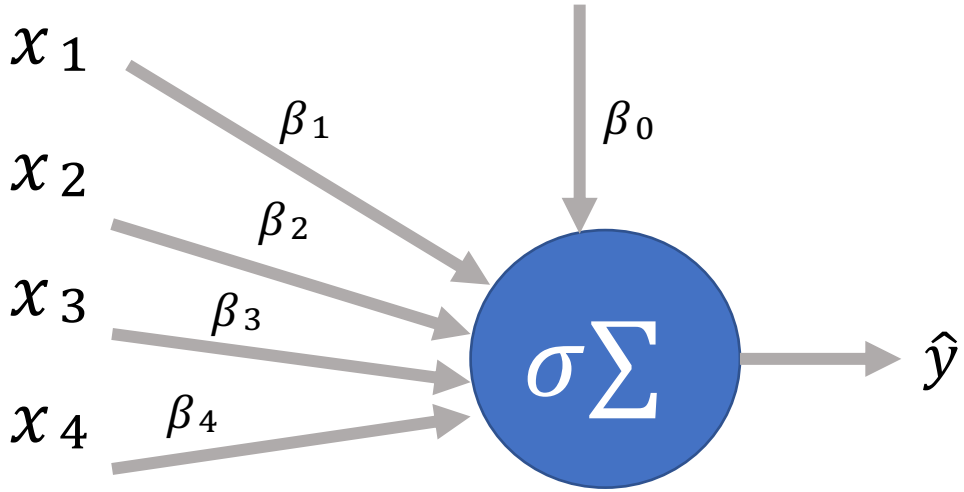


Updating weights

We want to minimize $\mathcal{L}(\hat{y}, y)$

For every weight we ask, how does changing the weight affect $\mathcal{L}(\hat{y}, y)$

Use partial derivatives to answer the question:



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$$\nabla\mathcal{L}(\hat{y}, y) = \begin{bmatrix} \frac{\partial\mathcal{L}(\hat{y},y)}{\partial\beta_1} \\ \frac{\partial\mathcal{L}(\hat{y},y)}{\partial\beta_2} \\ \frac{\partial\mathcal{L}(\hat{y},y)}{\partial\beta_3} \\ \frac{\partial\mathcal{L}(\hat{y},y)}{\partial\beta_4} \\ \frac{\partial\mathcal{L}(\hat{y},y)}{\partial\beta_0} \end{bmatrix}$$

New loss function:

$$L(a, b, c) = b(a + 2b + ac^2)$$

We want to minimize this L

For every parameter we ask, how does changing the parameter affect this loss

Use partial derivatives to answer the question:

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial a} \\ \frac{\partial \mathcal{L}}{\partial b} \\ \frac{\partial \mathcal{L}}{\partial c} \end{bmatrix}$$

New loss function:

$$L(a, b, c) = b(a + 2b + ac^2)$$

What is $\frac{\partial \mathcal{L}}{\partial a}$?

$(a + 2b + ac^2)$ (1 + 2c)

What is $\frac{\partial \mathcal{L}}{\partial b}$?

$(a + 2b + ac^2)(2)$

What is $\frac{\partial \mathcal{L}}{\partial c}$?

$(a + 2b + ac^2)(2c)$

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial a} \\ \frac{\partial \mathcal{L}}{\partial b} \\ \frac{\partial \mathcal{L}}{\partial c} \end{bmatrix}$$

New loss function:

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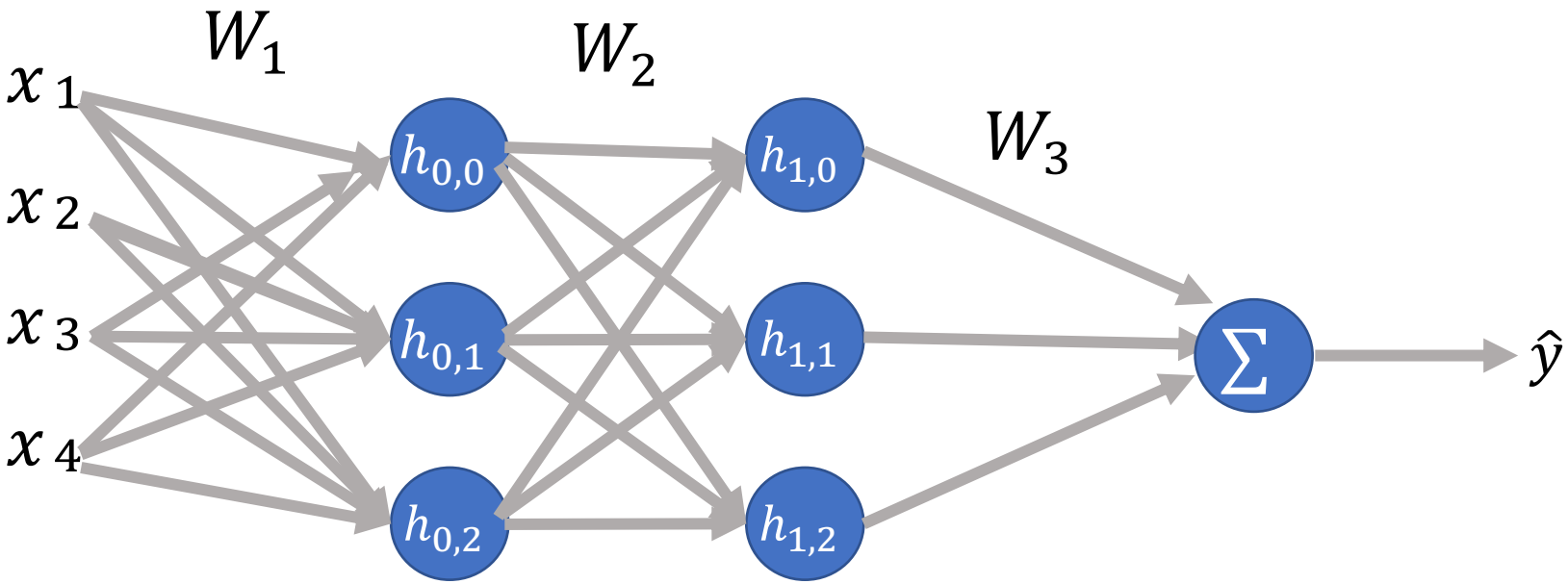
What is $\frac{\partial \mathcal{L}}{\partial c}$?

$(a + 2b + ac^2)(2c)$

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial a} \\ \frac{\partial \mathcal{L}}{\partial b} \\ \frac{\partial \mathcal{L}}{\partial c} \end{bmatrix}$$

$$= \begin{bmatrix} (a + 2b + ac^2)(1 + 2\mathrm{c}) \\ (a + 2b + ac^2)(2) \\ (a + 2b + ac^2)(2c) \end{bmatrix}$$

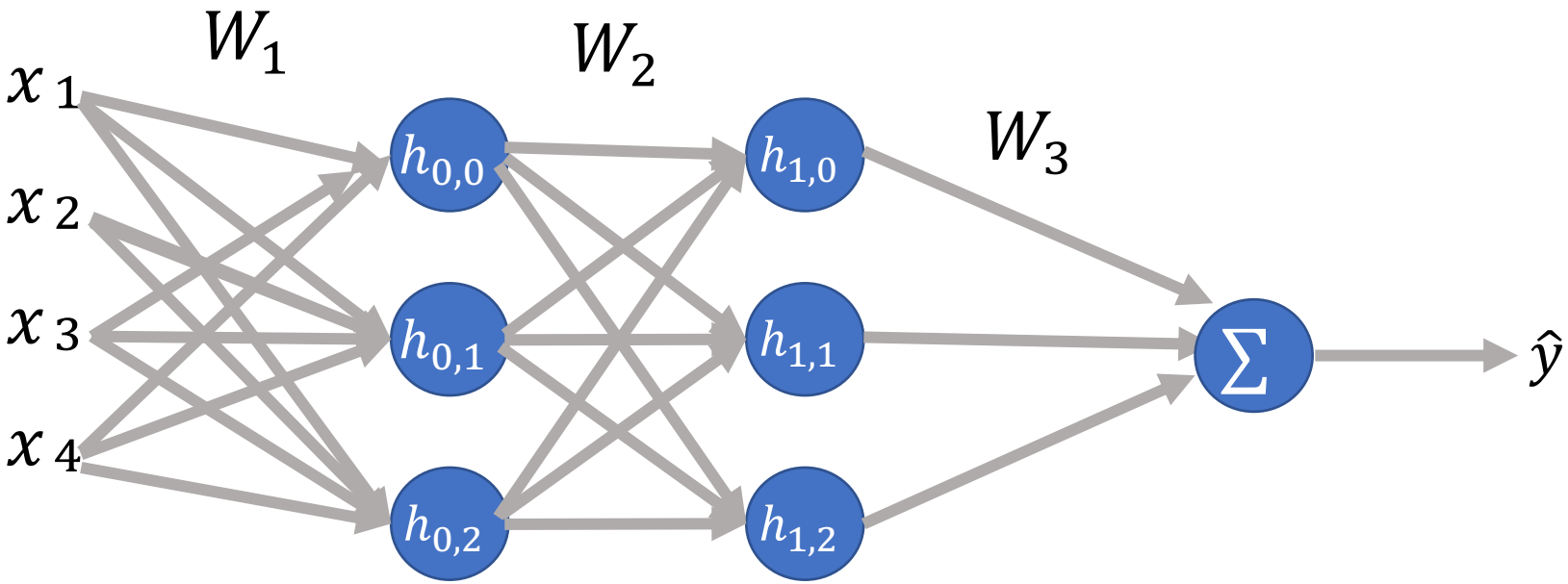
Feed forward NN



$$\boldsymbol{h}_0 = \sigma(xW_1)$$

$$L(\hat{y}, y) =$$

$$L(\sigma(\boldsymbol{W}_3\sigma(\boldsymbol{W}_2\,\sigma(\boldsymbol{x}\boldsymbol{W}_1 + \boldsymbol{b})) + \boldsymbol{b})), \boldsymbol{y})$$



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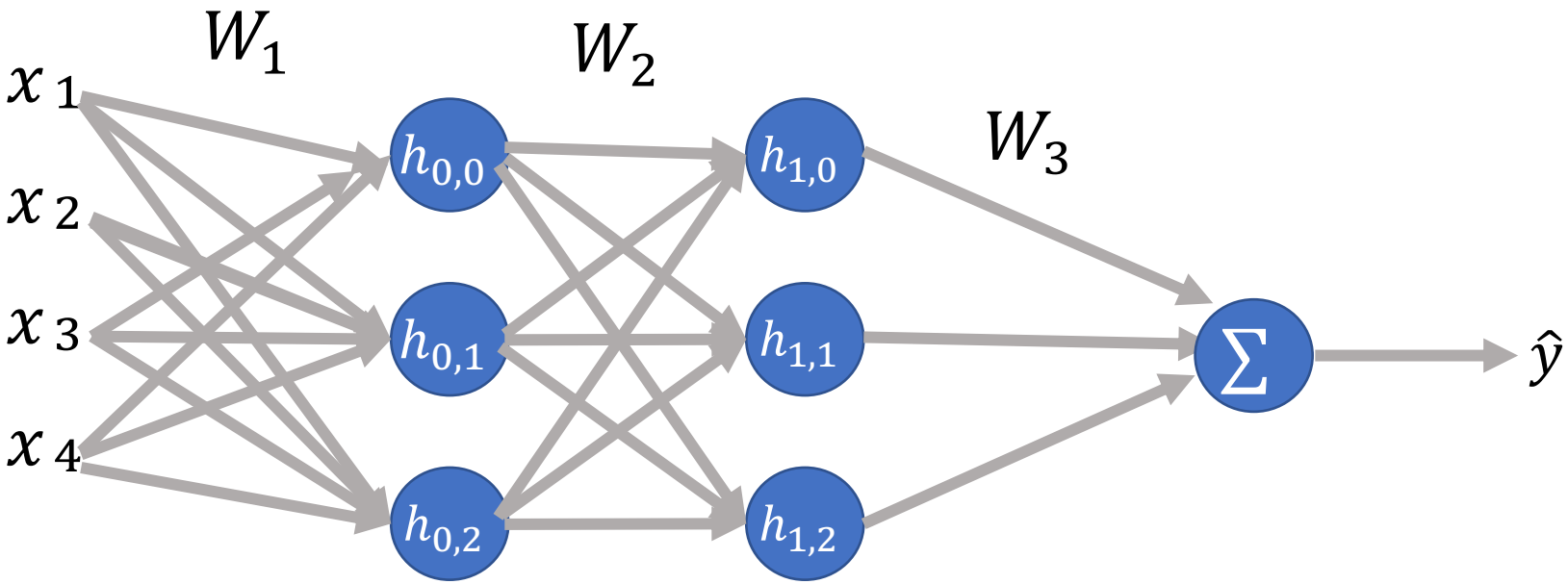

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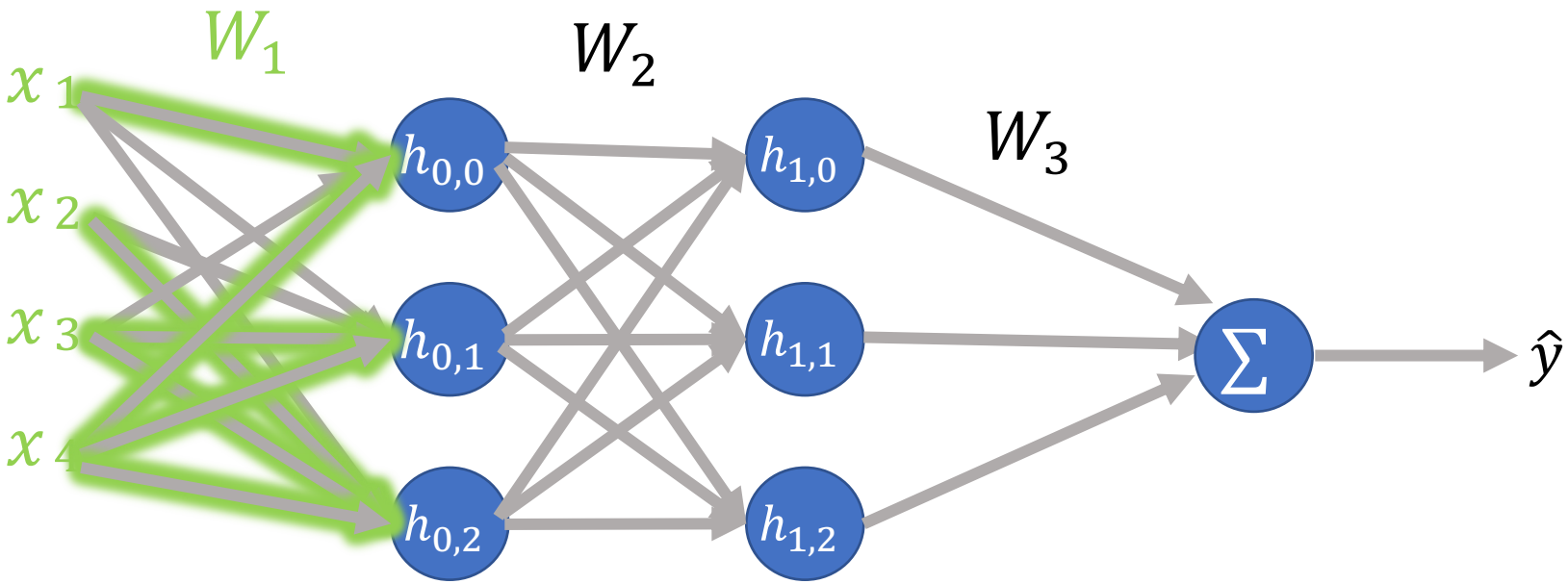

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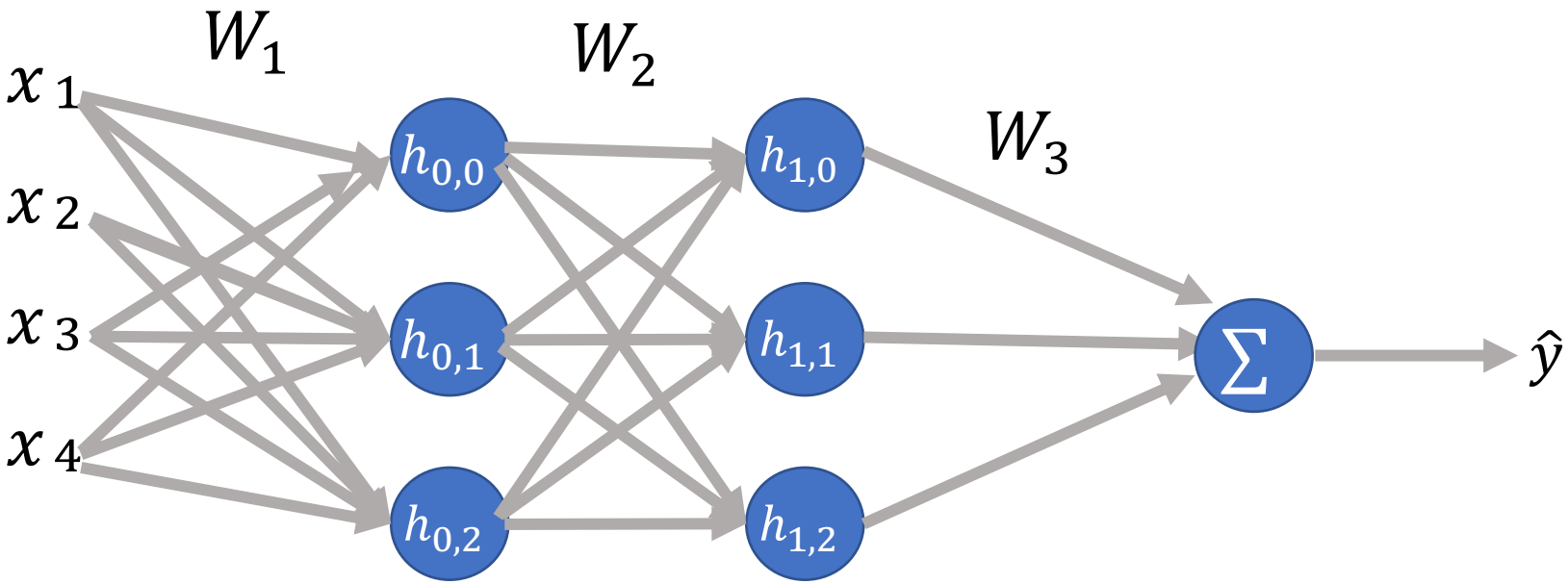

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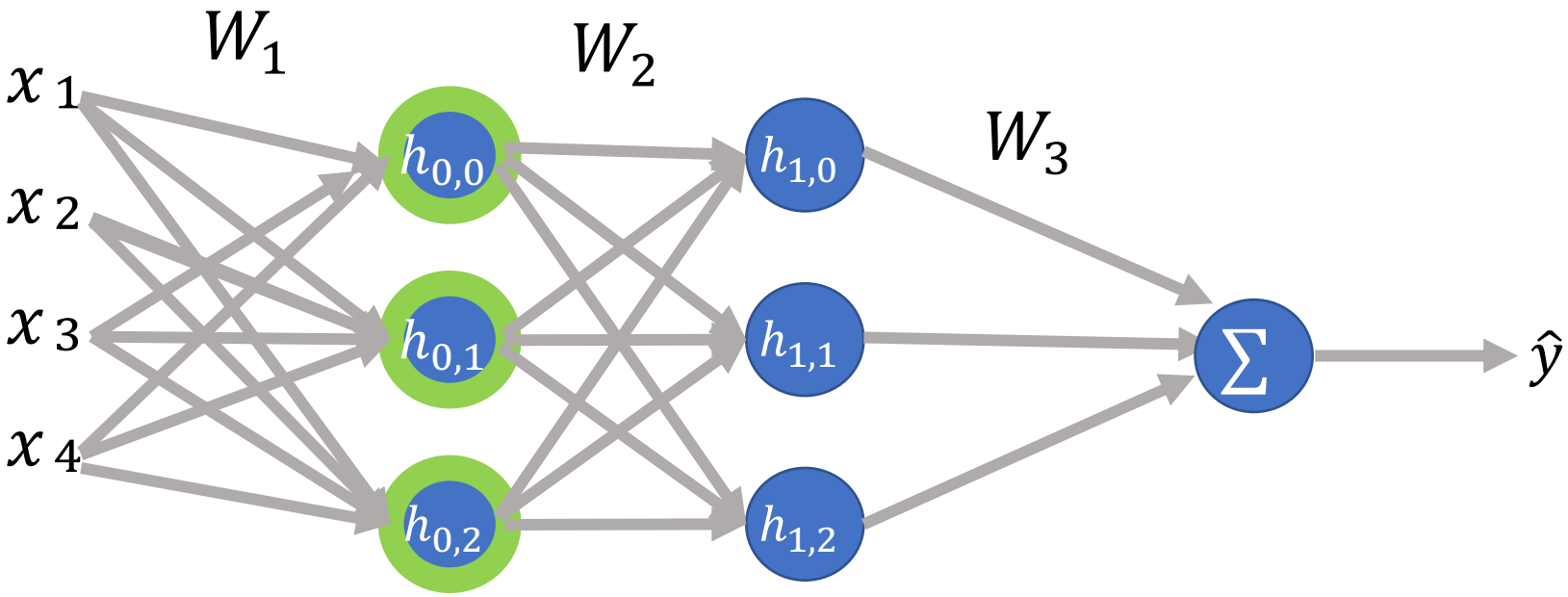

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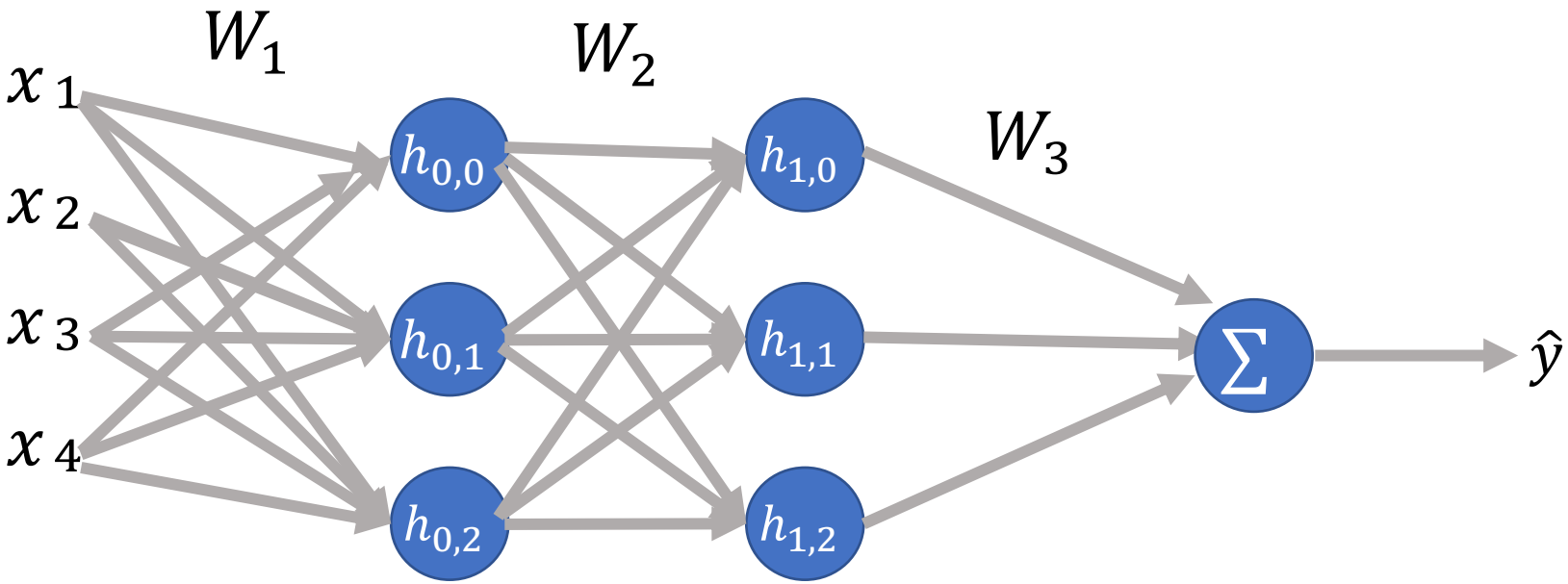

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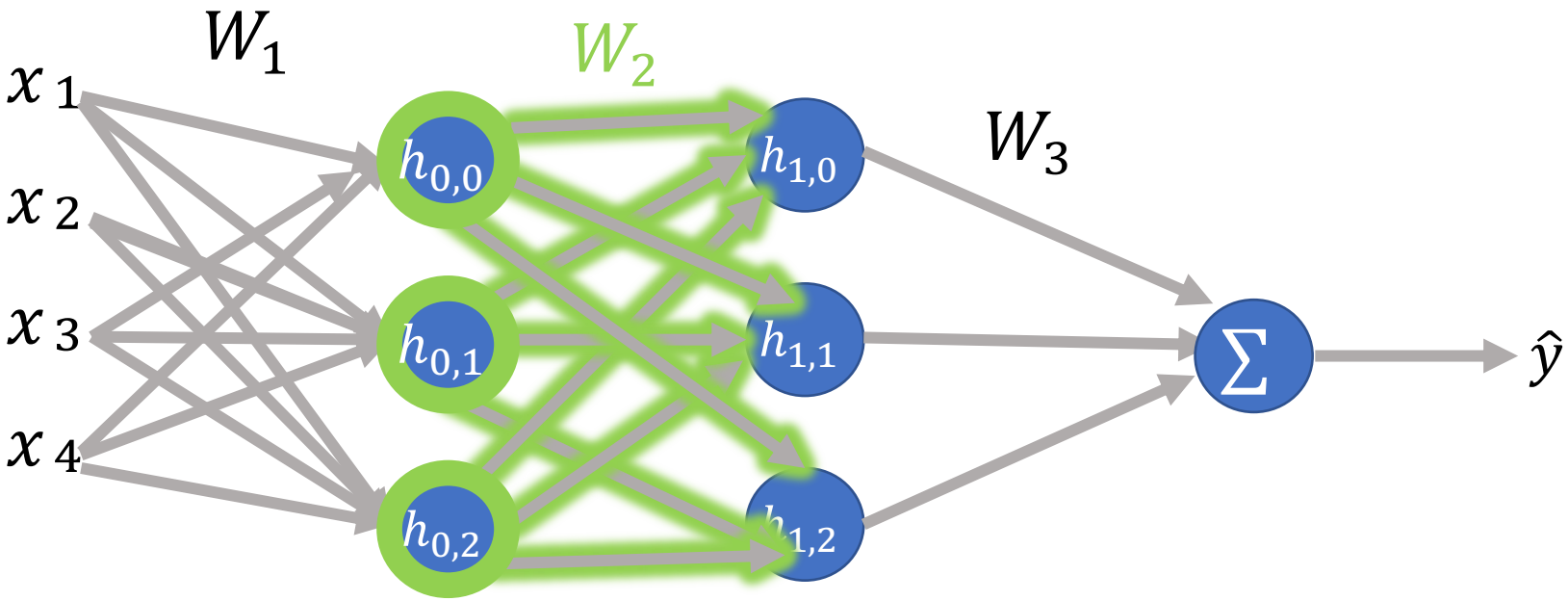

$$\boldsymbol{h}_0 = \sigma(xW_1)$$

Loss function:

$$L(\sigma(\boldsymbol{W}_3\sigma(\boldsymbol{W}_2\,\sigma(\boldsymbol{x}\boldsymbol{W}_1+\boldsymbol{b}))+\boldsymbol{b})),\boldsymbol{y})$$

How many parameters do we have?

W_1 is 4 x 3

W_2 is 3 x 3

W_3 is 3 x 1

3 bias terms

12 + 9 + 3 + 3 = 27 parameters

$$\nabla\mathcal{L}=\begin{bmatrix}\frac{\partial\mathcal{L}}{\partial\theta_1}\\ \frac{\partial\mathcal{L}}{\partial\theta_2}\\ \cdots\\ \cdots\\ \frac{\partial\mathcal{L}}{\partial\theta_{27}}\\ \frac{\partial\mathcal{L}}{\partial b_1}\end{bmatrix}$$

Outline

Updating weights in LR

Computation Graph

Backpropagation

2 layered Feed-Forward Neural Network

Issues when training NNs

Pytorch

Deep Averaging Neural Network.

Computation graph

$$L(a, b, c) = b(a + 2b + ac^2)$$

Computations:

$$d = c^2$$
$$\mathrm{e} = \mathrm{a} * \mathrm{d}$$
$$\mathrm{f} = 2 * \mathrm{b}$$
$$g = \mathrm{a} + \mathrm{f}$$
$$\mathrm{h} = \mathrm{g} + \mathrm{e}$$
$$\mathrm{i} = \mathrm{b} * \mathrm{h}$$

Computation graph

$L(a, b, c) = b(a + 2b + ac^2)$

Computations:

$d = c^2$

$e = a * d$

$f = 2 * b$

$g = a + f$

$h = g + e$

$i = b * h$

Computation graph

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$\mathrm{i} = h * b$

$d = c^2$

Computation graph

$L(a, b, c) = b(a + 2b + ac^2)$

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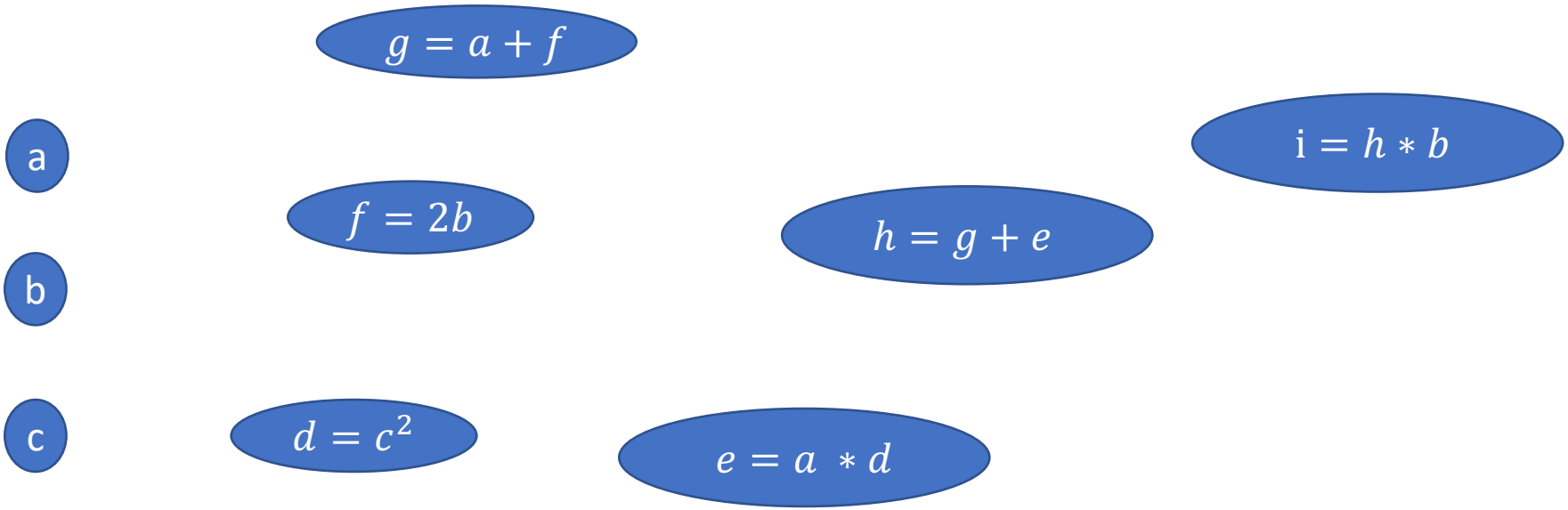
$g = a + f$

$h = g + e$

$i = b * h$



Computation graph

$L(a, b, c) = b(a + 2b + ac^2)$

Computations:

$d = c^2$

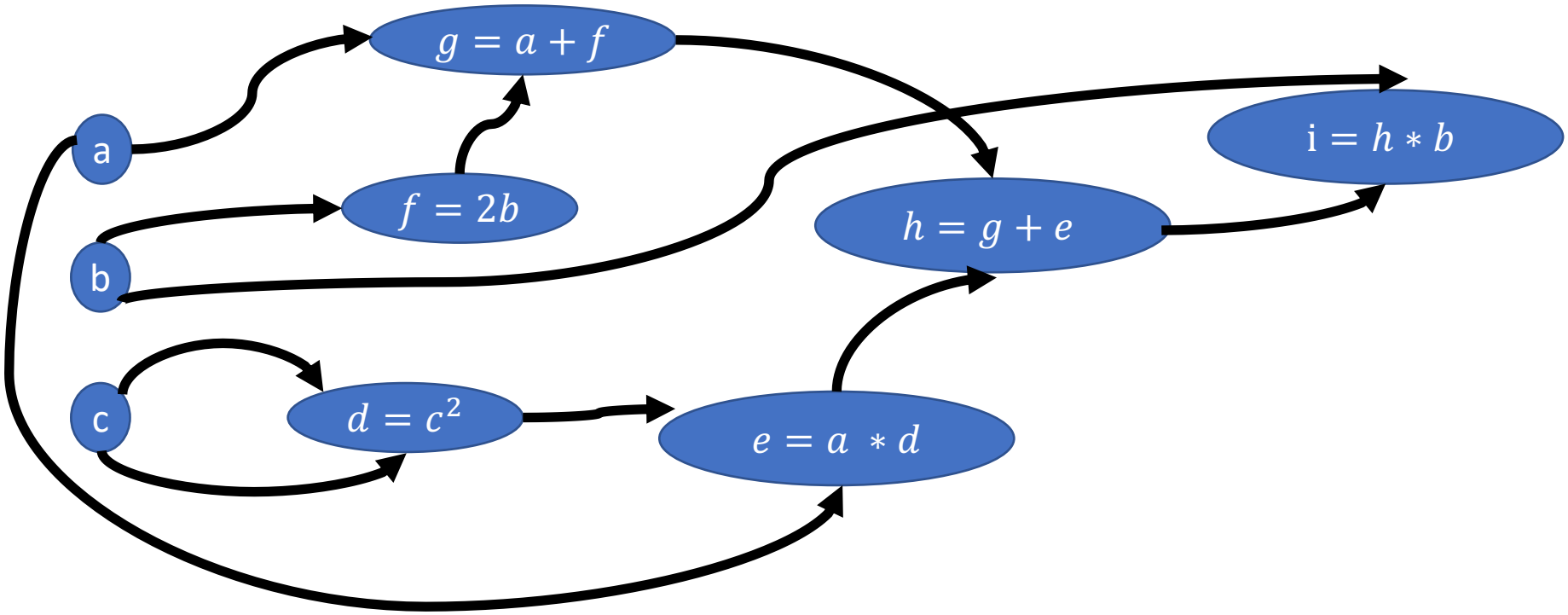
e = a ∗ d

f = 2 ∗ b

g = a + f

h = g + e

i = b ∗ h



Computation graph

$L(a, b, c) = b(a + 2b + ac^2)$

$L(3, 1, -2) = b(a + 2b + ac^2)$

Computations:

$d = c^2$

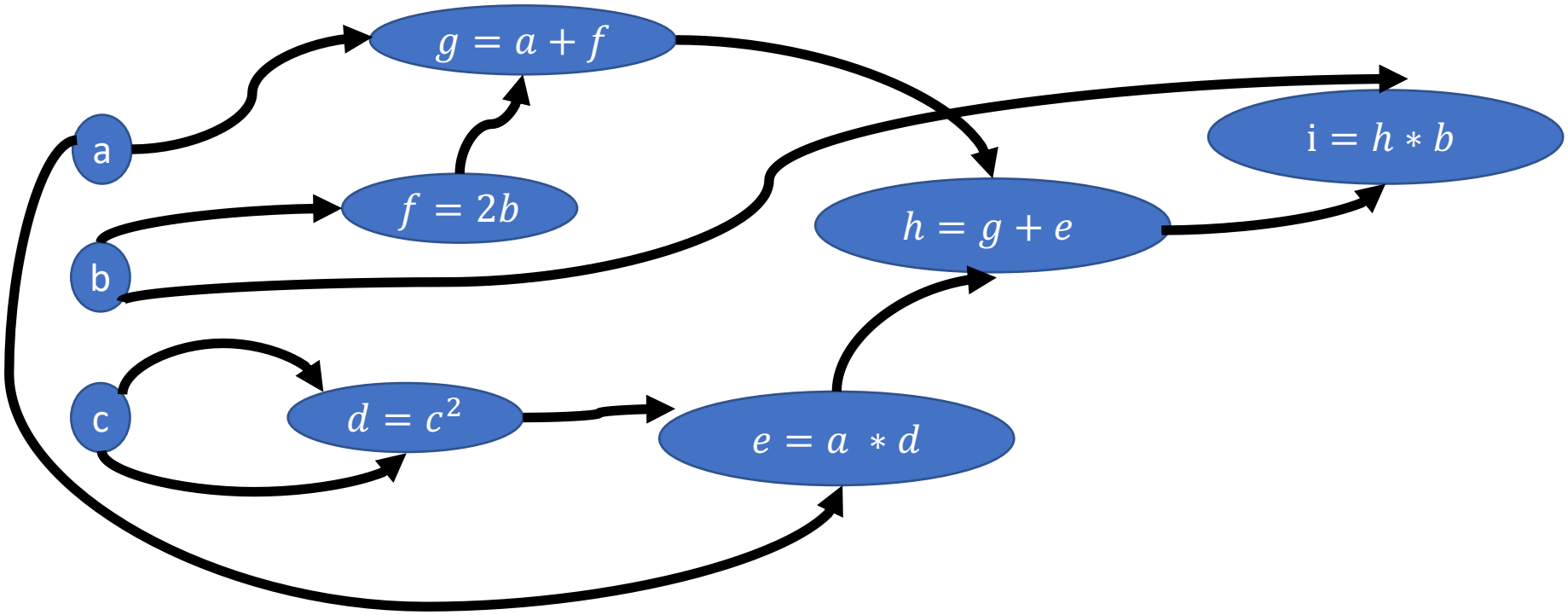
$e = a * d$

$f = 2 * b$

$g = a + f$

$h = g + e$

$i = h * b$



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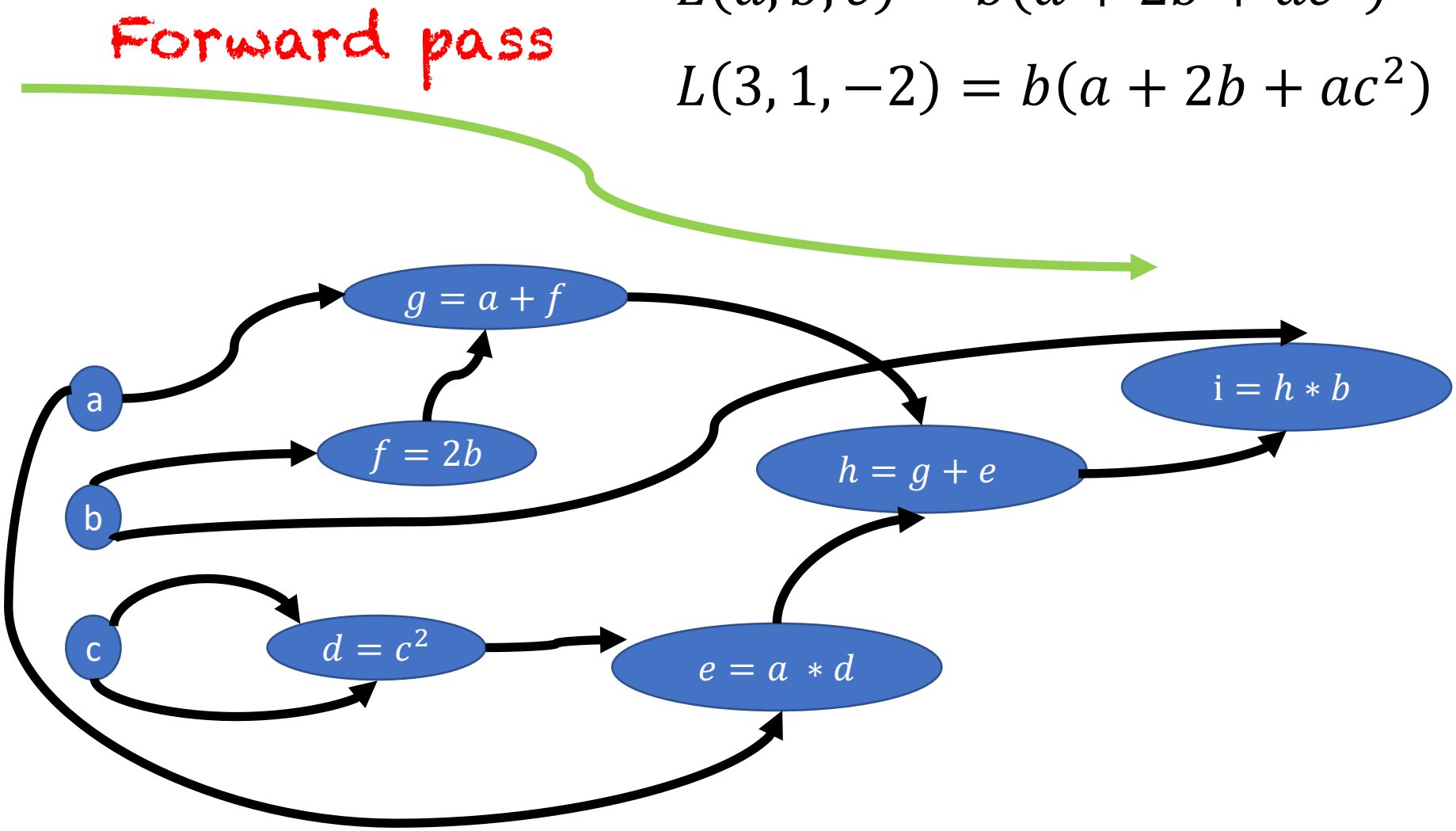

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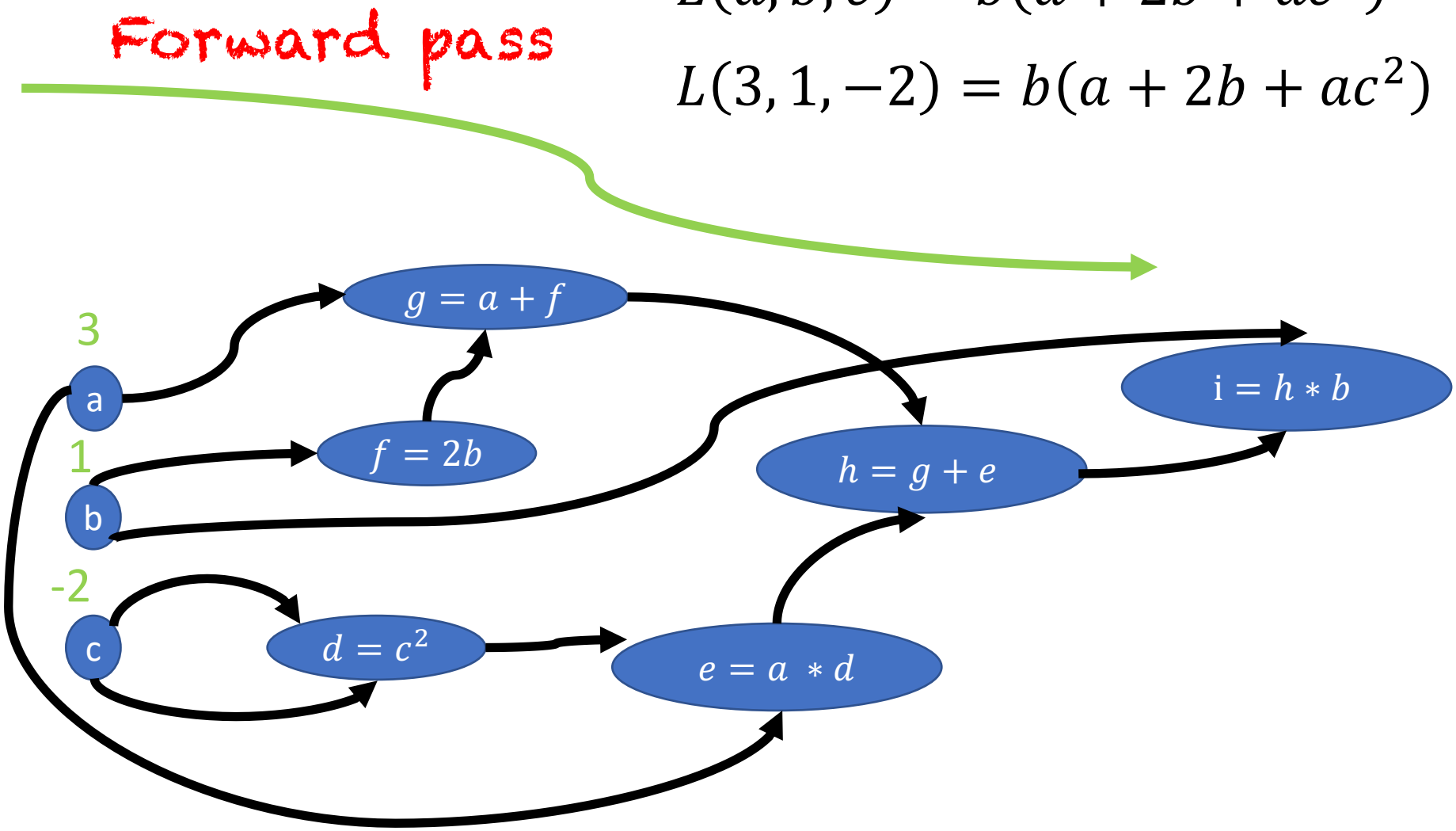
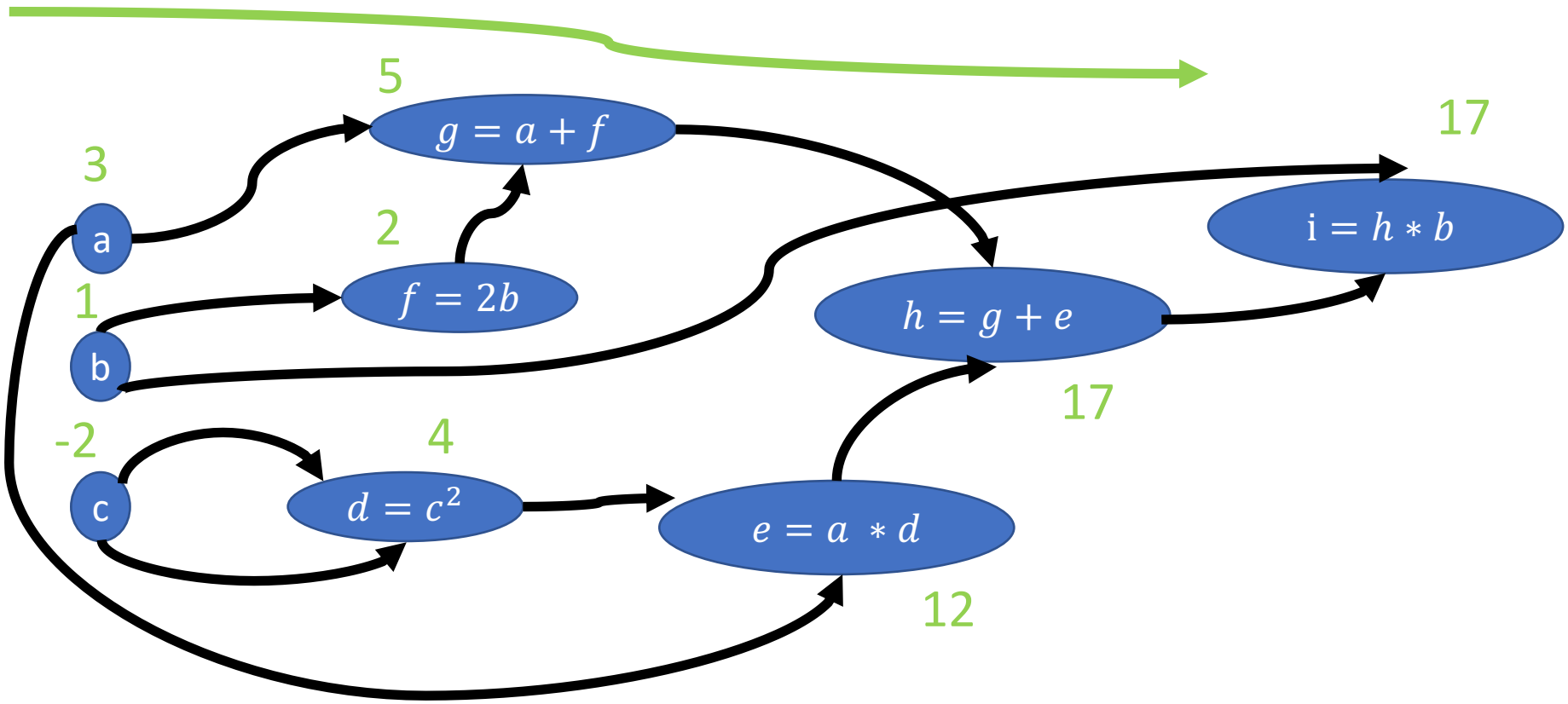


Computation graph $L(a,b,c) = b(a + 2b + ac^2)$

$$L(3,1,-2) = b(a + 2b + ac^2)$$

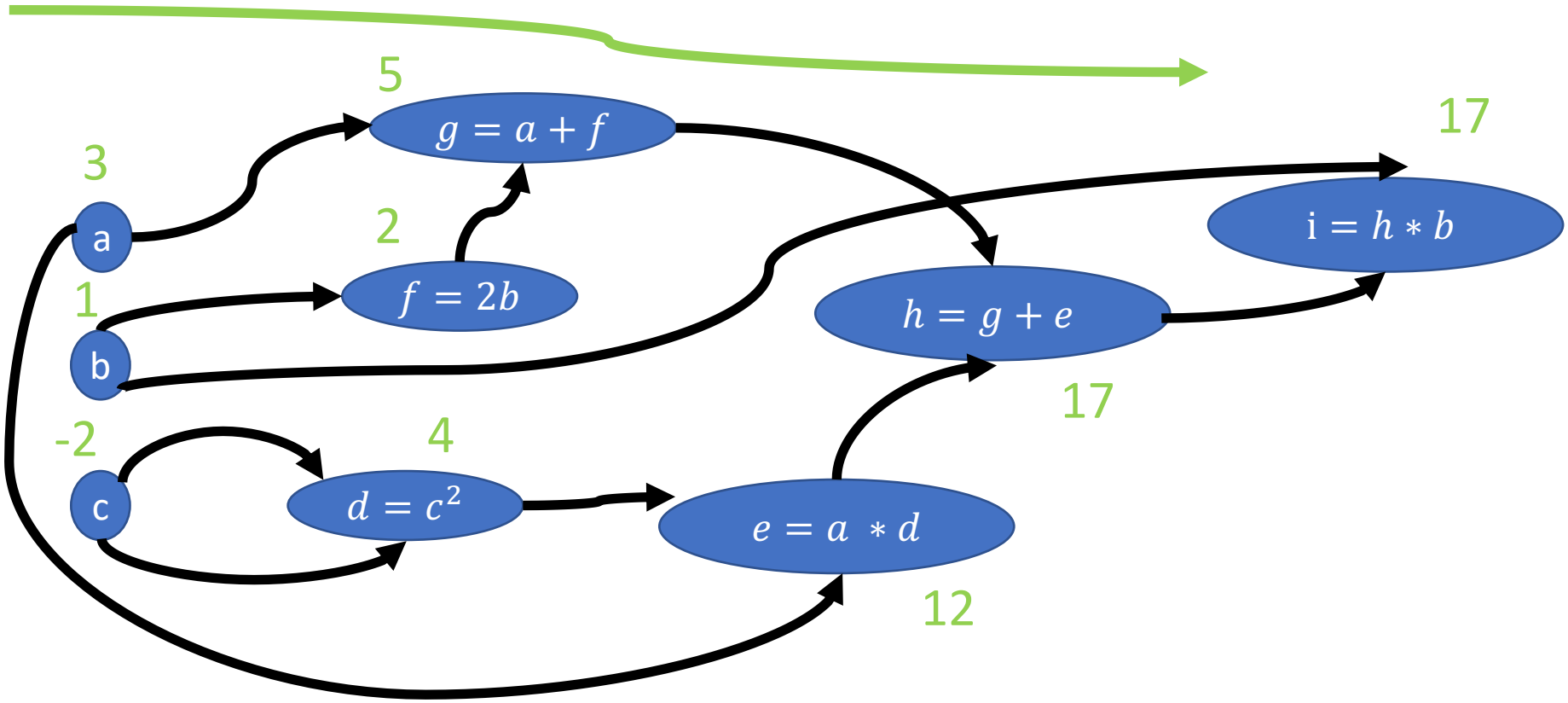
Forward pass



Computation graph $L(a,b,c) = b(a + 2b + ac^2)$

Forward pass

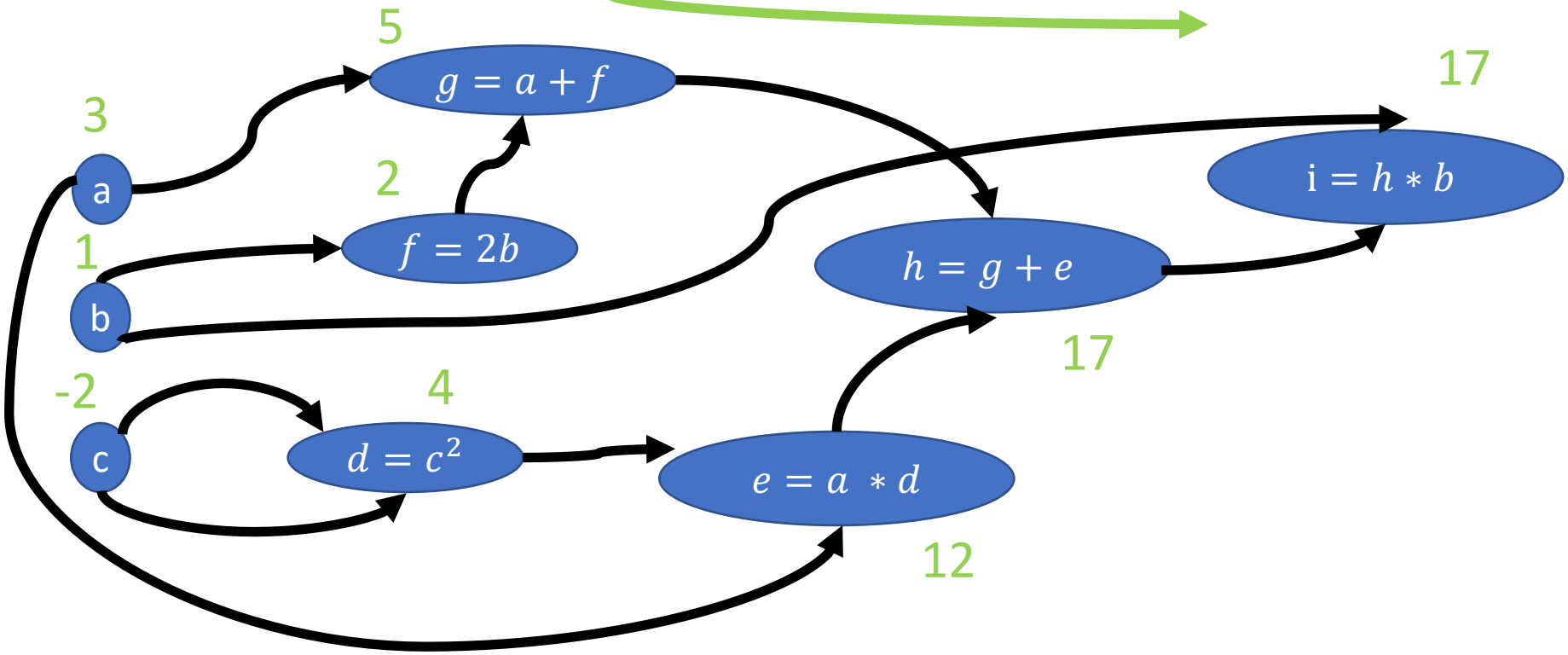
$$L(3,1,-2) = b(a + 2b + ac^2)$$
$$L(3,1,-2) = 1(3 + 2(1) + 3(-2)^2)$$



Computation graph $L(a,b,c) = b(a + 2b + ac^2)$

Forward pass

$$L(3,1,-2) = b(a + 2b + ac^2)$$
$$L(3,1,-2) = 1(3 + 2(1) + 3(-2)^2)$$
$$L(3,1,-2) = 1(3 + 2 + 12) = 17$$



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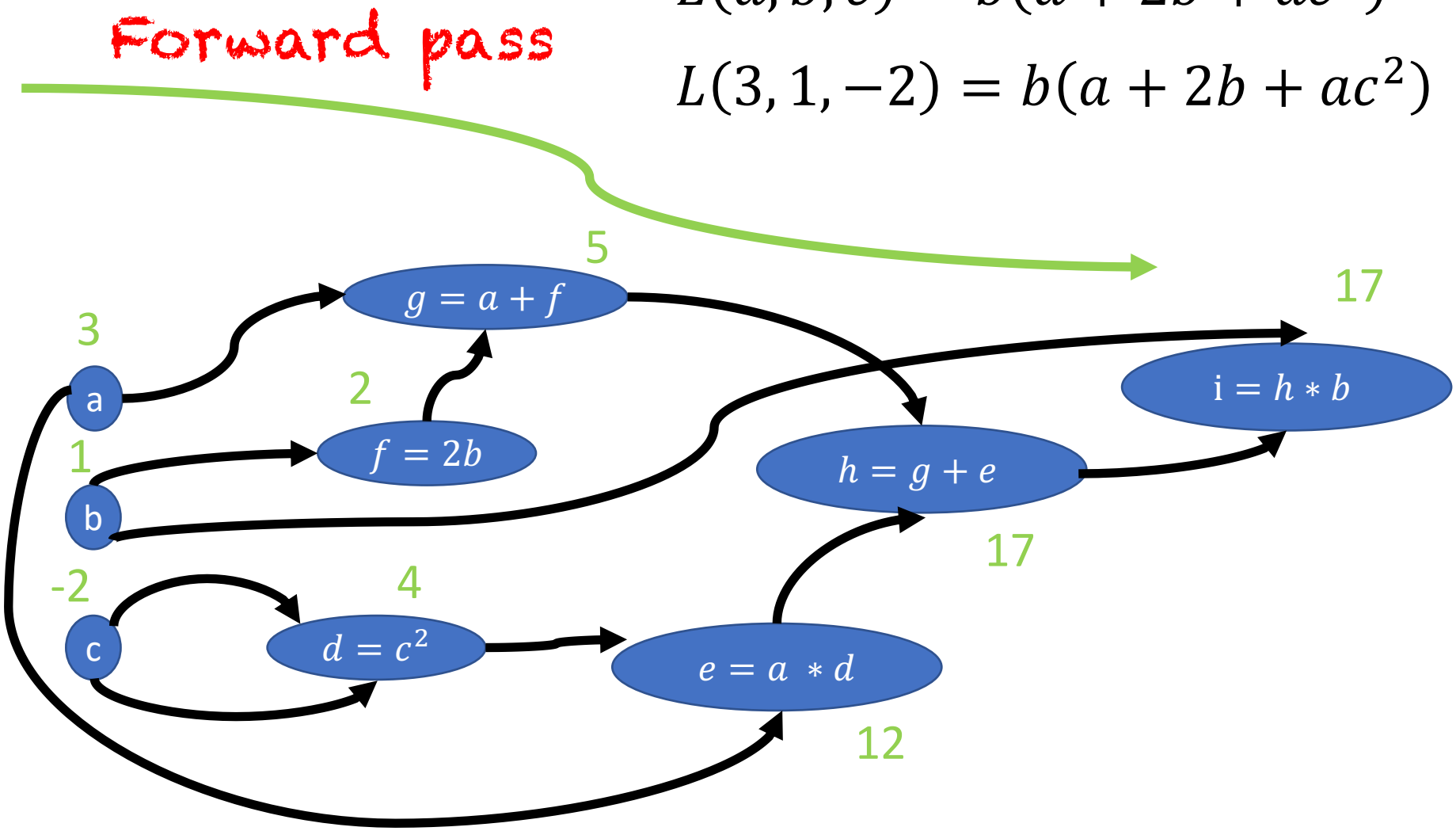


One node view

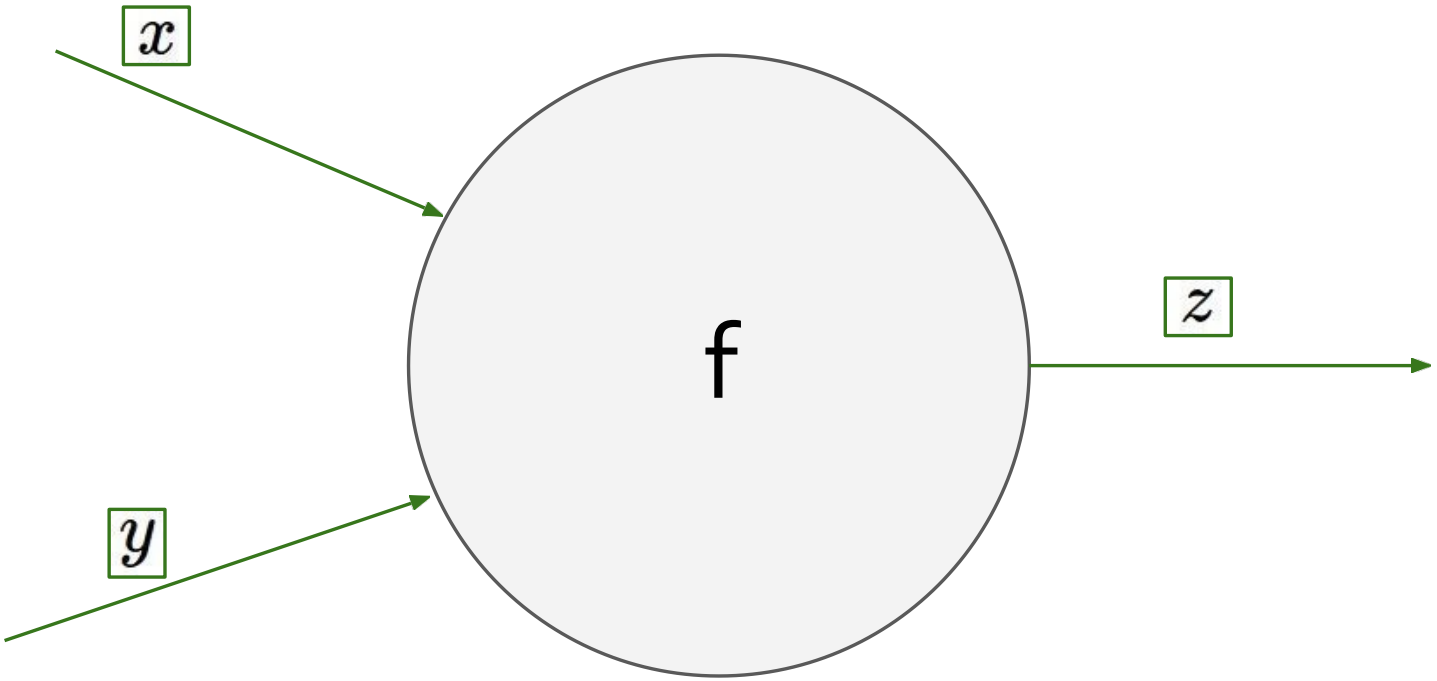
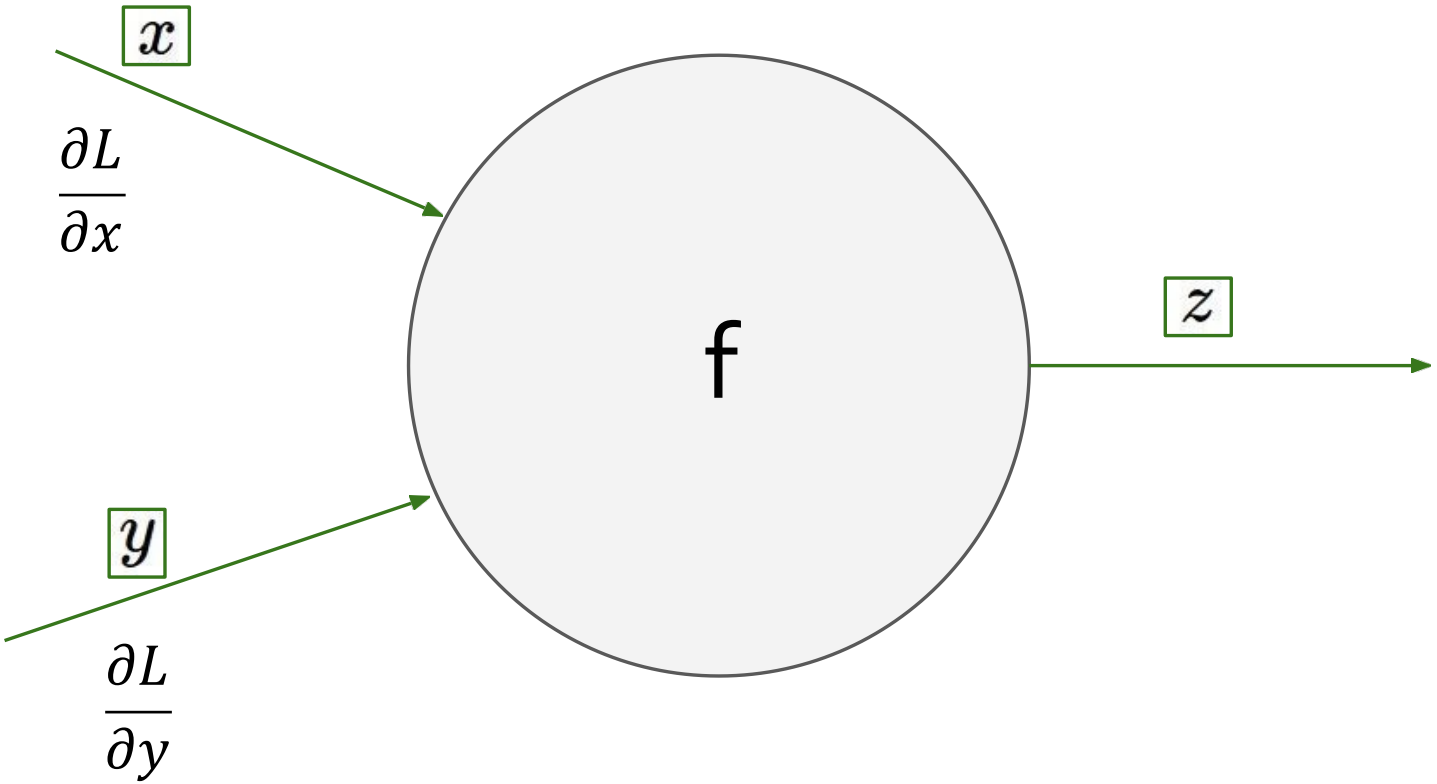


Figure from Andrej Karpathy

One node view



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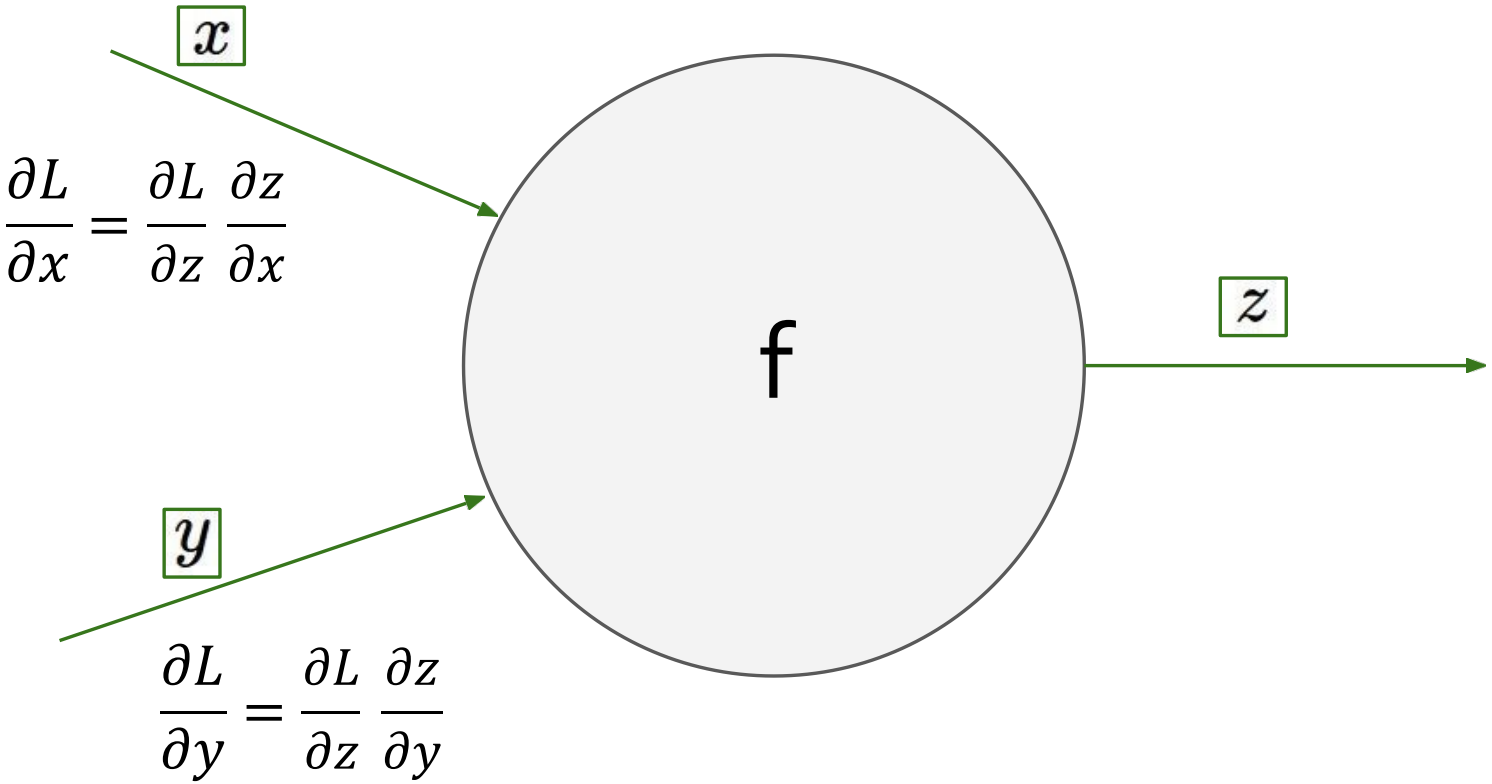
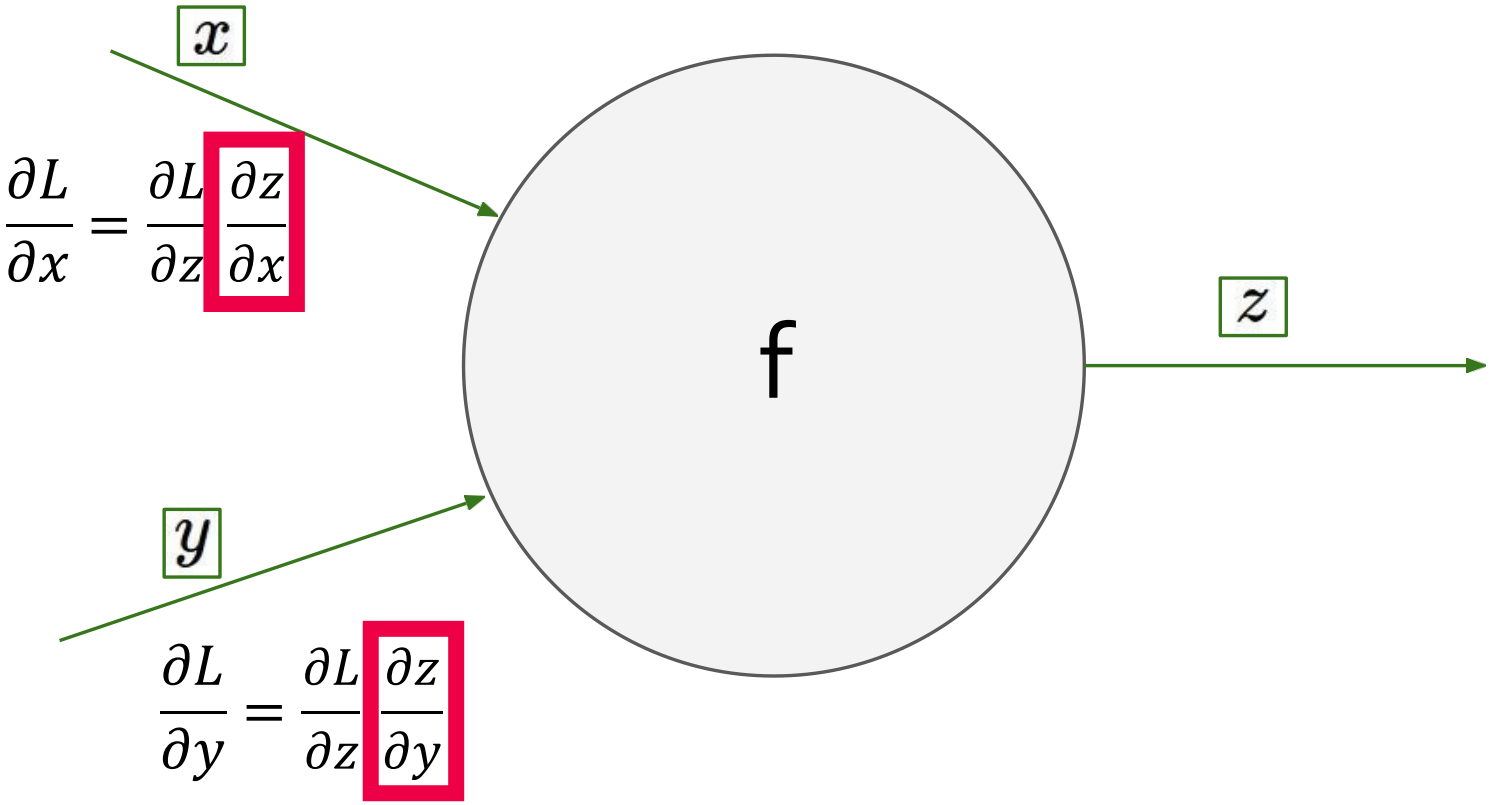


Figure from Andrej Karpathy

One node view

Local gradient



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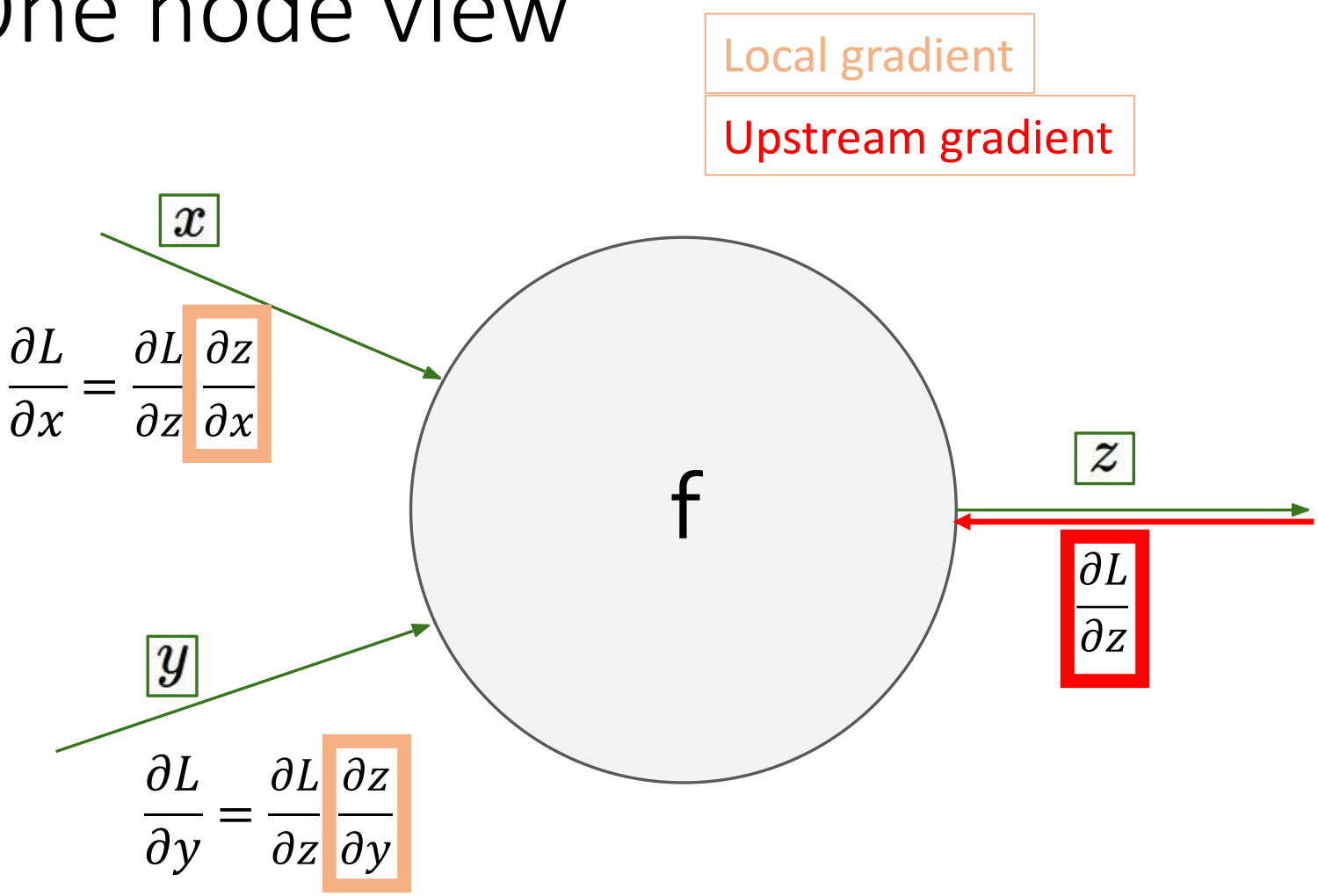

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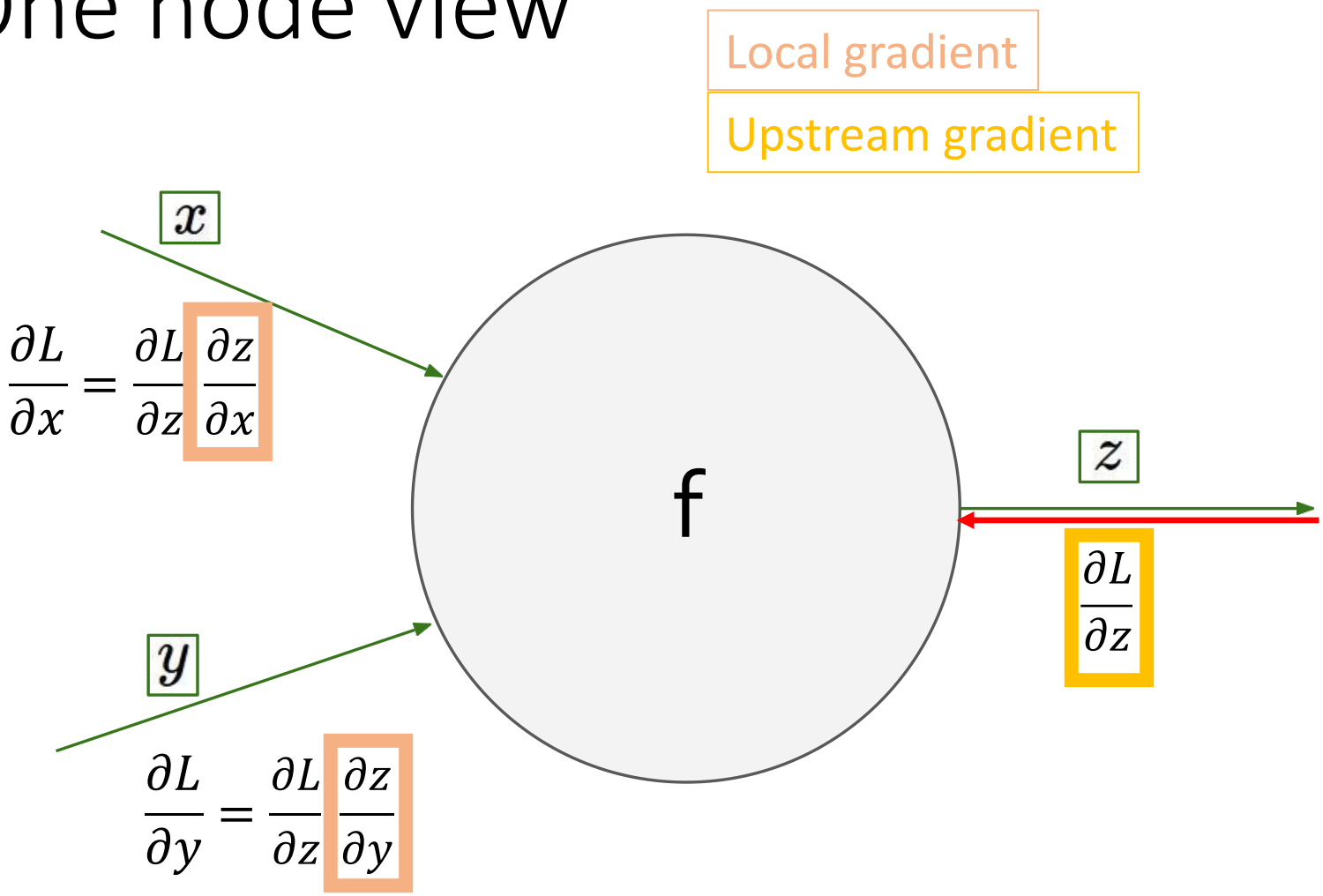

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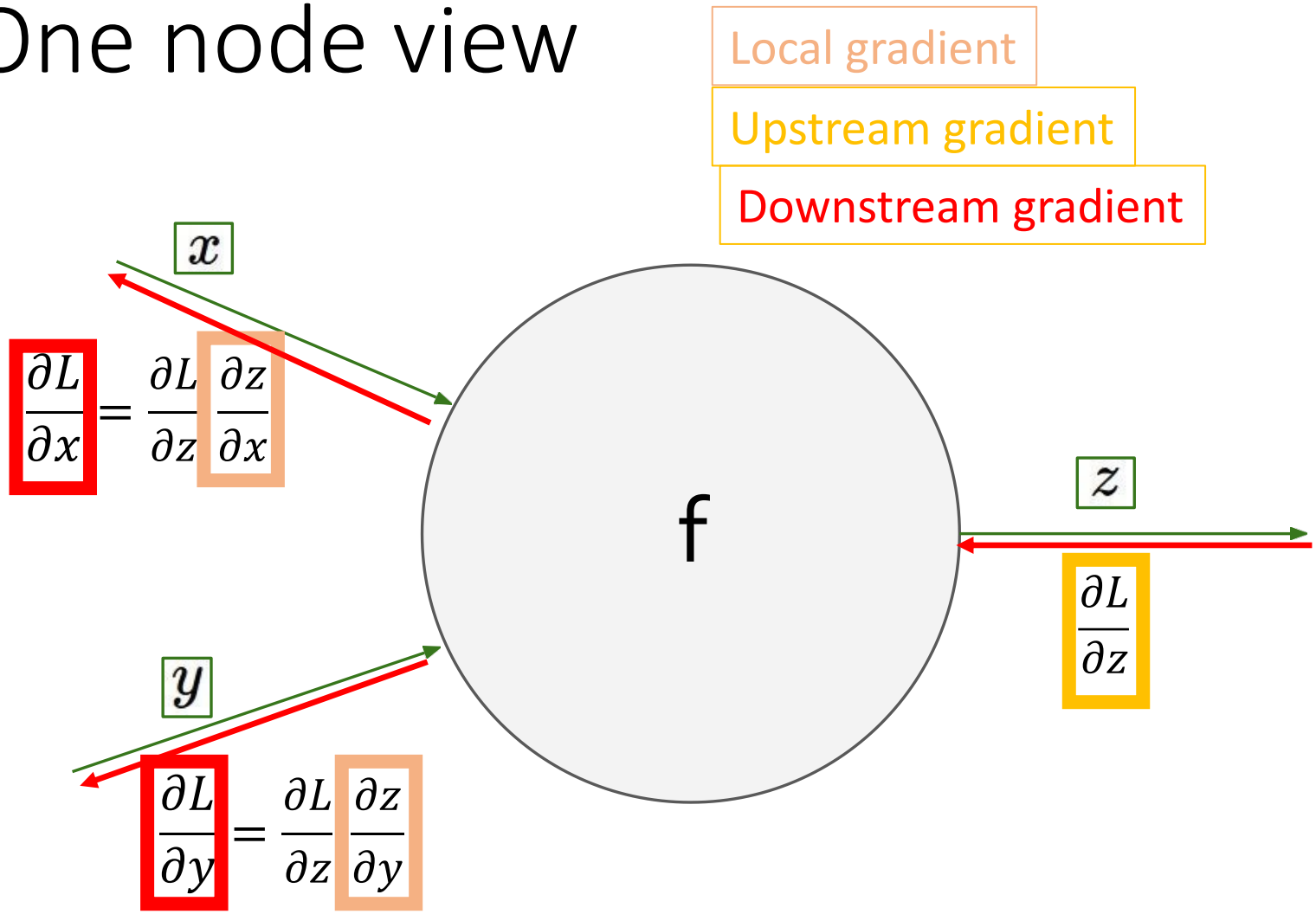

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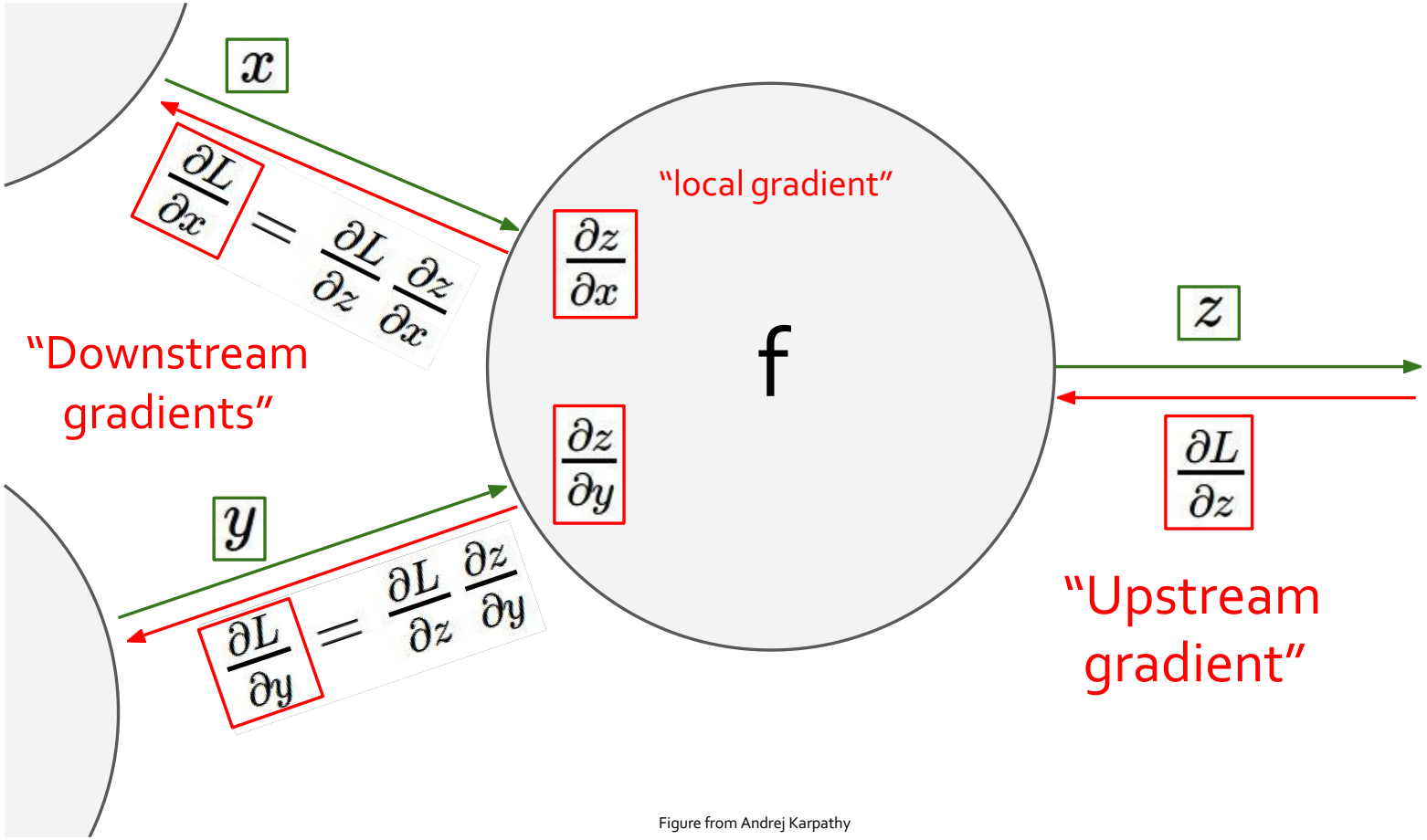

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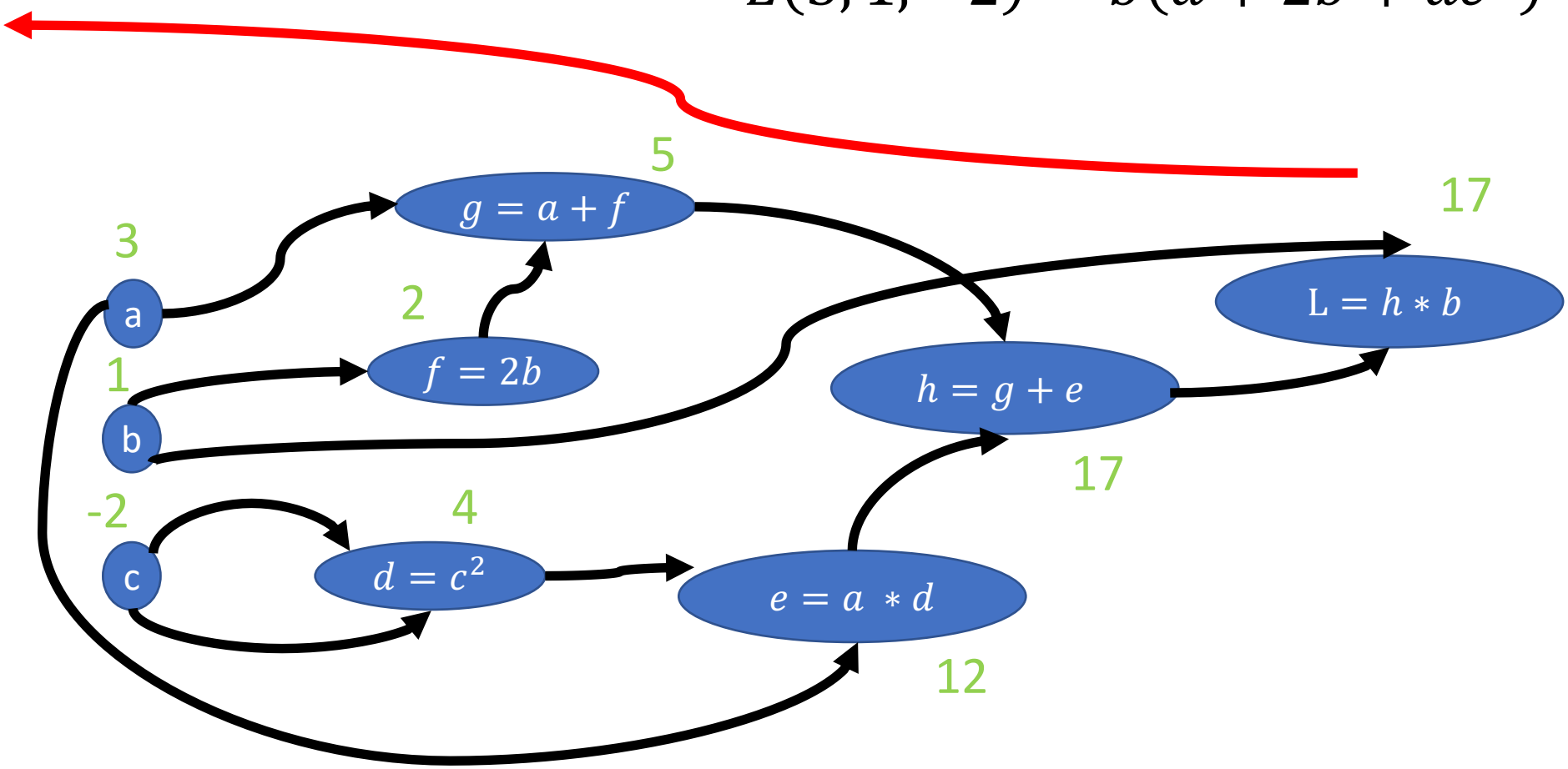
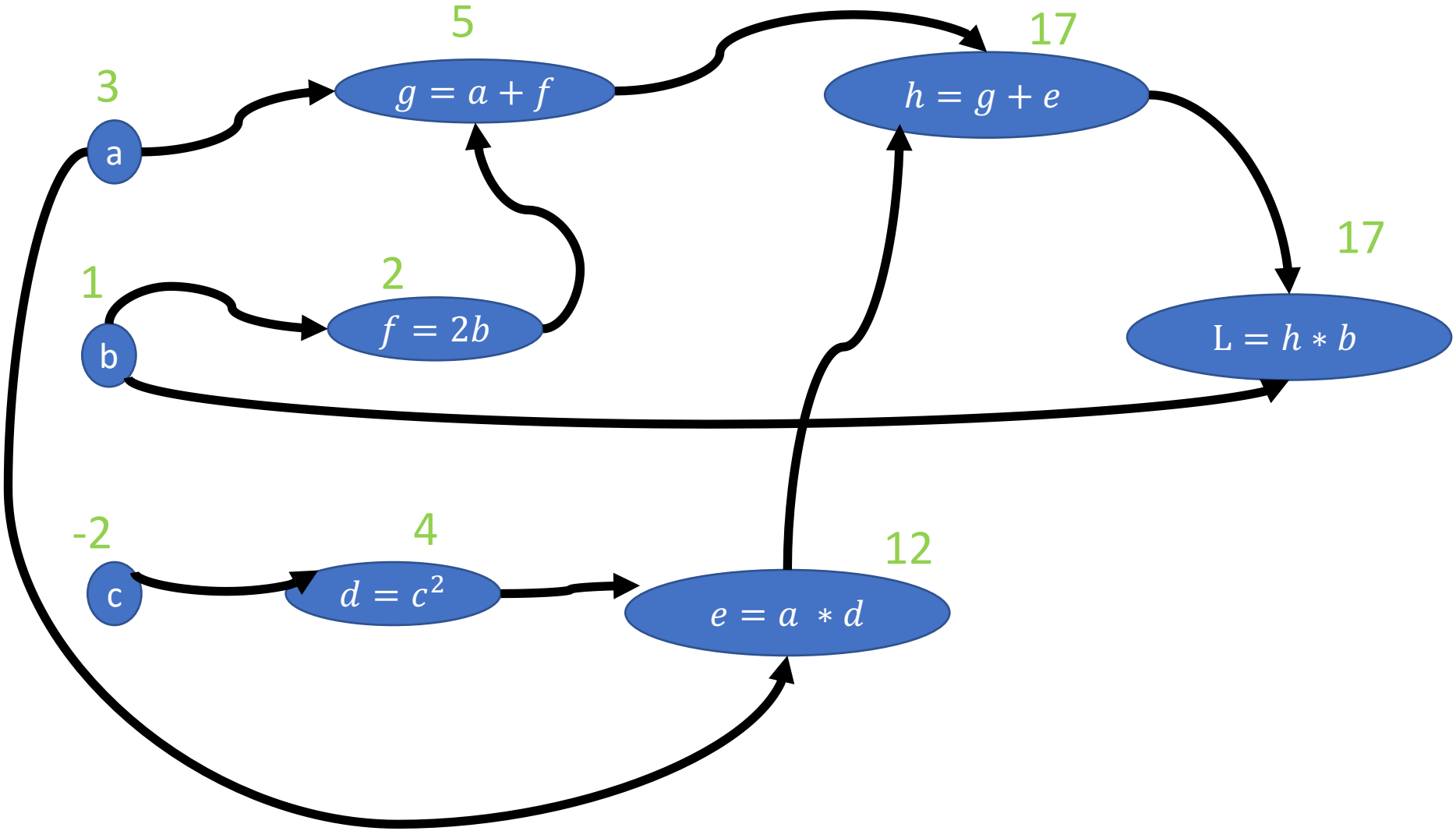


backward pass



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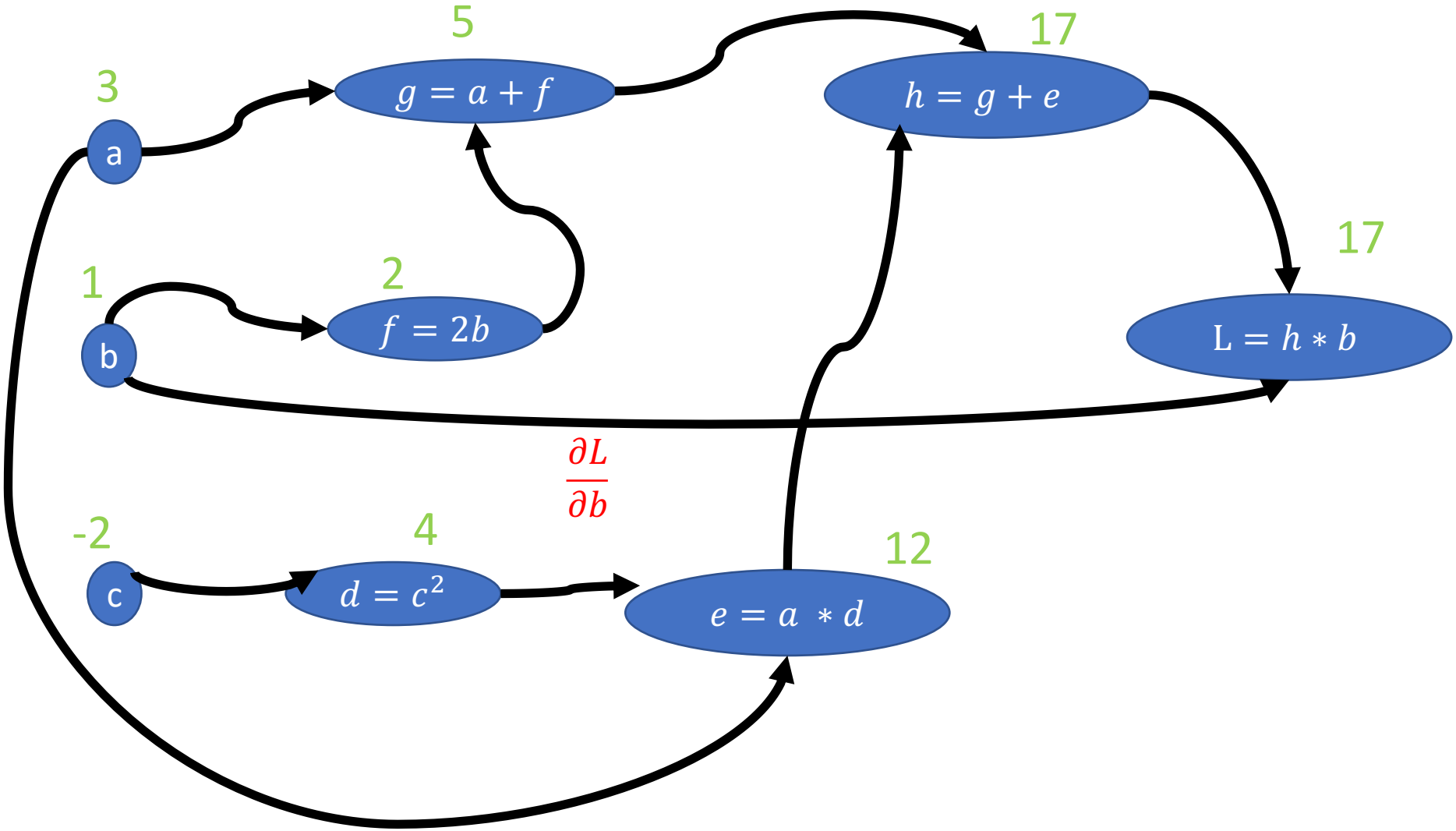
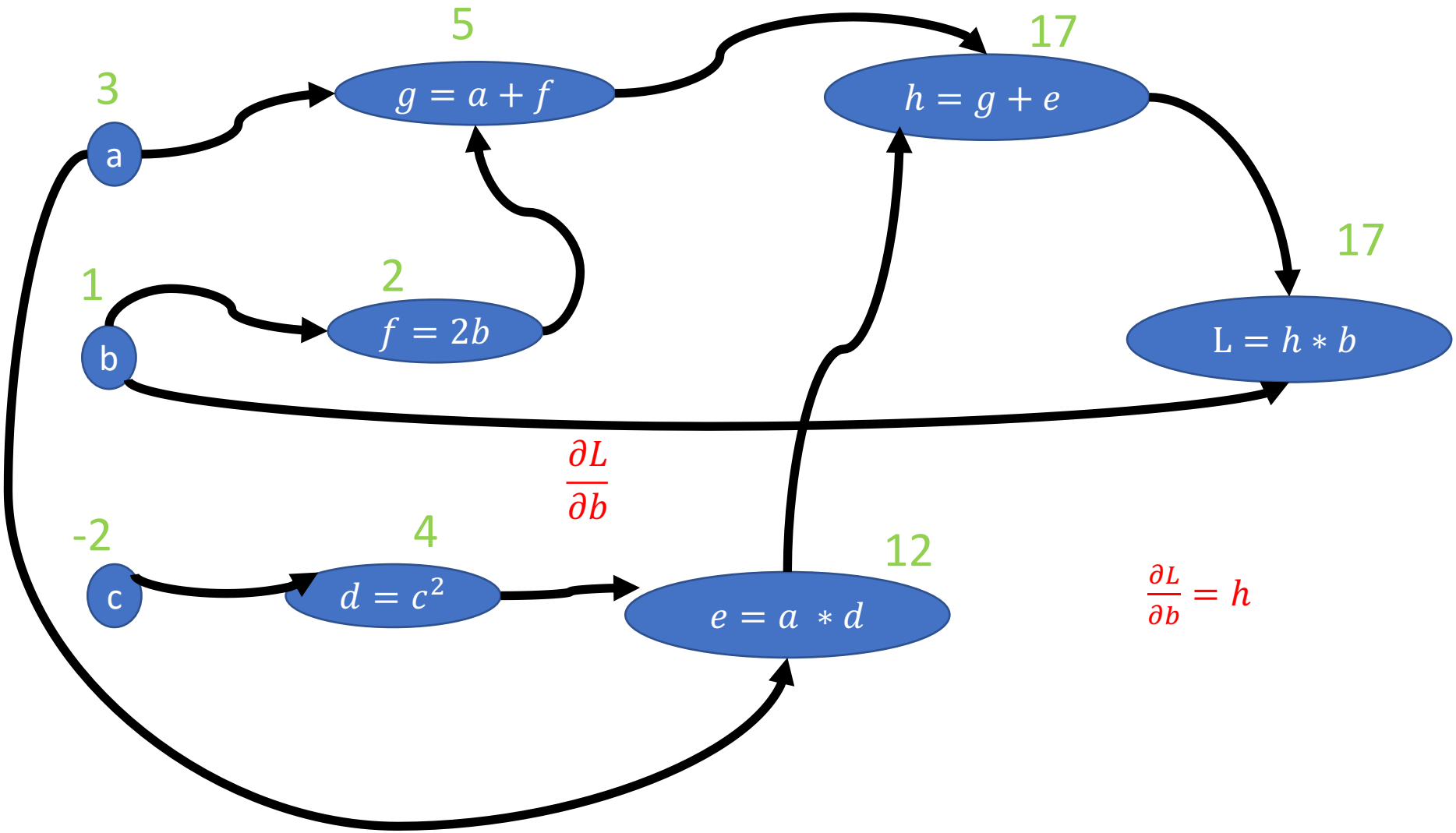
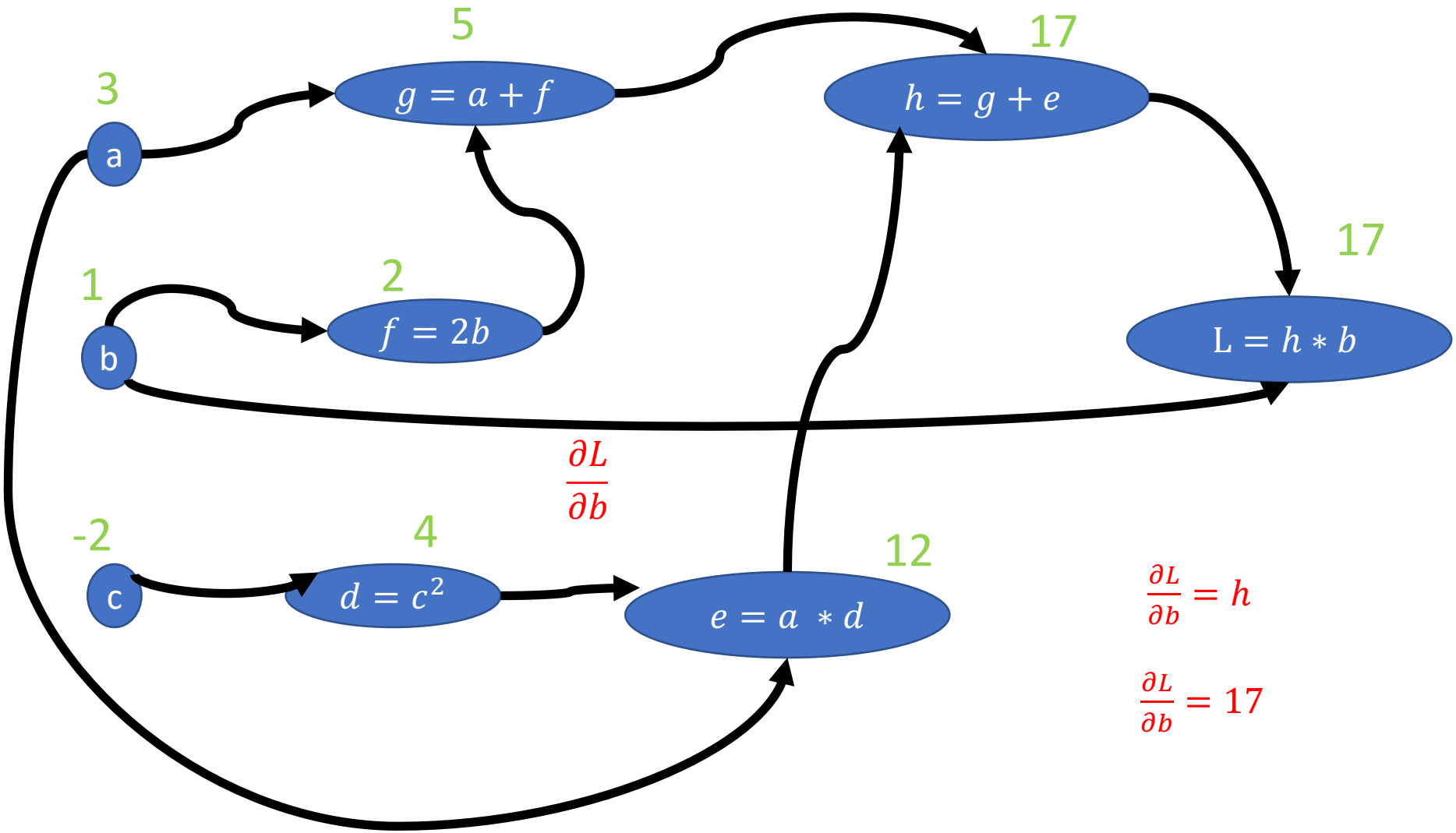


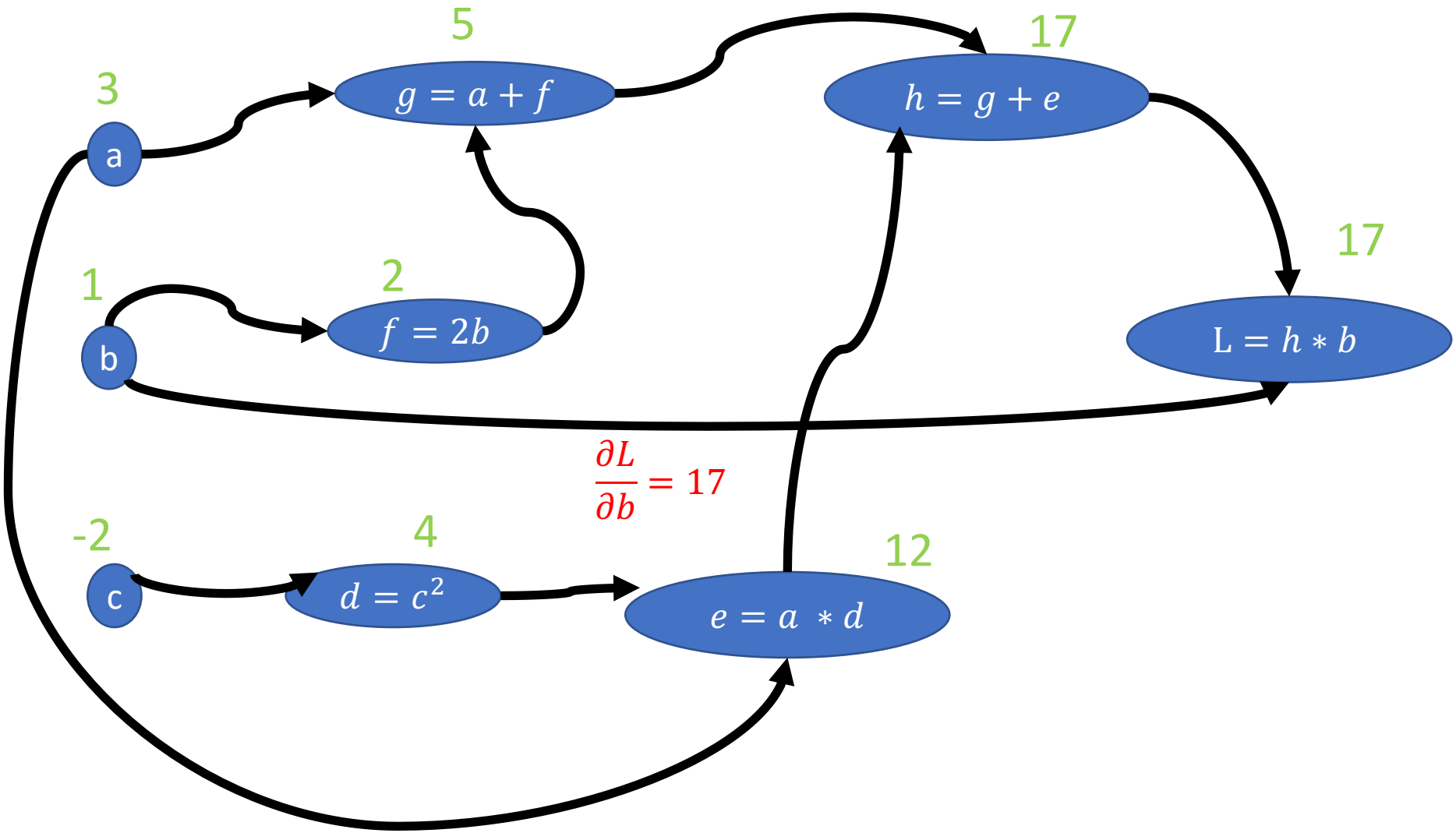
backward pass



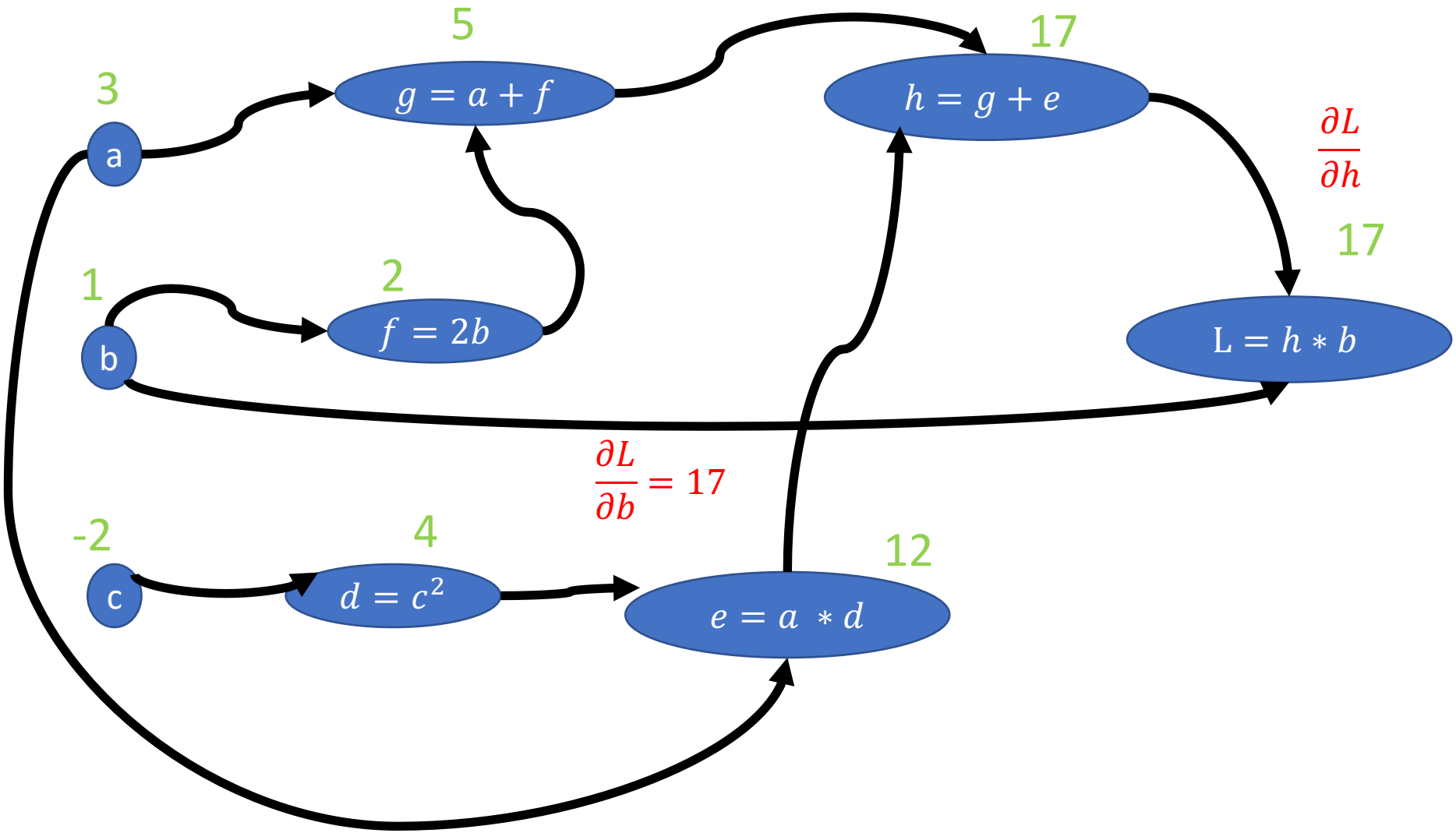
backward pass



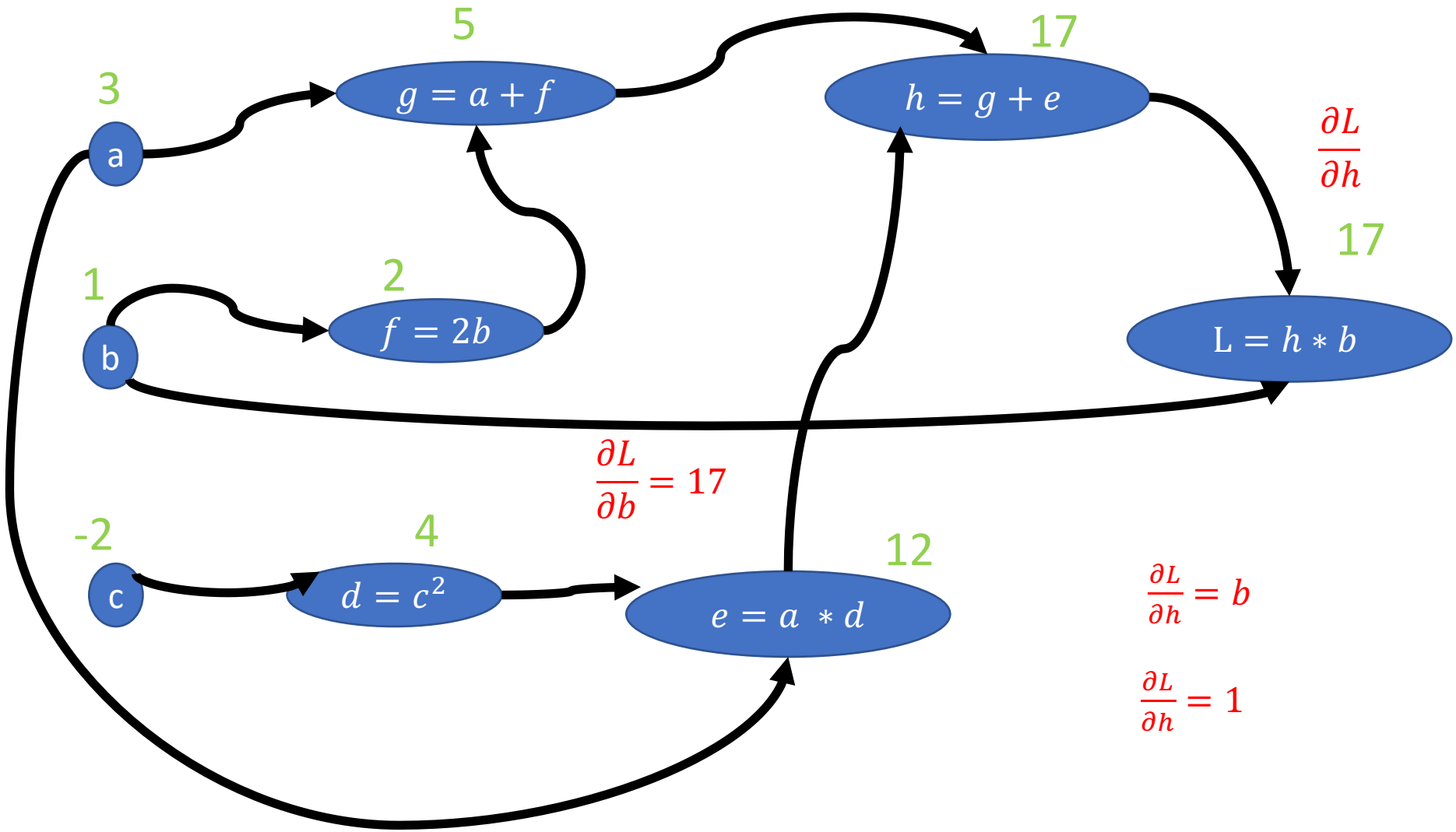
backward pass



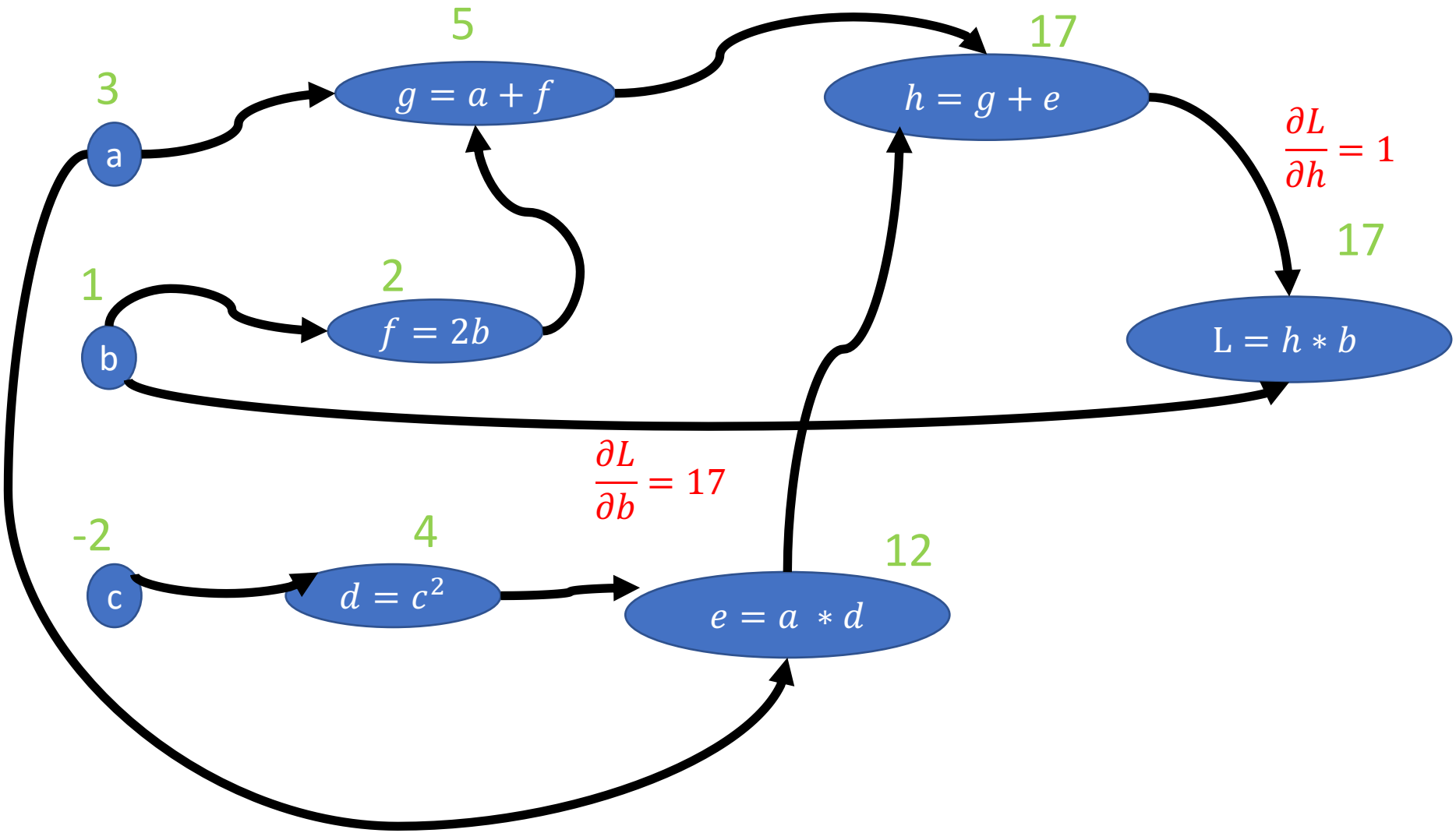
backward pass

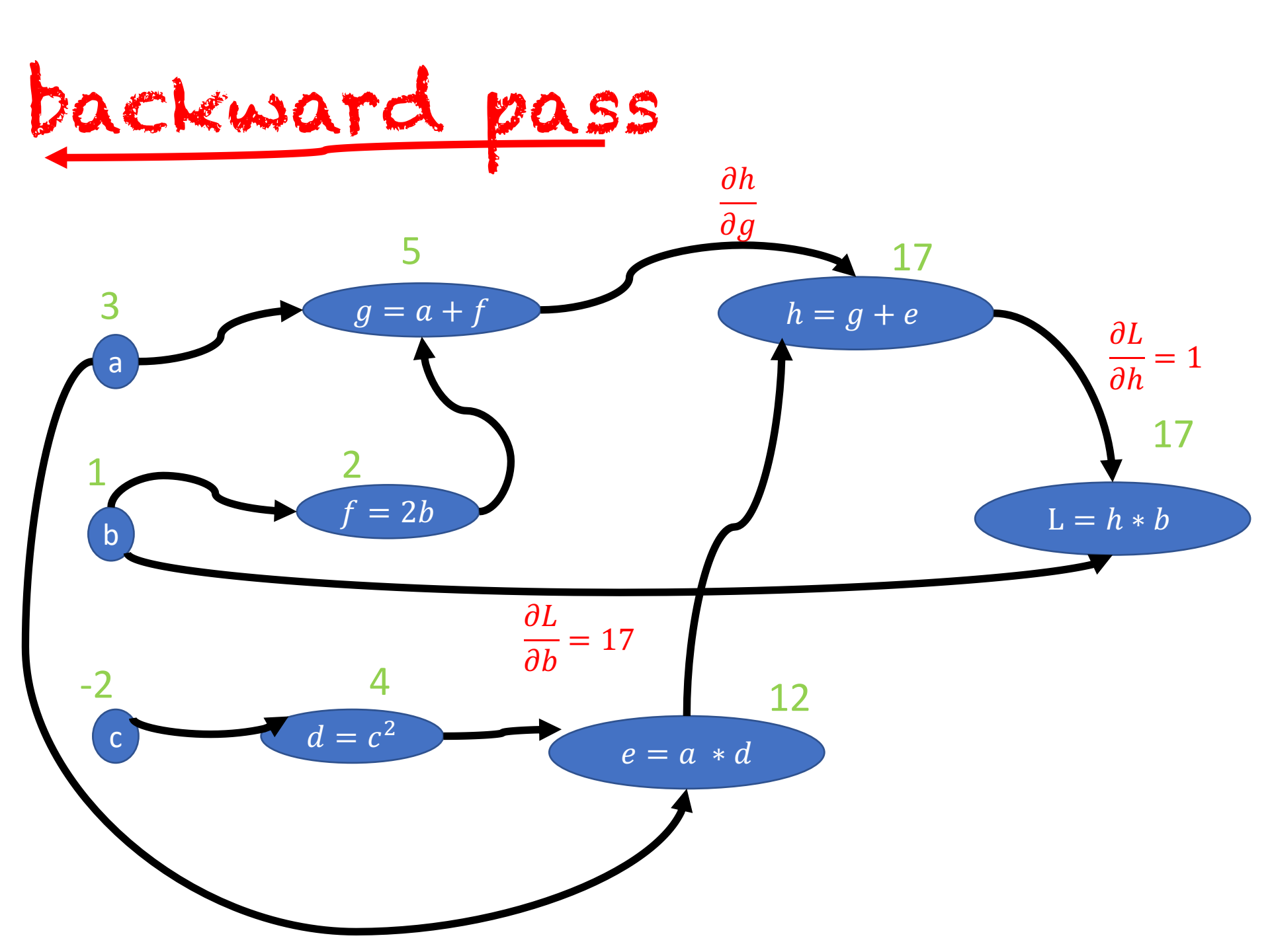


backward pass

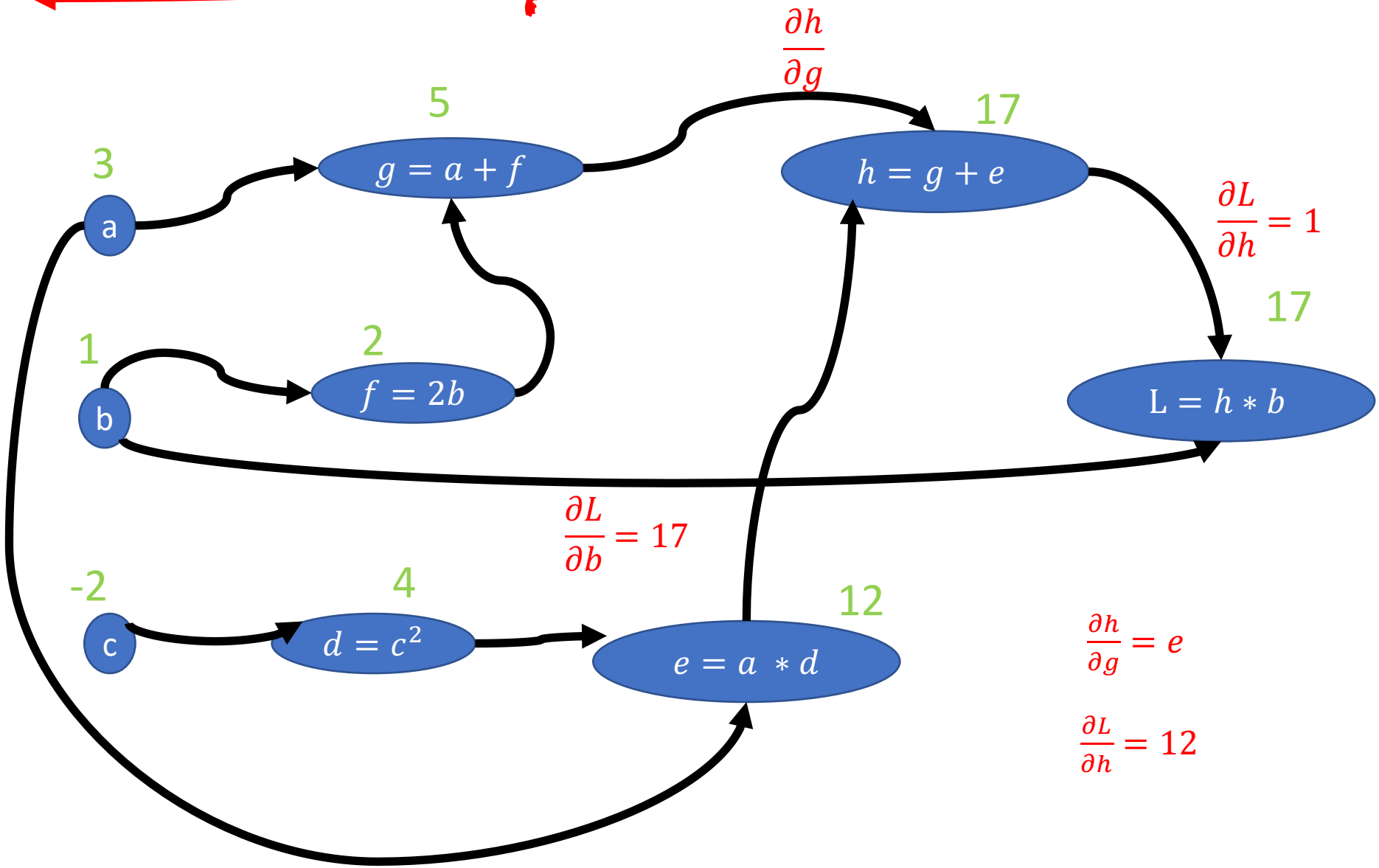


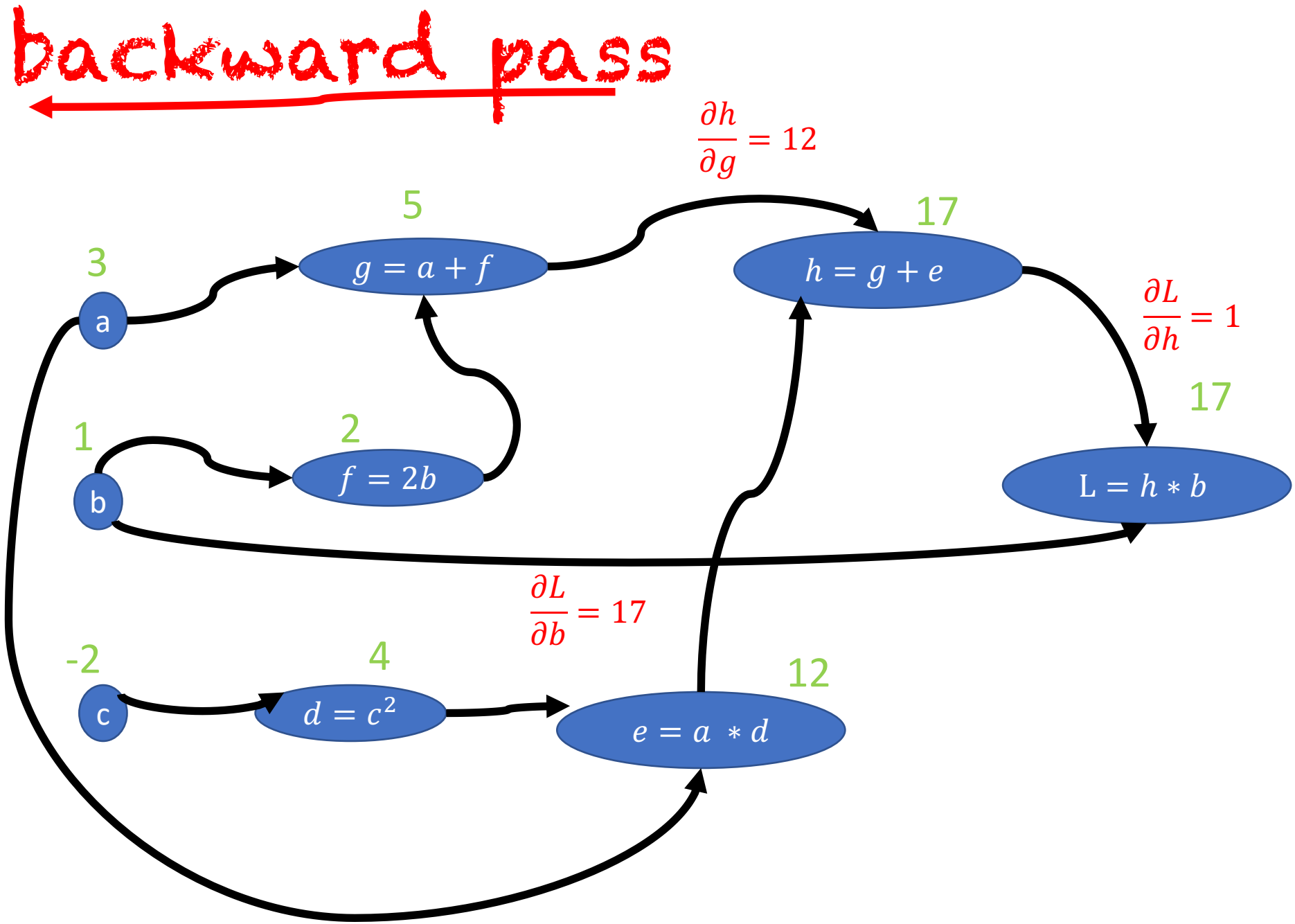
backward pass

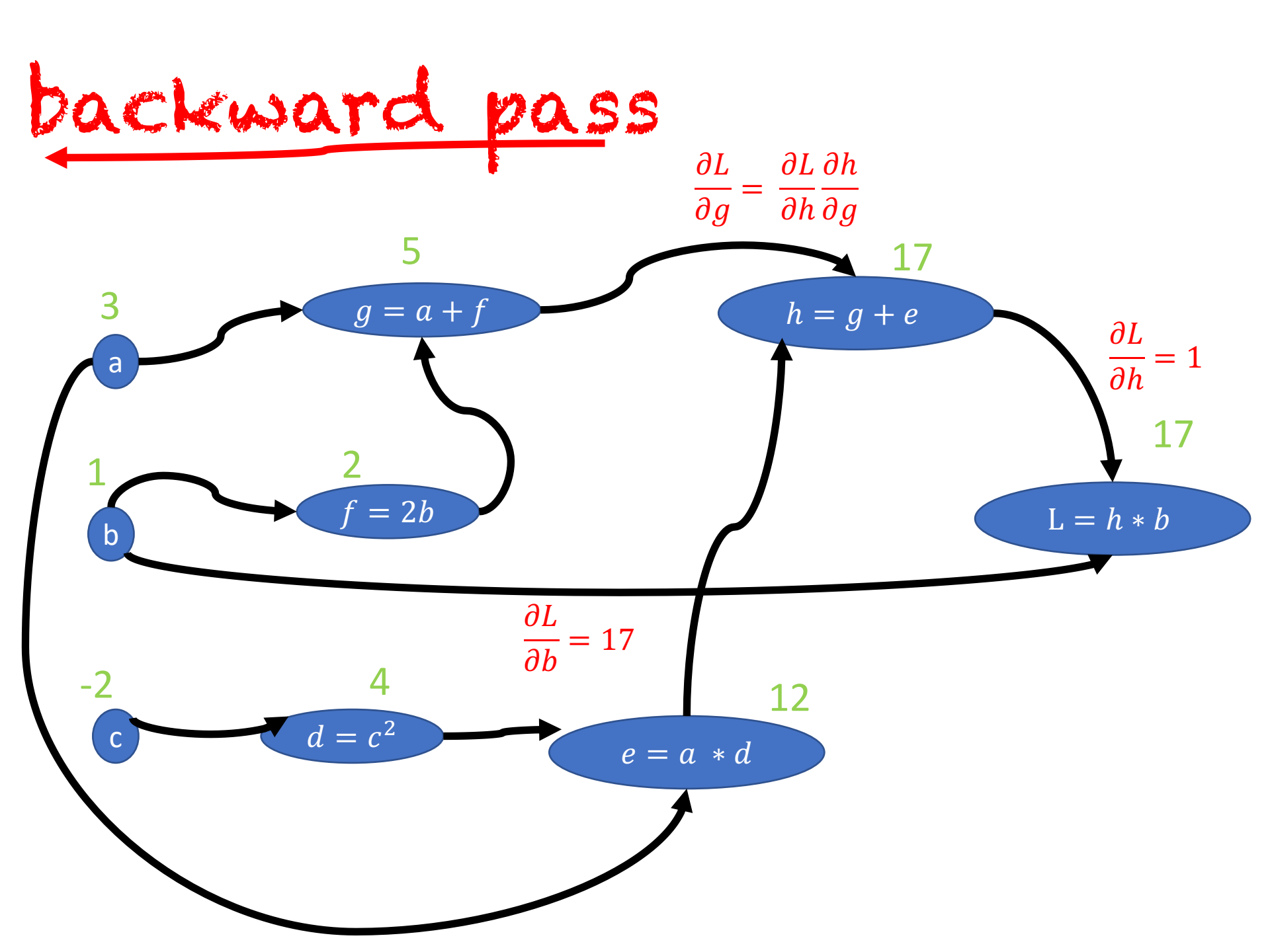


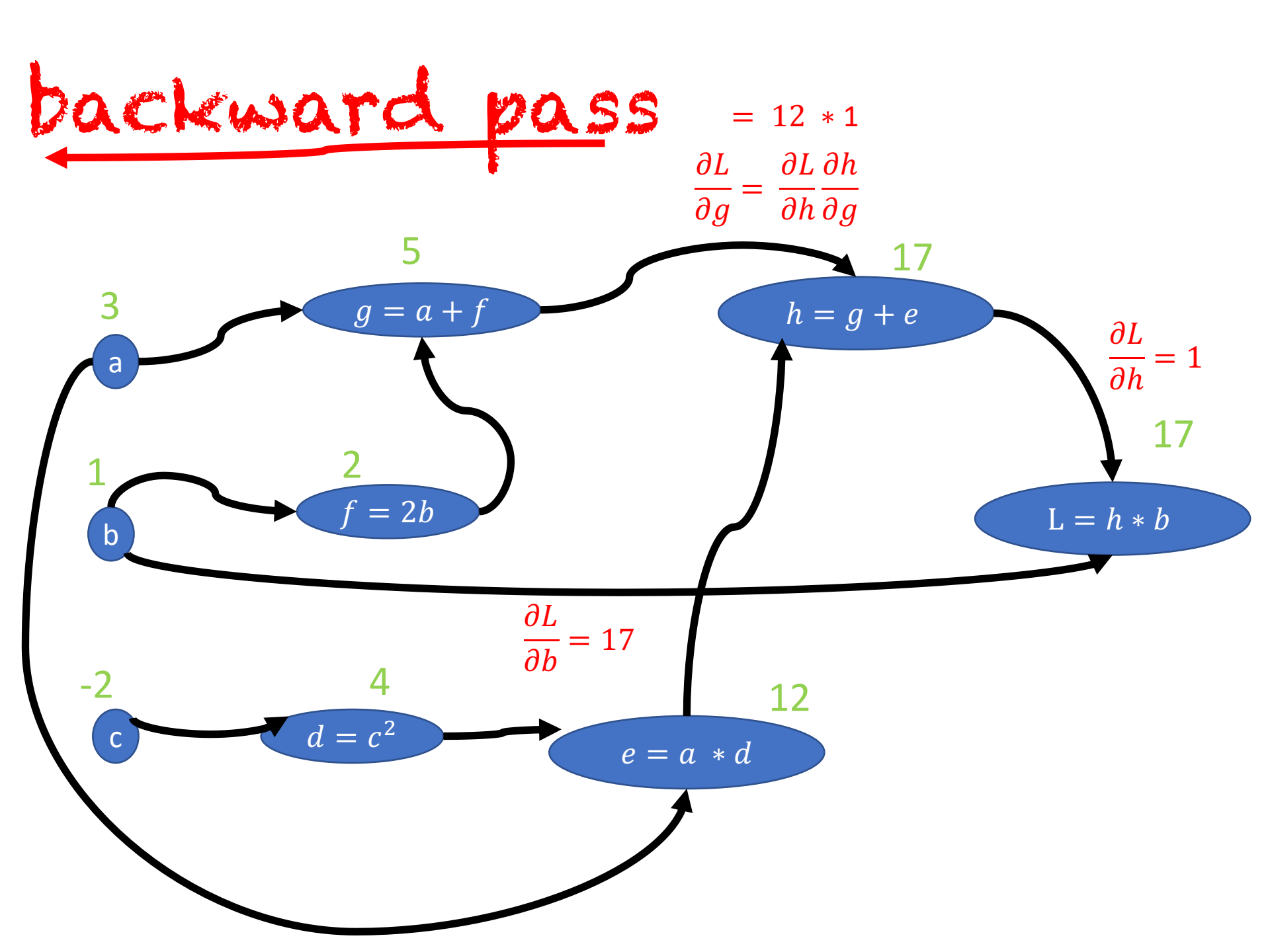

backward pass
$\frac{\partial h}{\partial g}$
5
17
3
$g = a + f$
$h = g + e$
$\frac{\partial L}{\partial h} = 1$
a
17
1
2
$f = 2b$
$L = h * b$
b
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$

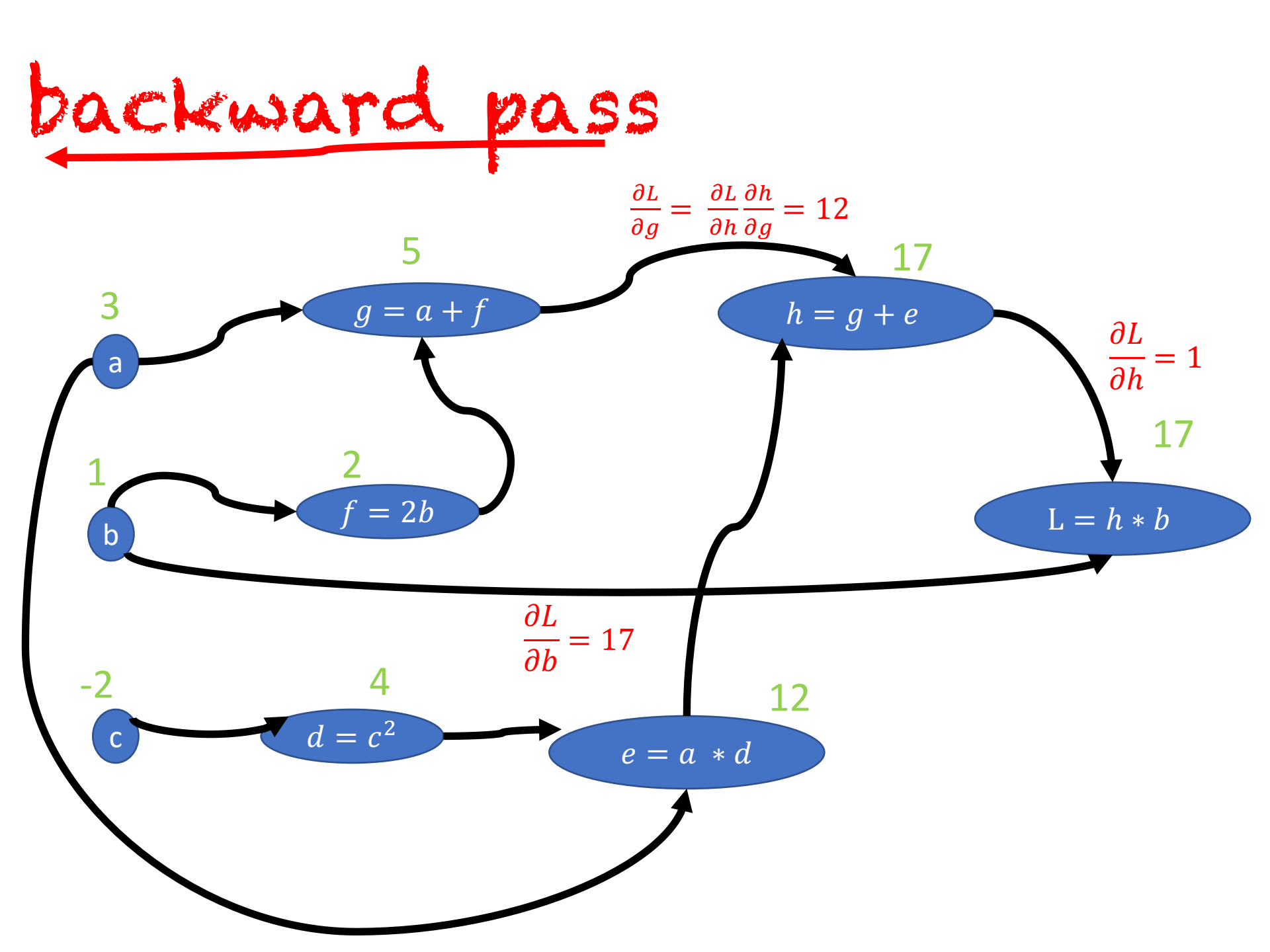
backward pass

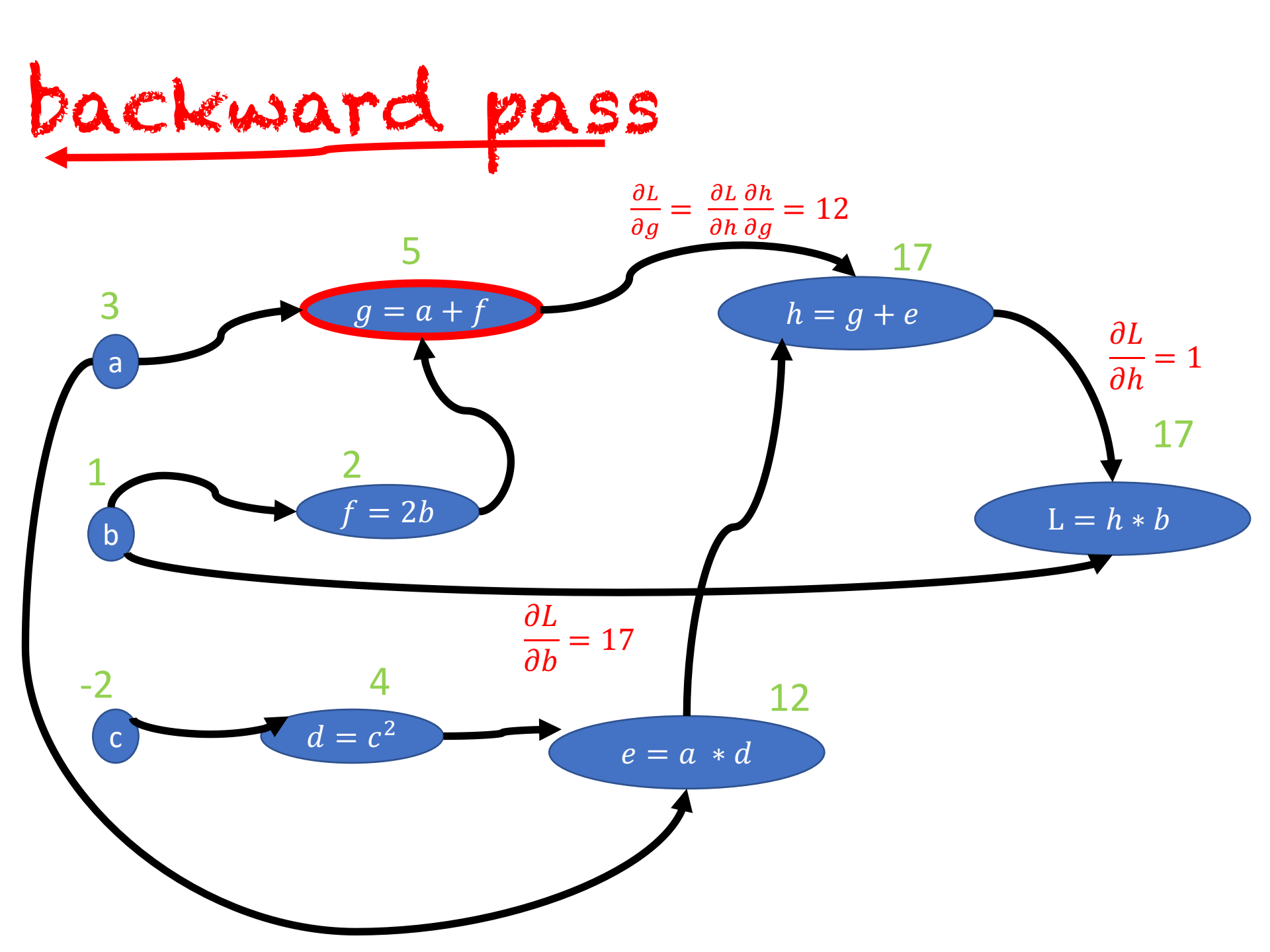


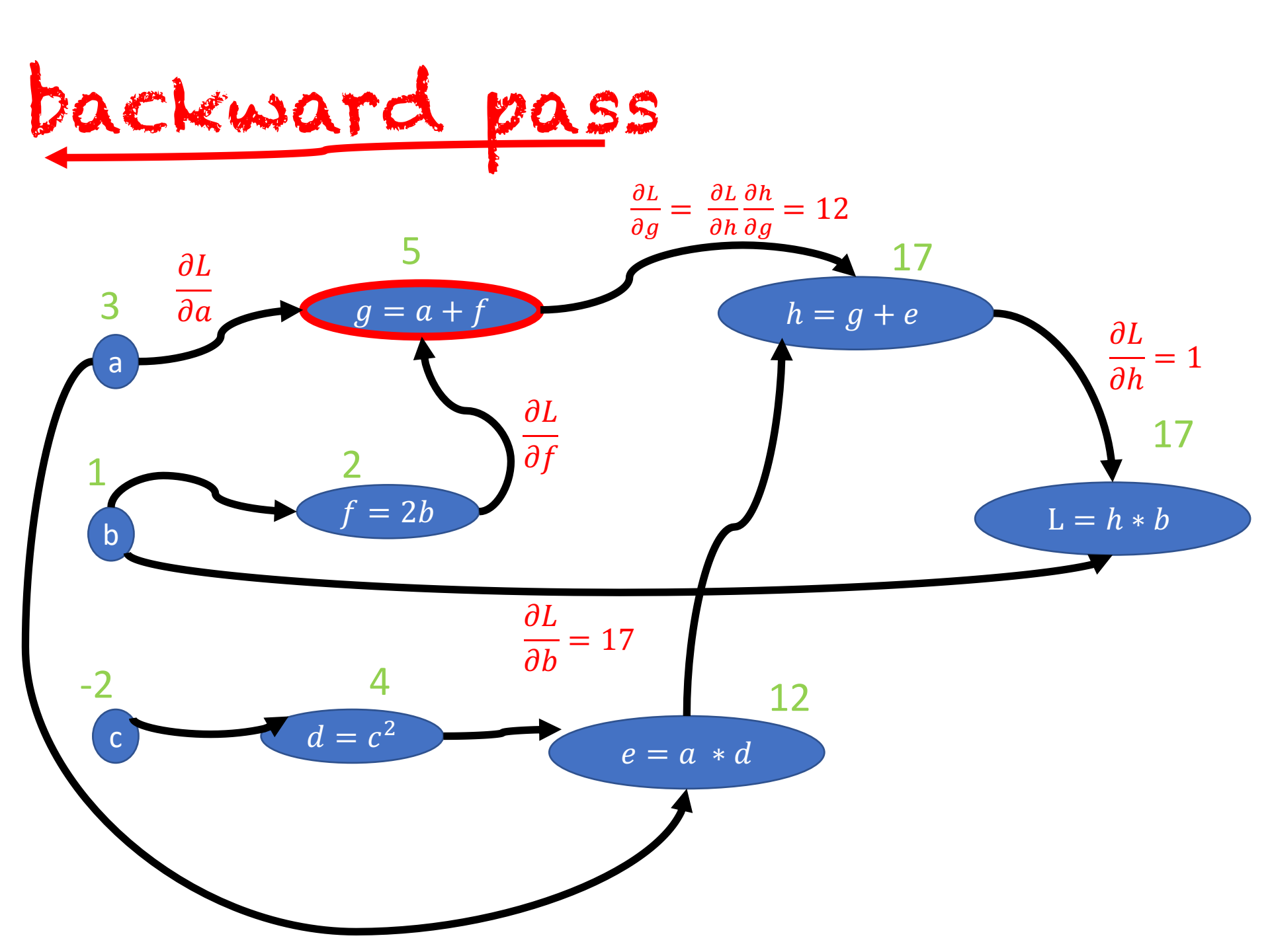

backward pass
$\frac{\partial h}{\partial g} = 12$
5
17
3
$g = a + f$
$h = g + e$
a
$\frac{\partial L}{\partial h} = 1$
17
1
2
$f = 2b$
$L = h * b$
b
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$

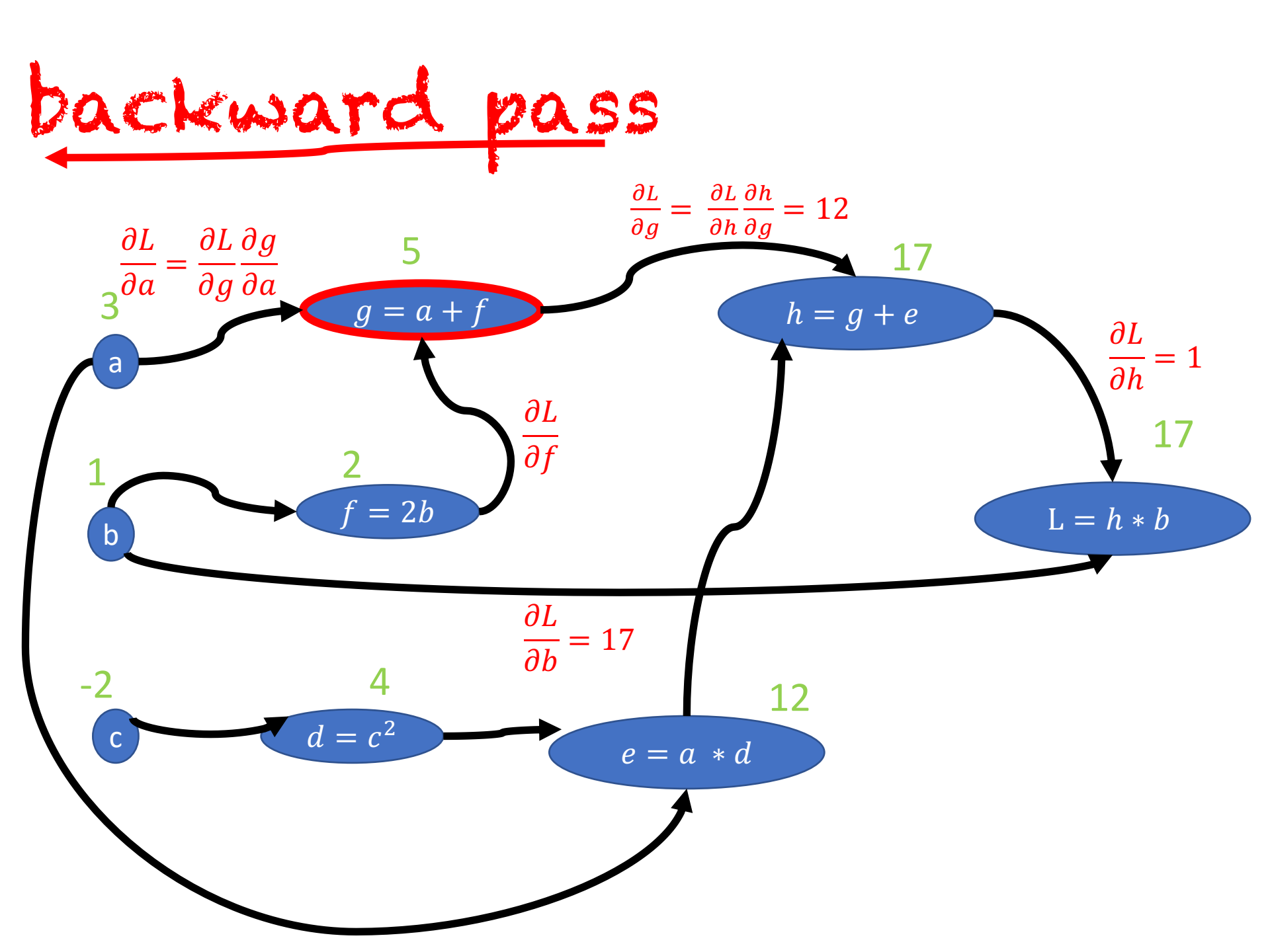

backward pass
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g}$
5
17
3
$g = a + f$
$h = g + e$
a
$\frac{\partial L}{\partial h} = 1$
17
1
2
$f = 2b$
$L = h * b$
b
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$


backward pass
$= 12 * 1$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g}$
5
17
3
$g = a + f$
$h = g + e$
$\frac{\partial L}{\partial h} = 1$
a
17
1
2
$f = 2b$
$L = h * b$
b
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
5
17
3
$g = a + f$
$h = g + e$
a
$\frac{\partial L}{\partial h} = 1$
17
1
2
$f = 2b$
b
$L = h * b$
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
5
17
3
$g = a + f$
$h = g + e$
$\frac{\partial L}{\partial h} = 1$
17
1
2
$f = 2b$
$L = h * b$
b
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
5
17
$\frac{\partial L}{\partial a}$
3
a
$g = a + f$
$h = g + e$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial f}$
17
1
2
$f = 2b$
b
$L = h * b$
$\frac{\partial L}{\partial b} = 17$
-2
4
12
c
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a}$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial f}$
$\frac{\partial L}{\partial b} = 17$
3
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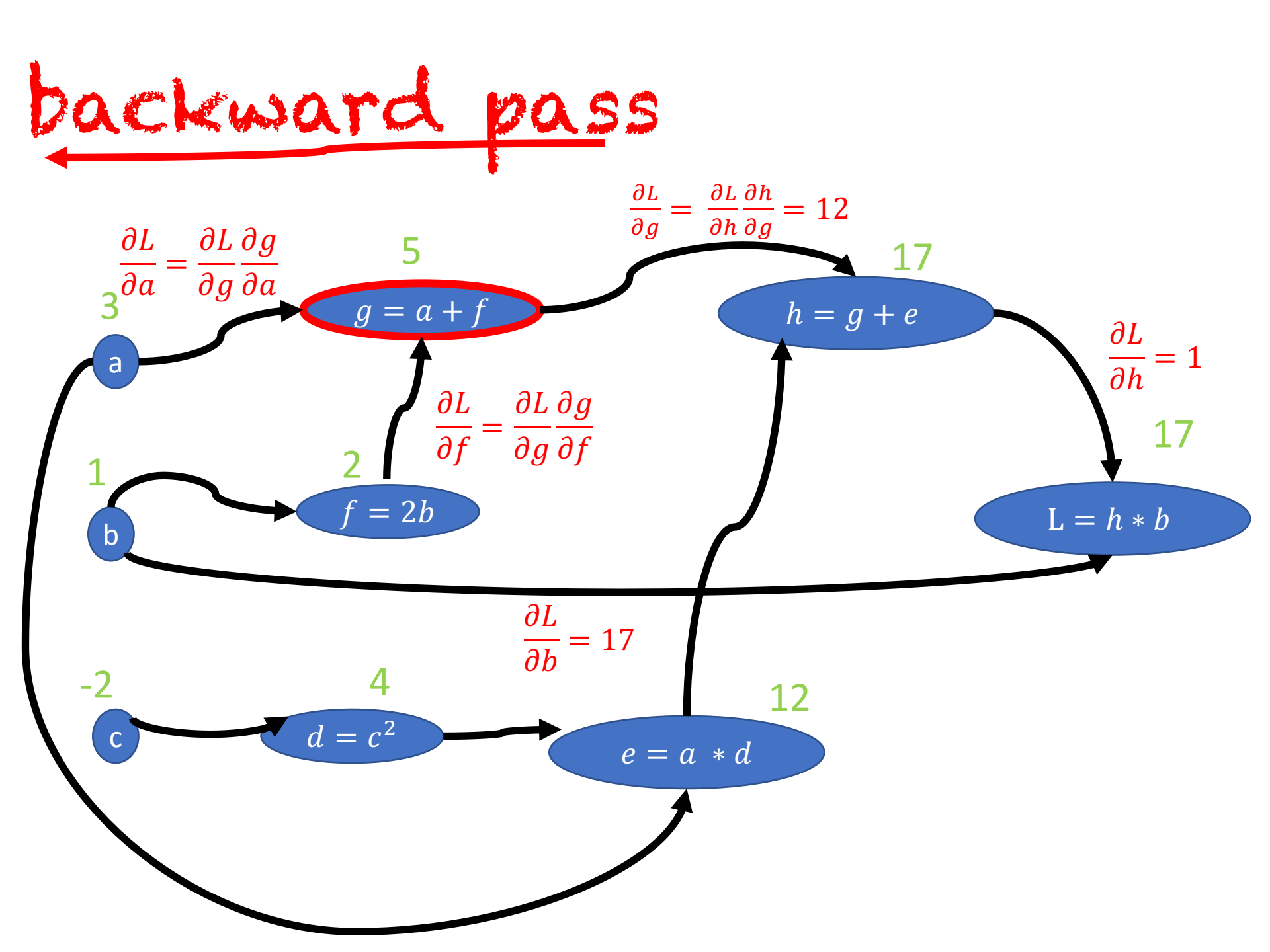
17
1
2
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$d = c^2$
$e = a * d$
$L = h * b$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a}$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f}$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
3
5
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17
1
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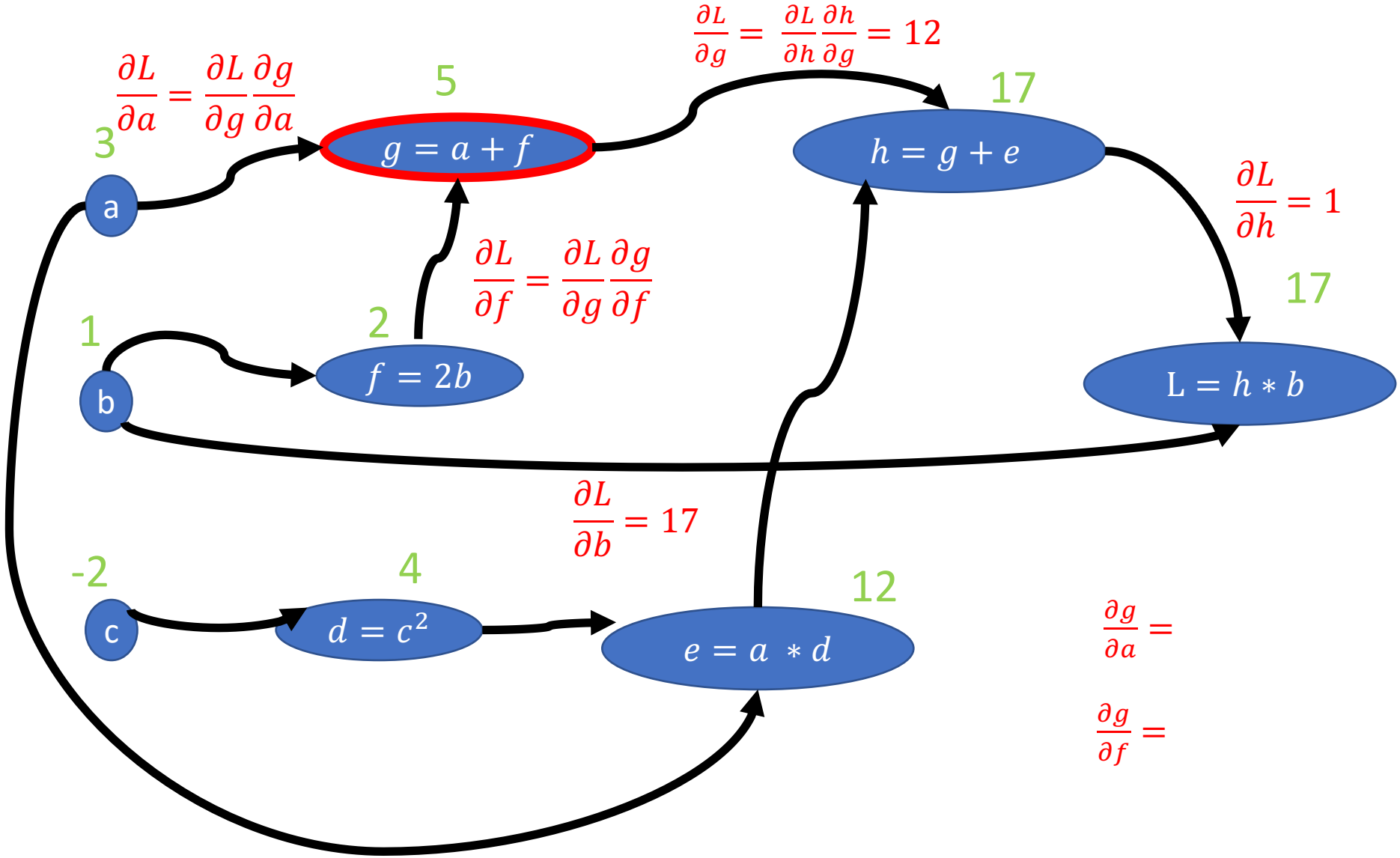
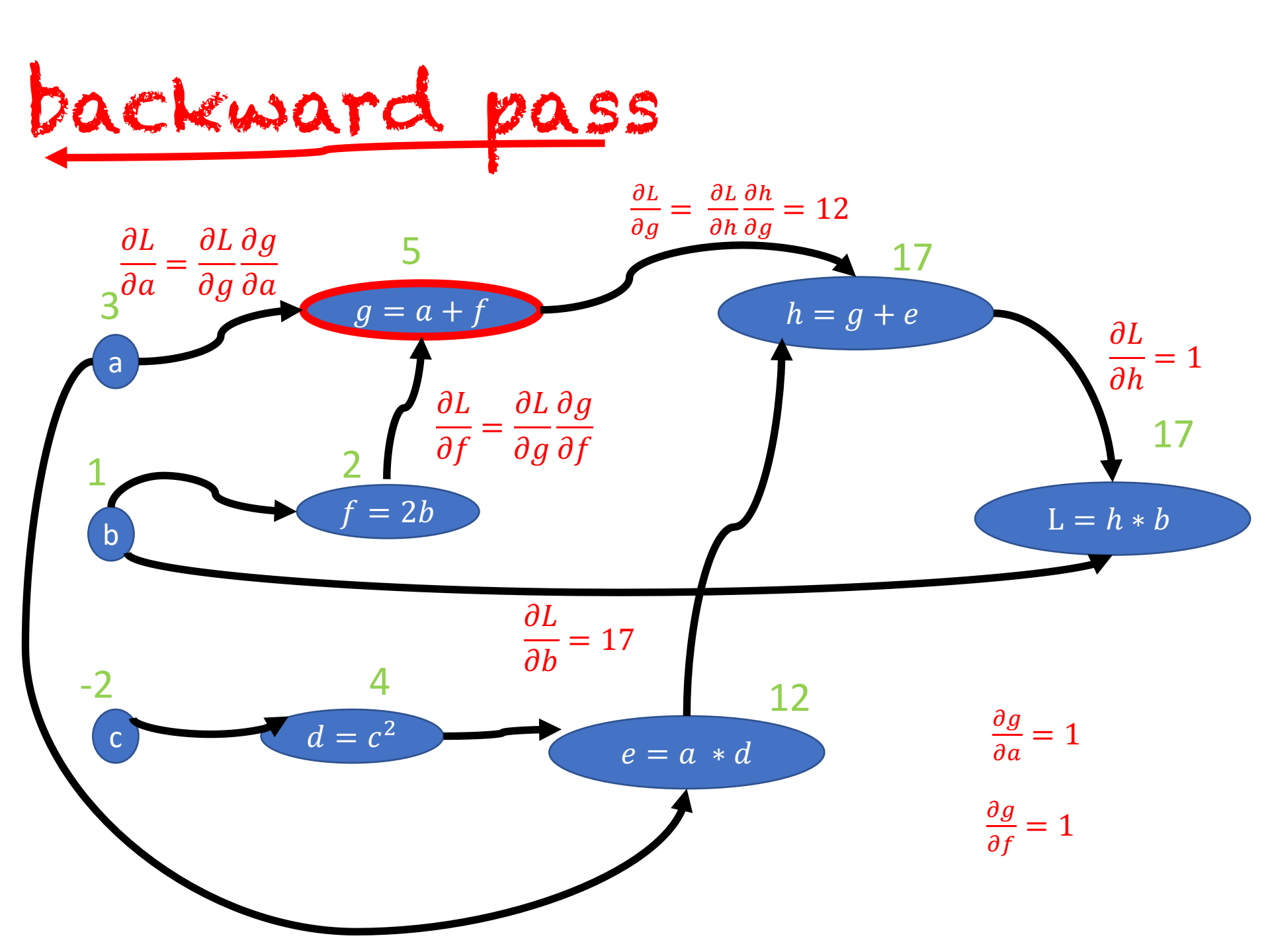
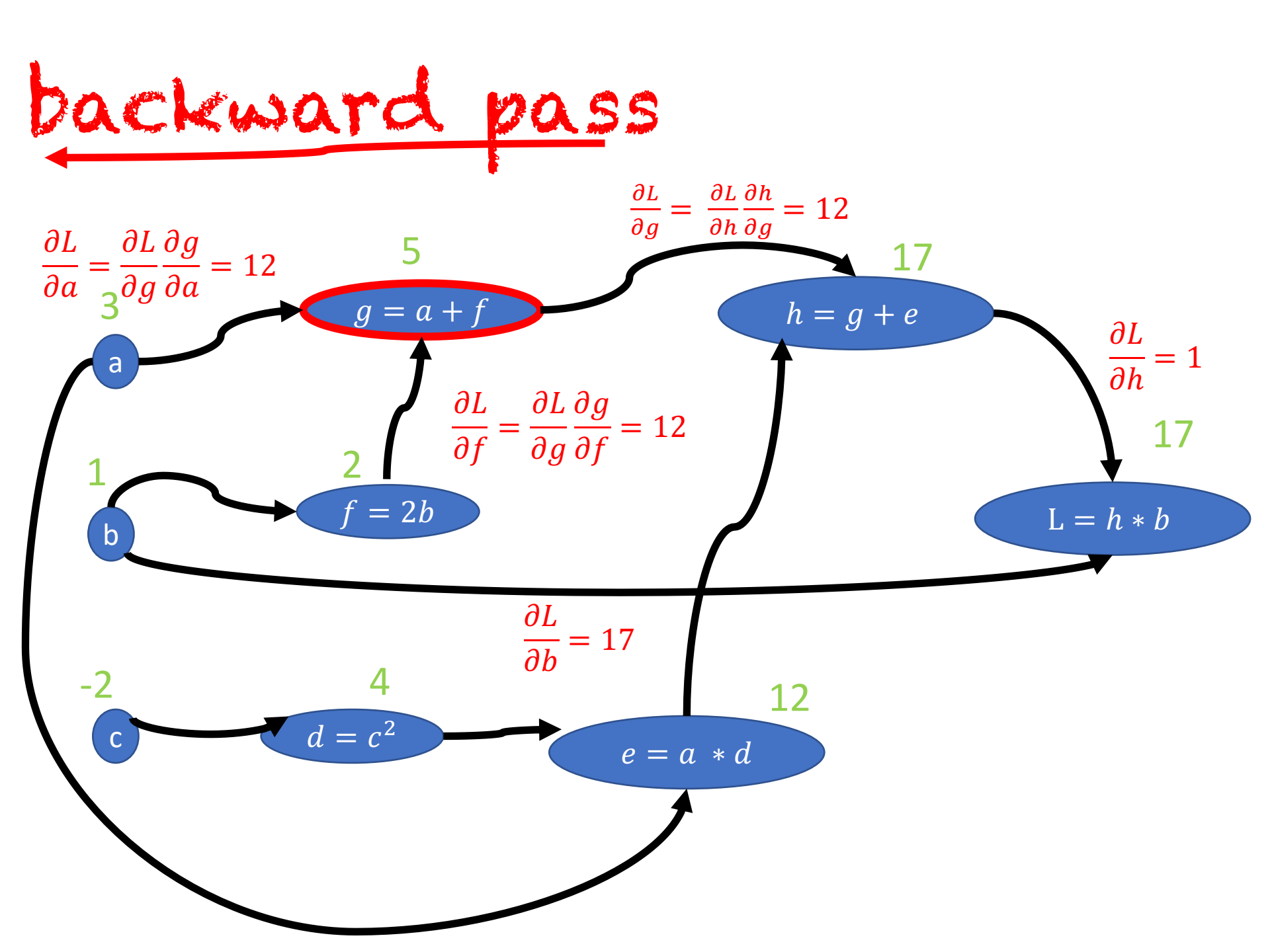
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$\mathrm{L} = h * b$
$d = c^2$
$e = a * d$

backward pass




backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a}$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f}$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial g}{\partial a} = 1$
$\frac{\partial g}{\partial f} = 1$
3
5
17
17
1
2
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
3
5
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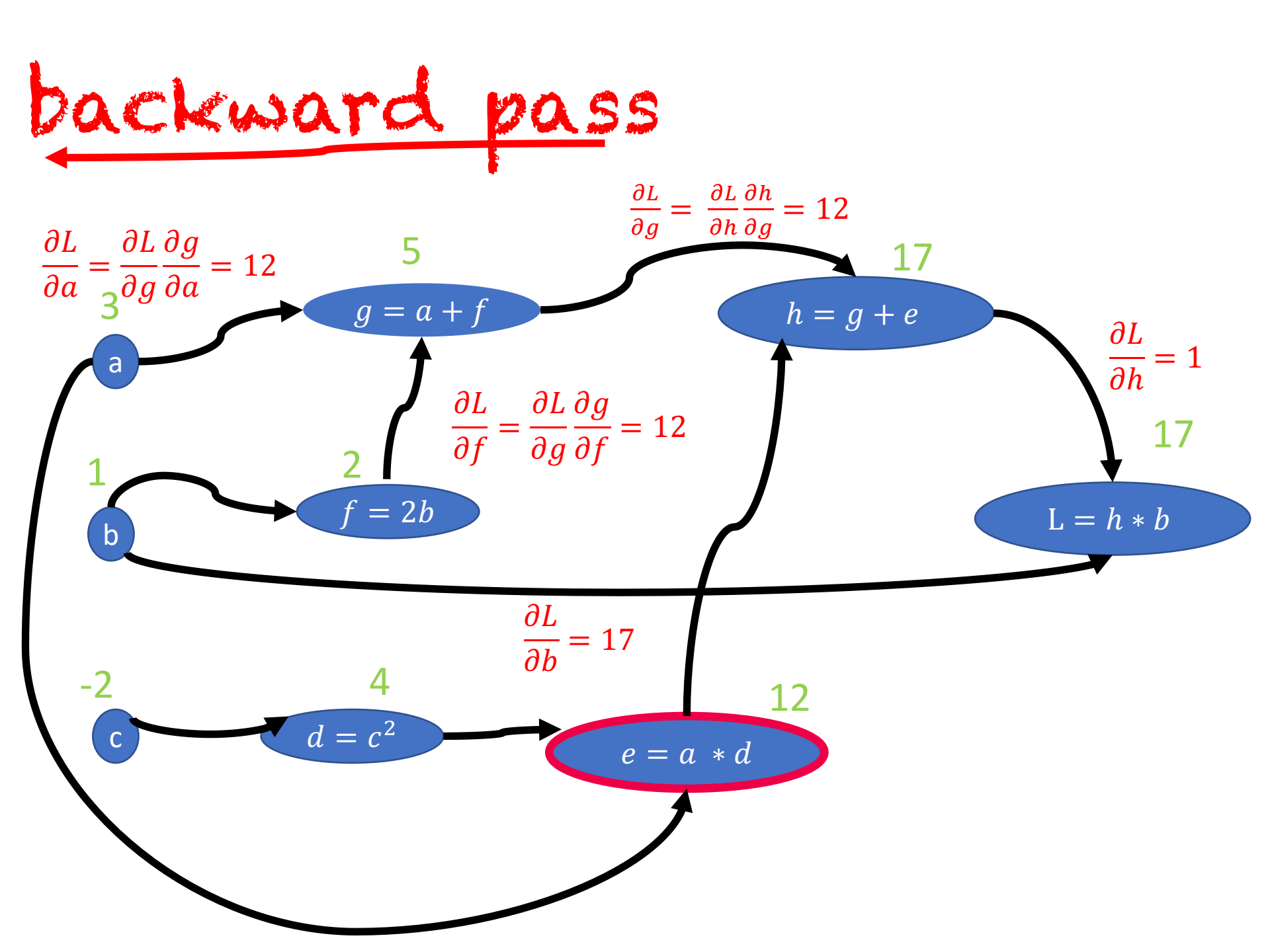
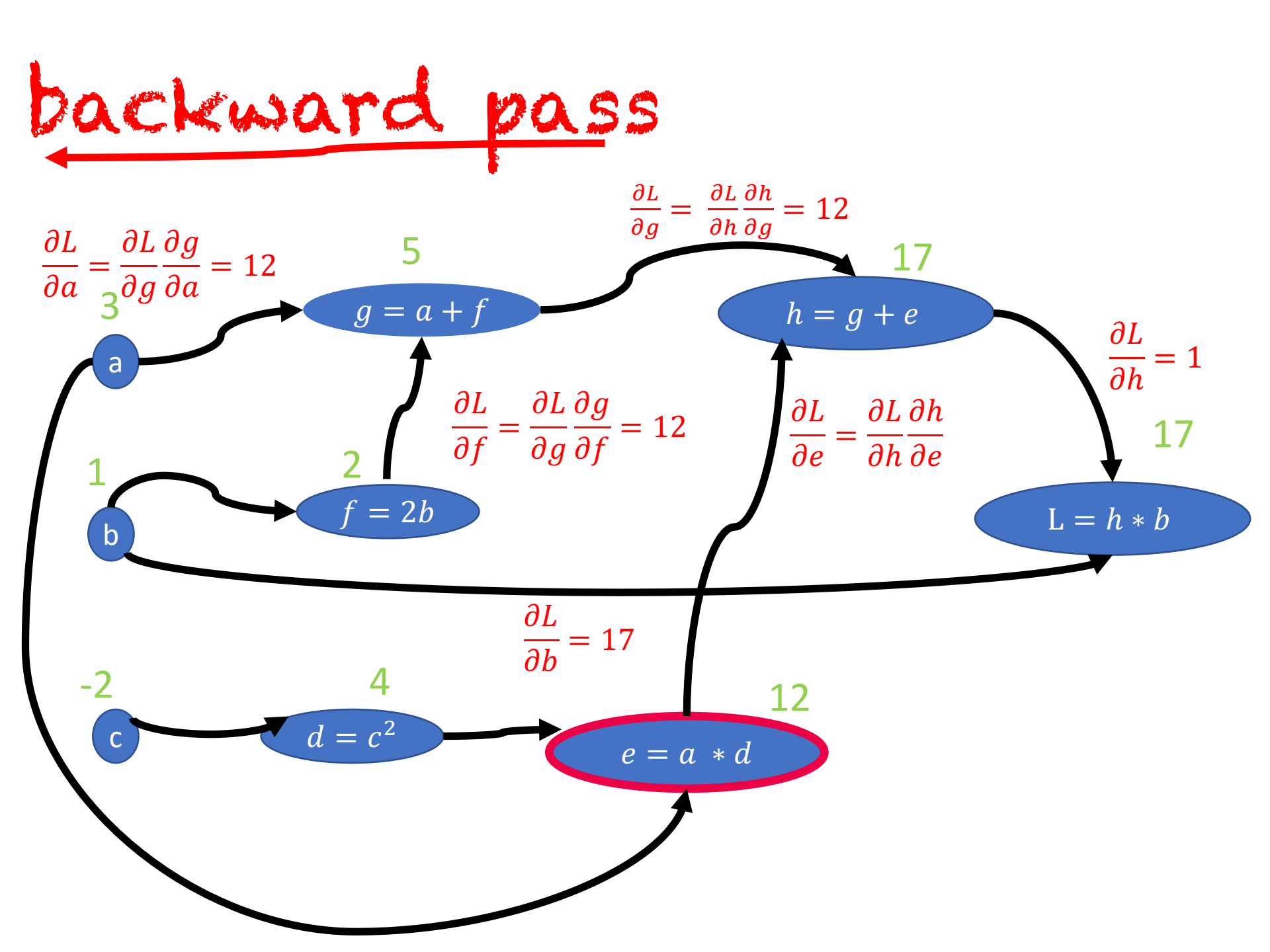
17
1
2
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
3
5
17
17
1
2
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial e}$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
3
5
17
17
1
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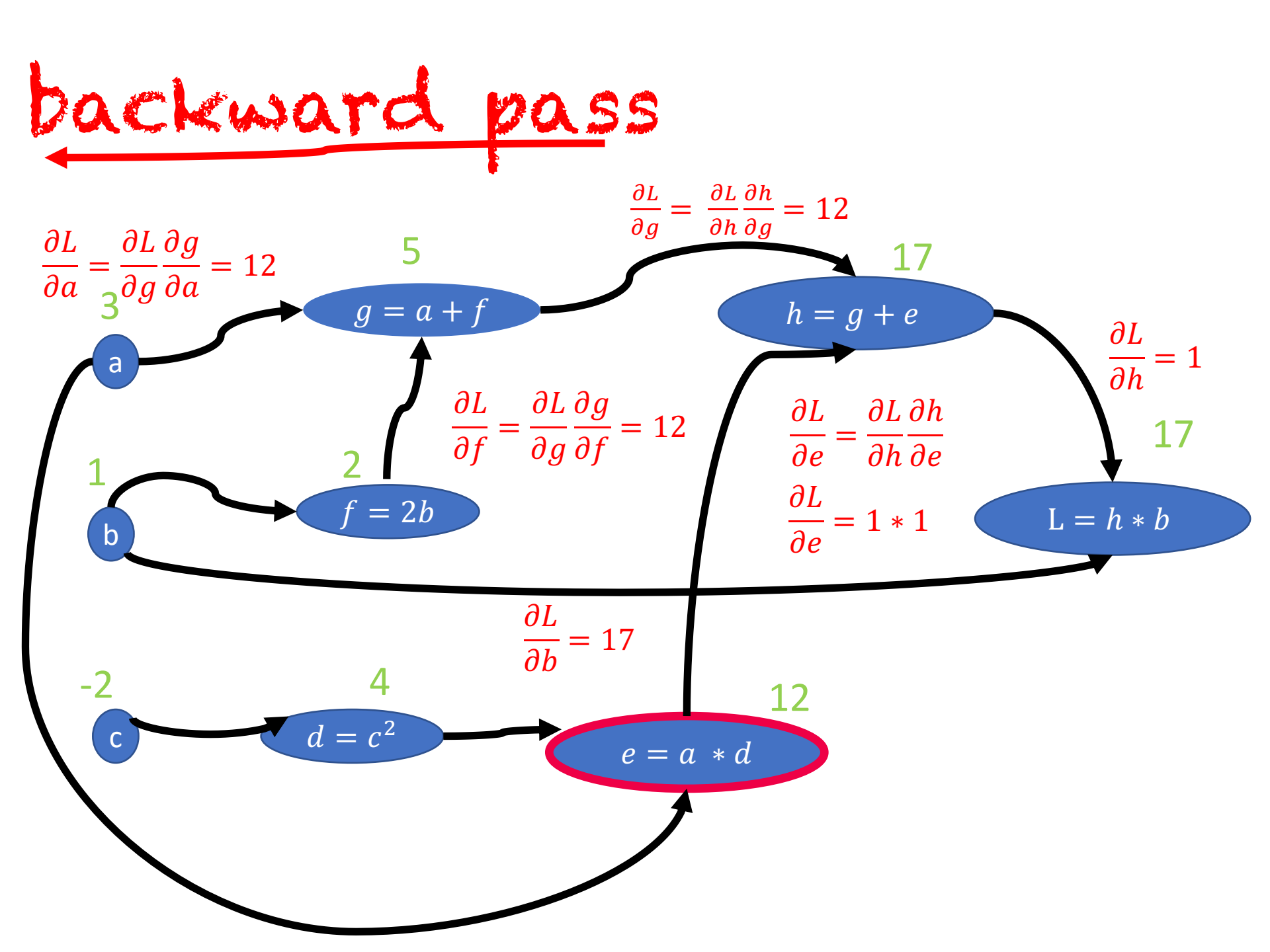
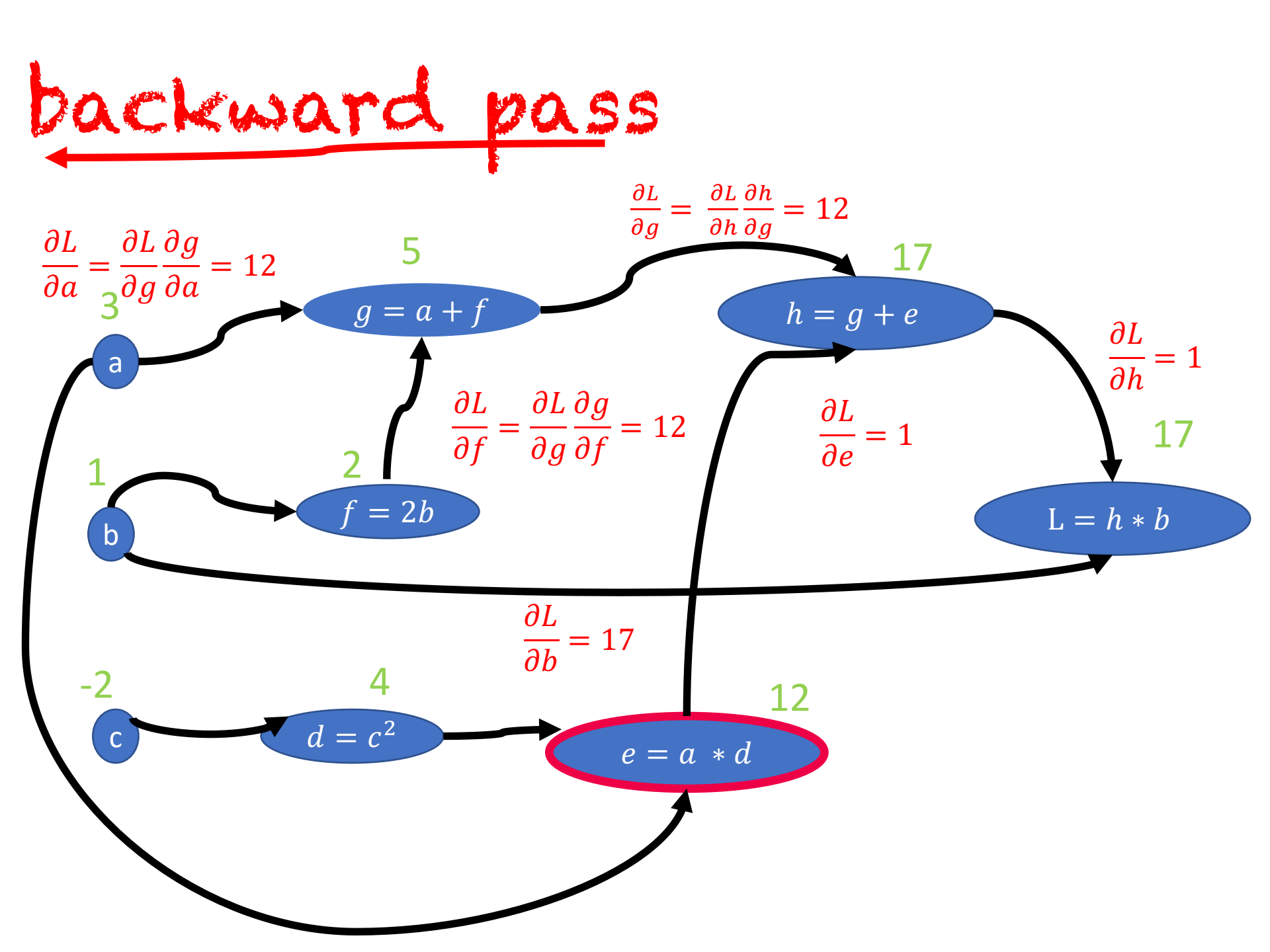
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial e} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial e}$
$\frac{\partial L}{\partial e} = 1 * 1$
$\frac{\partial L}{\partial b} = 17$
3
5
17
1
2
17
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$\mathrm{L} = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
3
5
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1
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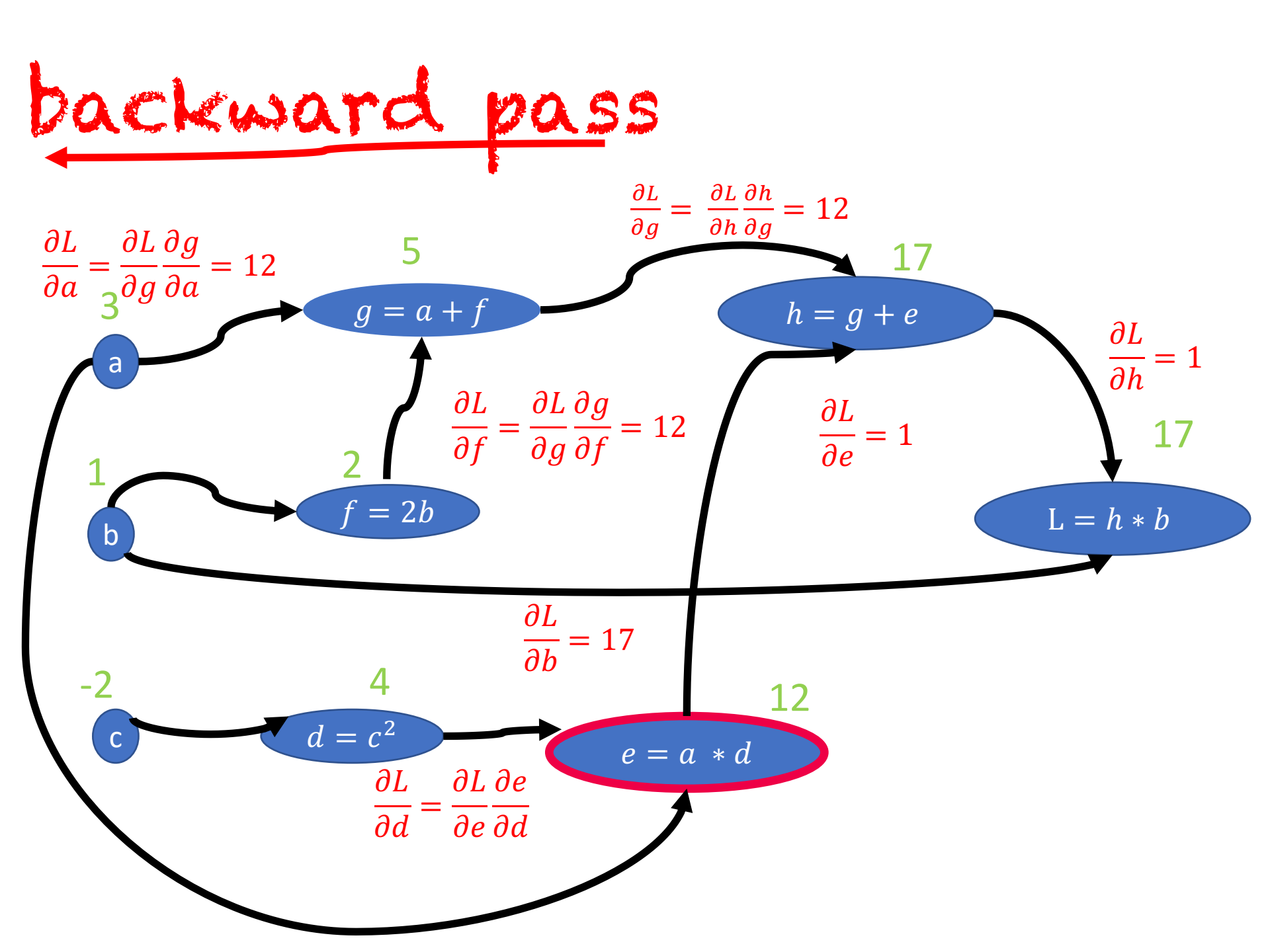
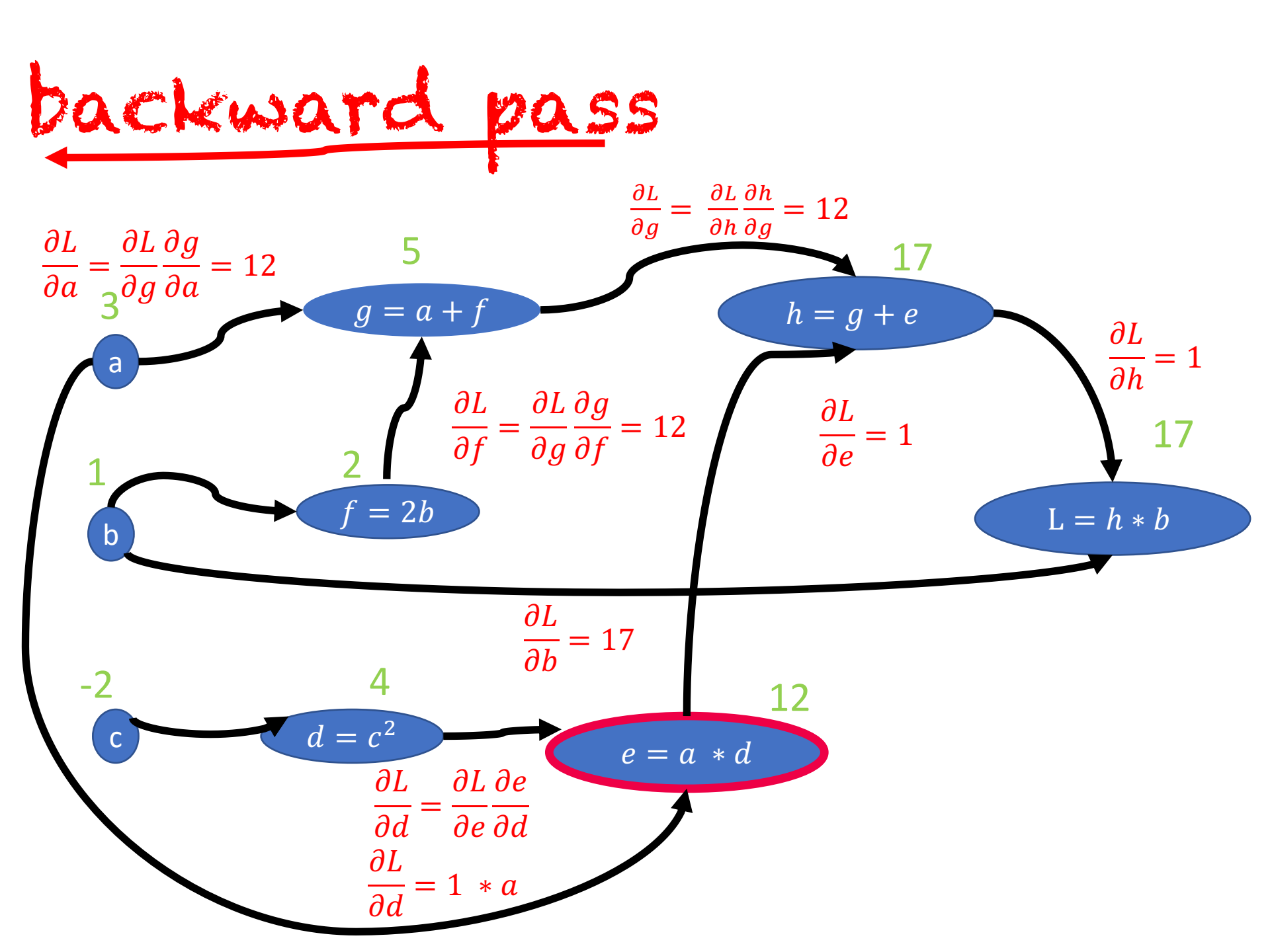
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial d} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}$
3
5
17
1
2
17
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial d} = \frac{\partial L}{\partial e}\frac{\partial e}{\partial d}$
$\frac{\partial L}{\partial d} = 1 * a$
3
5
17
1
2
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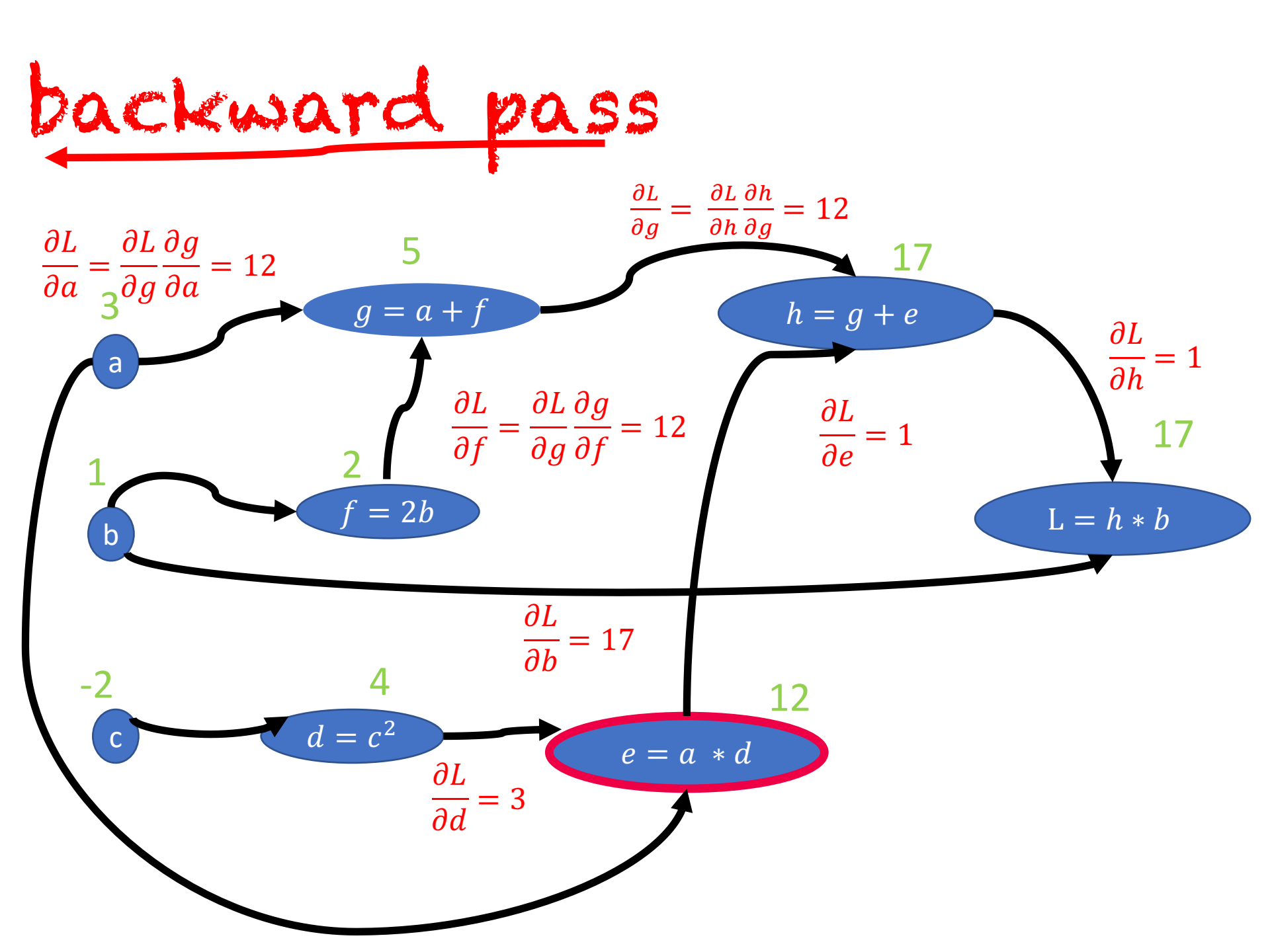
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial d} = 3$
3
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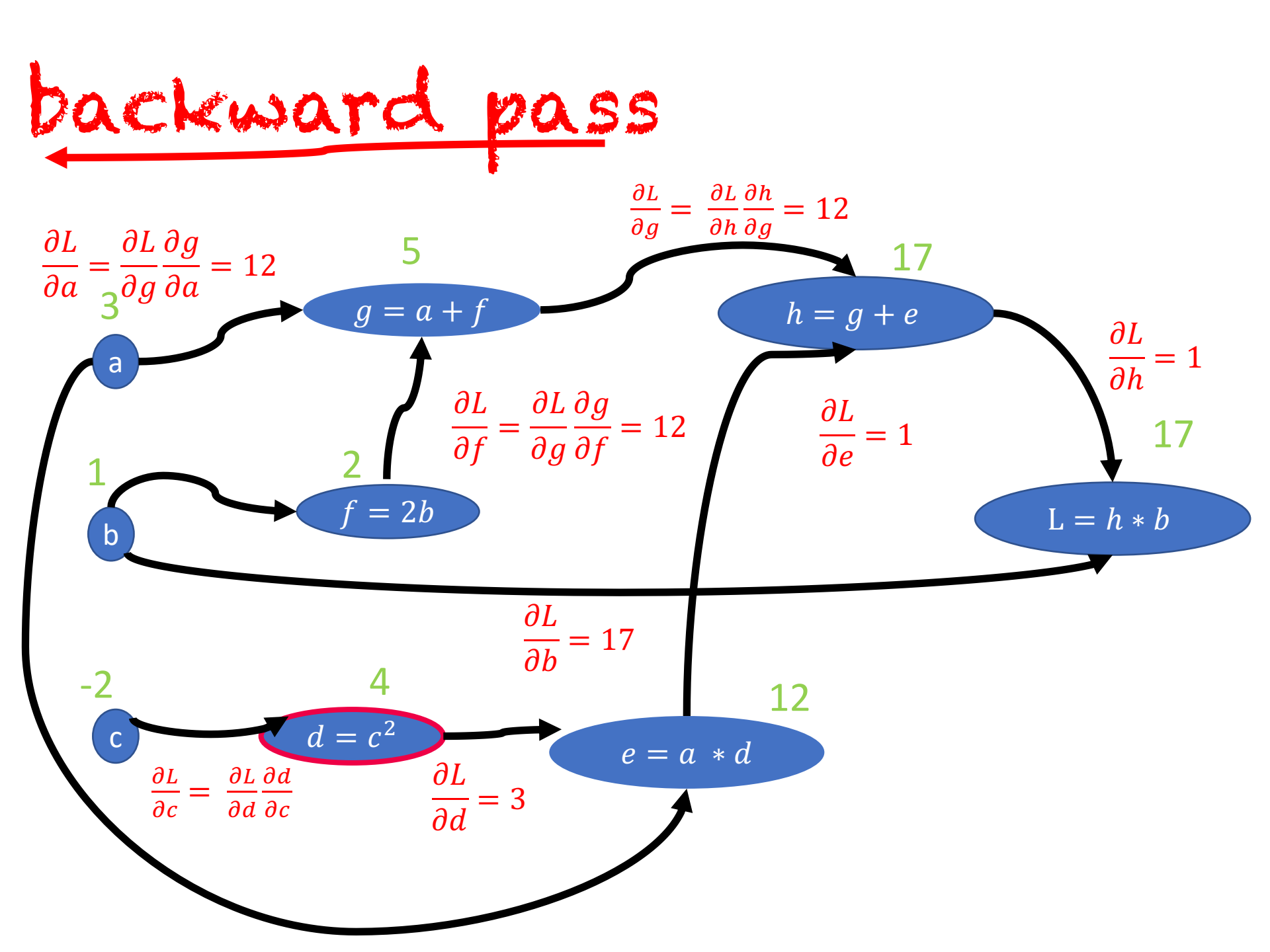
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial c} = \frac{\partial L}{\partial d}\frac{\partial d}{\partial c}$
$\frac{\partial L}{\partial d} = 3$
3
5
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4
12
a
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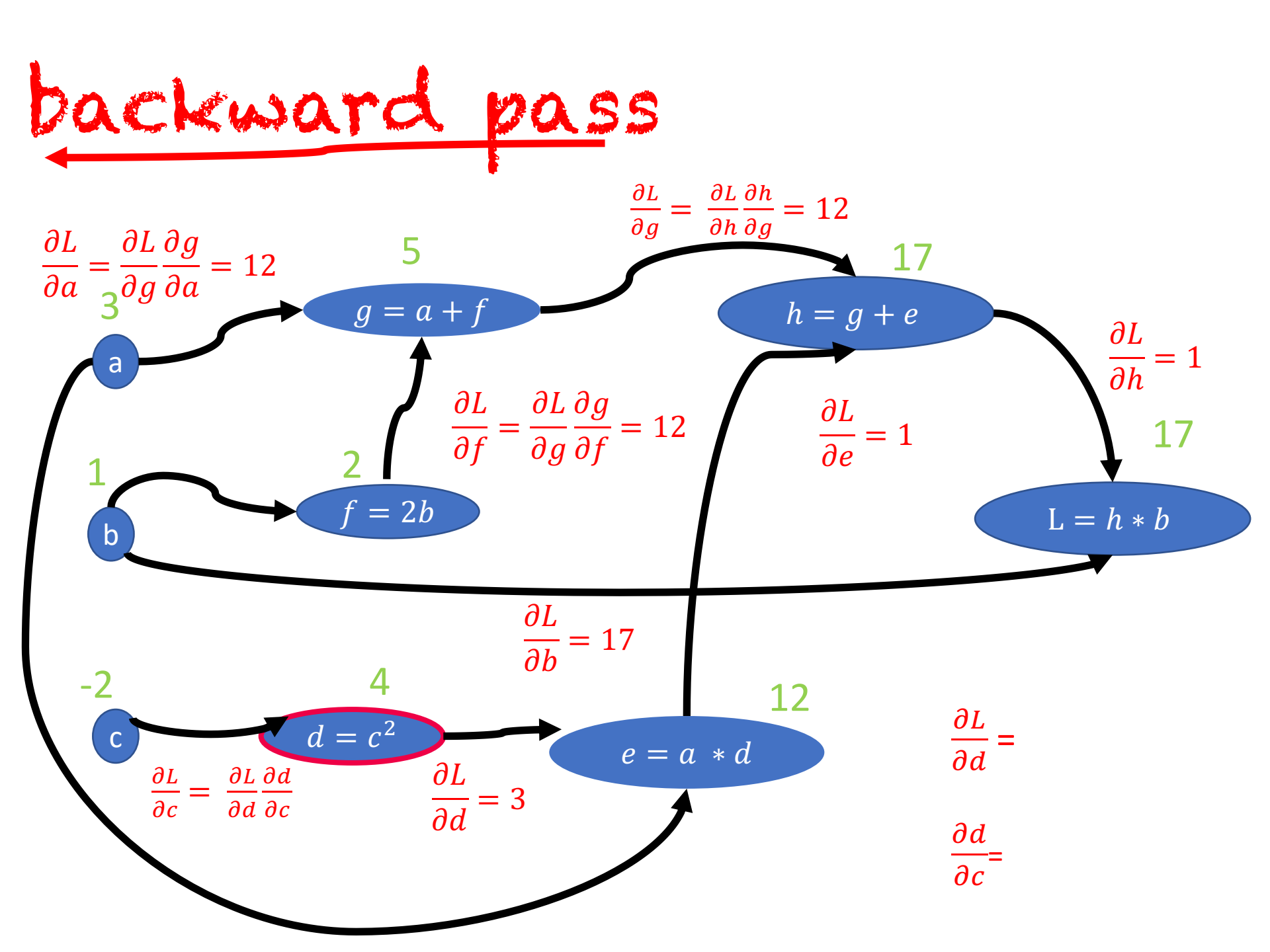
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
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$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial c} = \frac{\partial L}{\partial d}\frac{\partial d}{\partial c}$
$\frac{\partial L}{\partial d} = 3$
$\frac{\partial L}{\partial d} =$
$\frac{\partial d}{\partial c} =$
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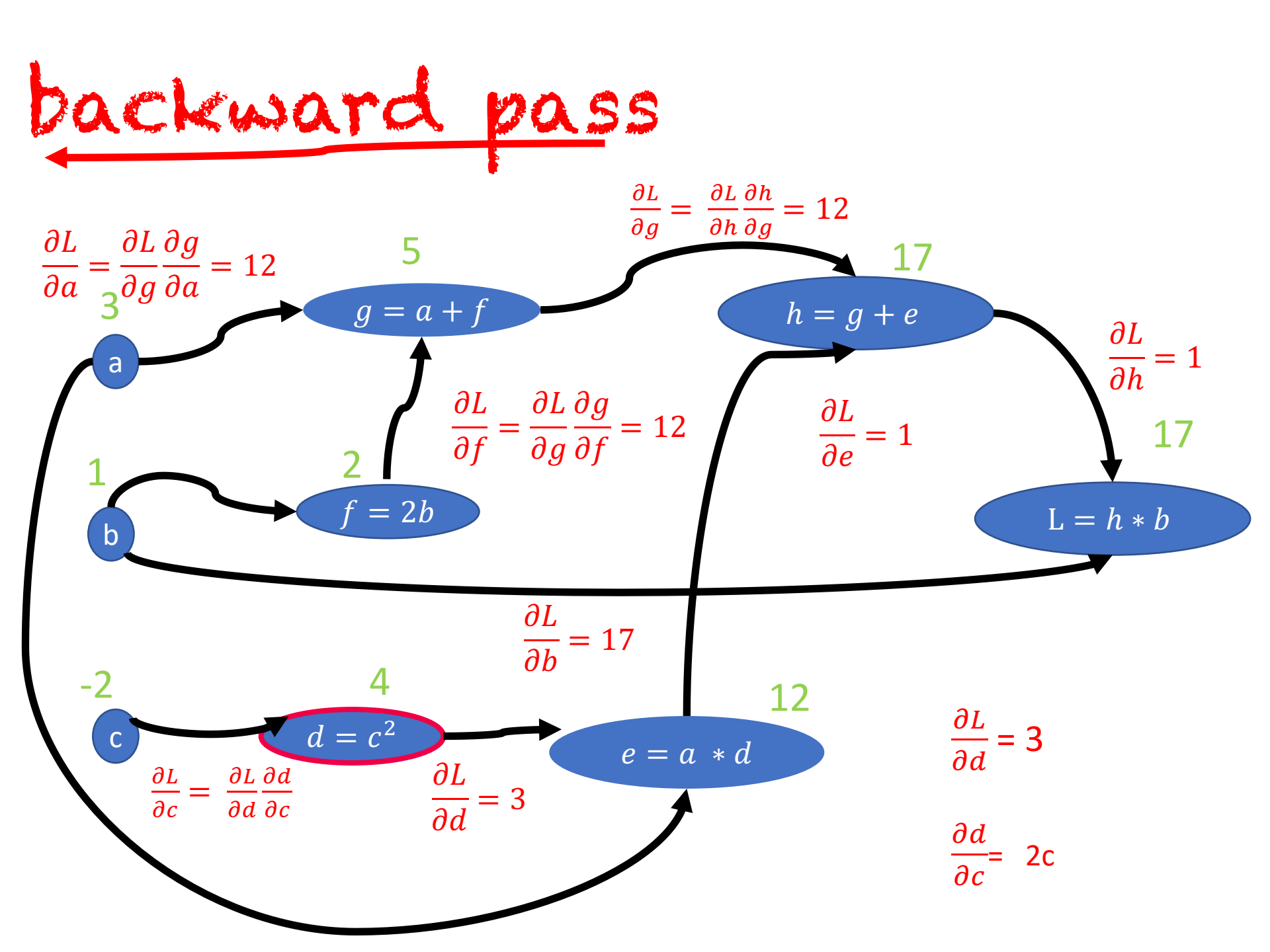
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial a} = 12$
$\frac{\partial L}{\partial g} = \frac{\partial L}{\partial h}\frac{\partial h}{\partial g} = 12$
$\frac{\partial L}{\partial f} = \frac{\partial L}{\partial g}\frac{\partial g}{\partial f} = 12$
$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial c} = \frac{\partial L}{\partial d}\frac{\partial d}{\partial c}$
$\frac{\partial L}{\partial d} = 3$
$\frac{\partial L}{\partial d} = 3$
$\frac{\partial d}{\partial c} = 2c$
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17
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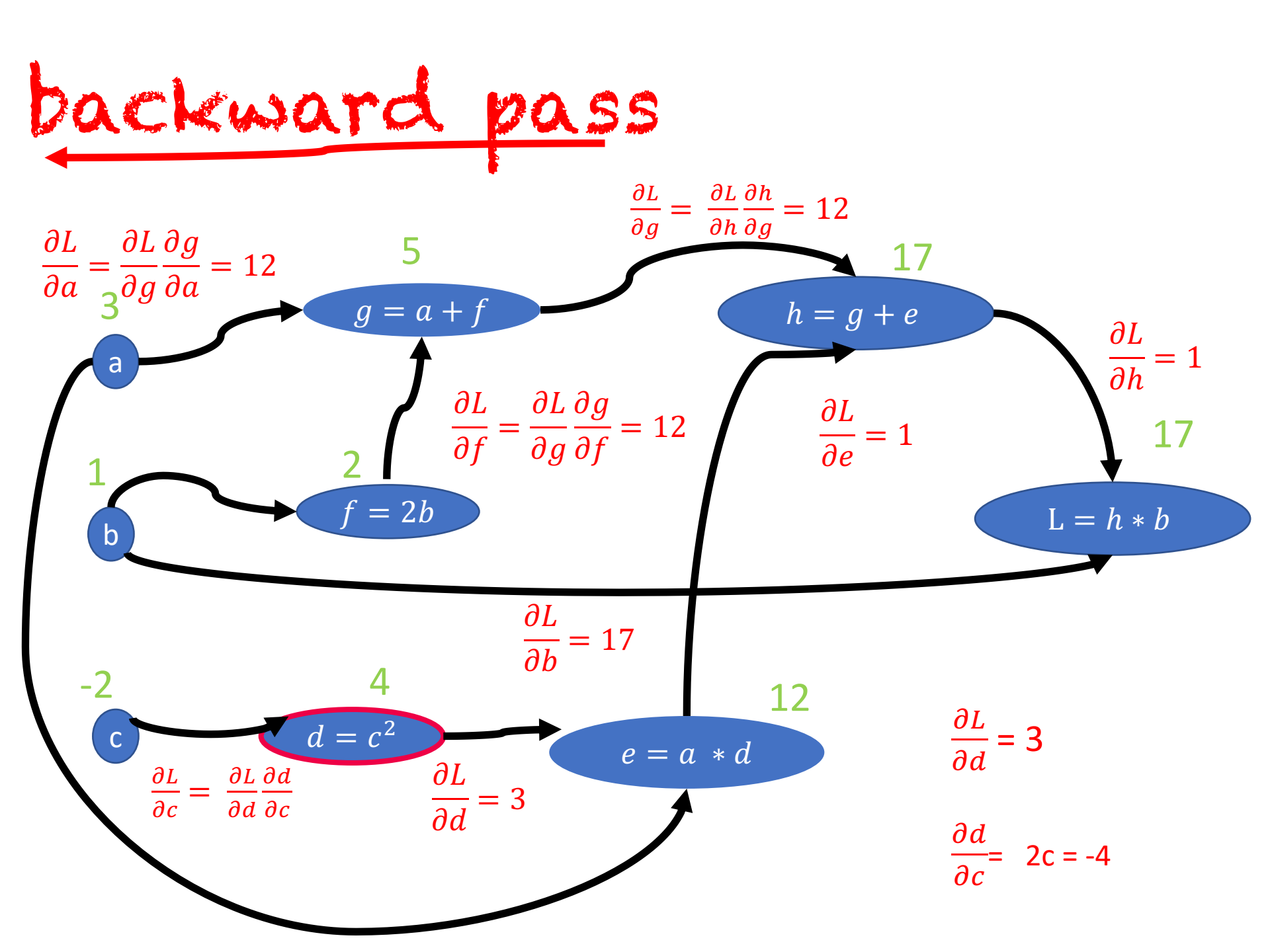
2
17
-2
4
12
a
b
c
$g = a + f$
$h = g + e$
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
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$\frac{\partial L}{\partial e} = 1$
$\frac{\partial L}{\partial h} = 1$
$\frac{\partial L}{\partial b} = 17$
$\frac{\partial L}{\partial c} = \frac{\partial L}{\partial d}\frac{\partial d}{\partial c}$
$\frac{\partial L}{\partial d} = 3$
$\frac{\partial d}{\partial c} = 2c = -4$
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c
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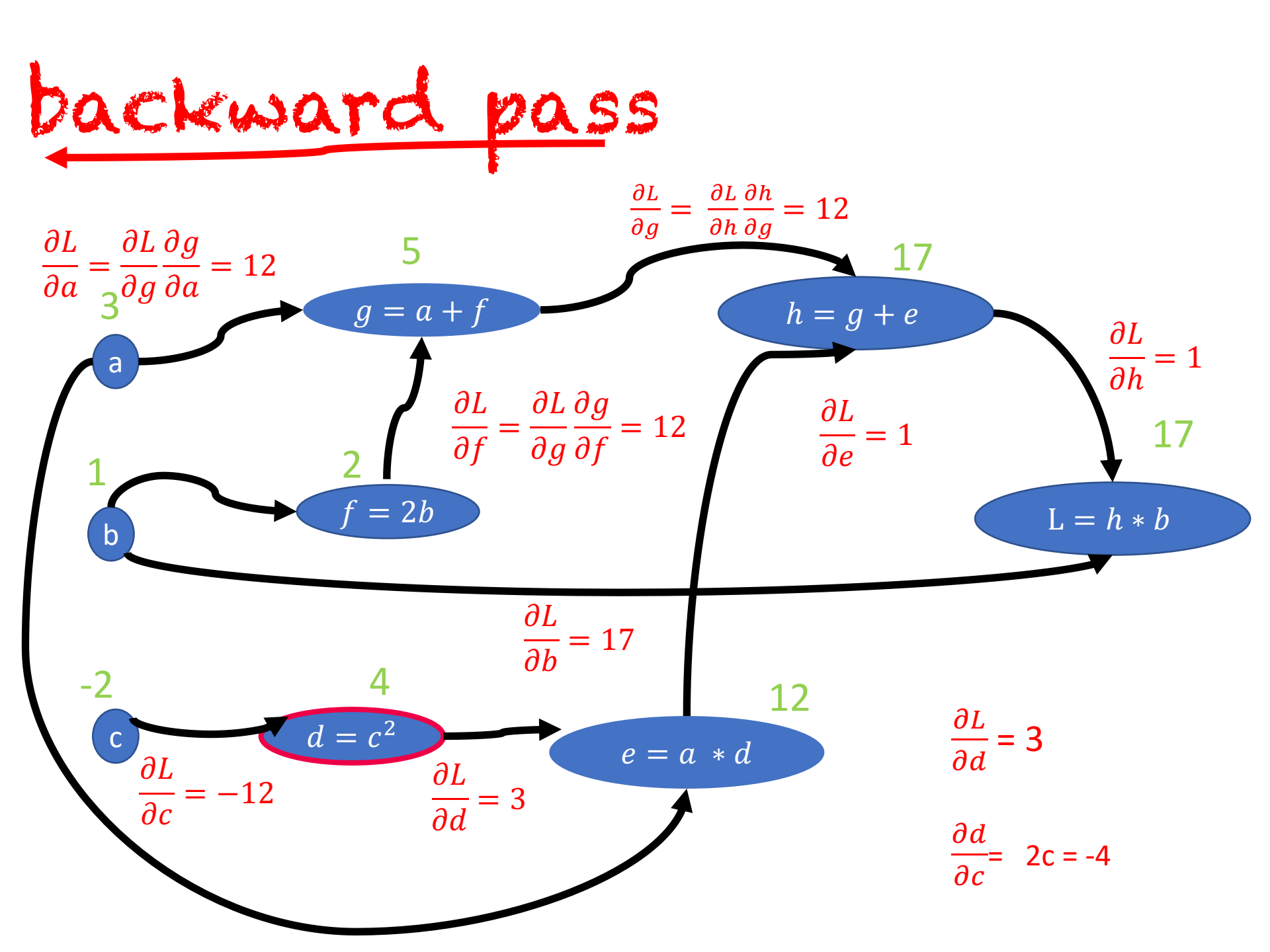
$f = 2b$
$L = h * b$
$d = c^2$
$e = a * d$


backward pass
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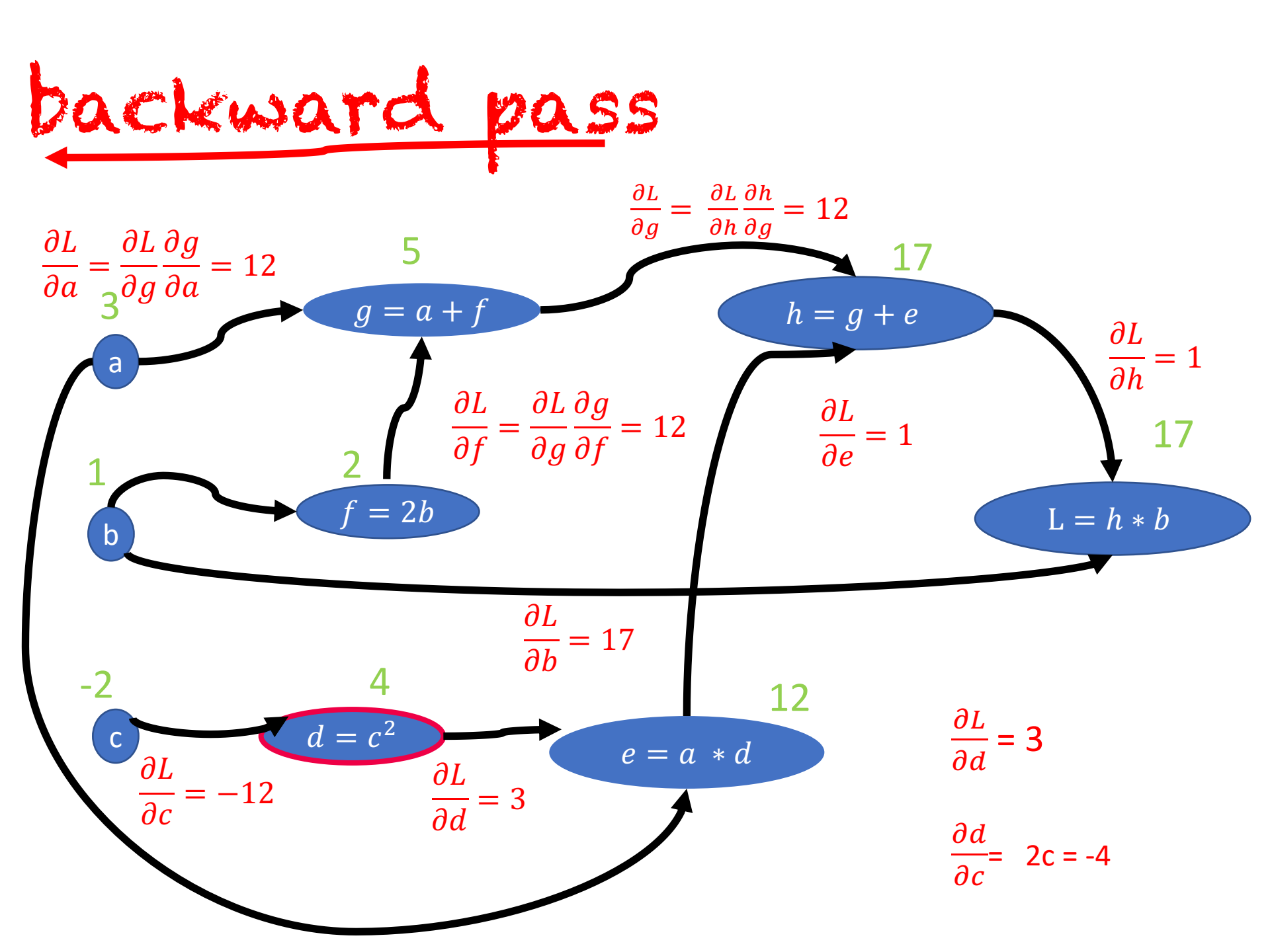
$d = c^2$
$e = a * d$

Backwards differentiation in computation graphs

- The importance of the computation graph comes from the backward pass
- This is used to compute the derivatives that we'll need for the weight update.
- For training, we need the derivative of the loss with respect to weights in early layers of the network
- But loss is computed only at the very end of the network!
- Solution: **backward differentiation**
- Given a computation graph and the derivatives of all the functions in it we can automatically compute the derivative of the loss with respect to these early weights.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$d = c^2$
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Exploding gradient

The gradient can accumulate, becoming very big

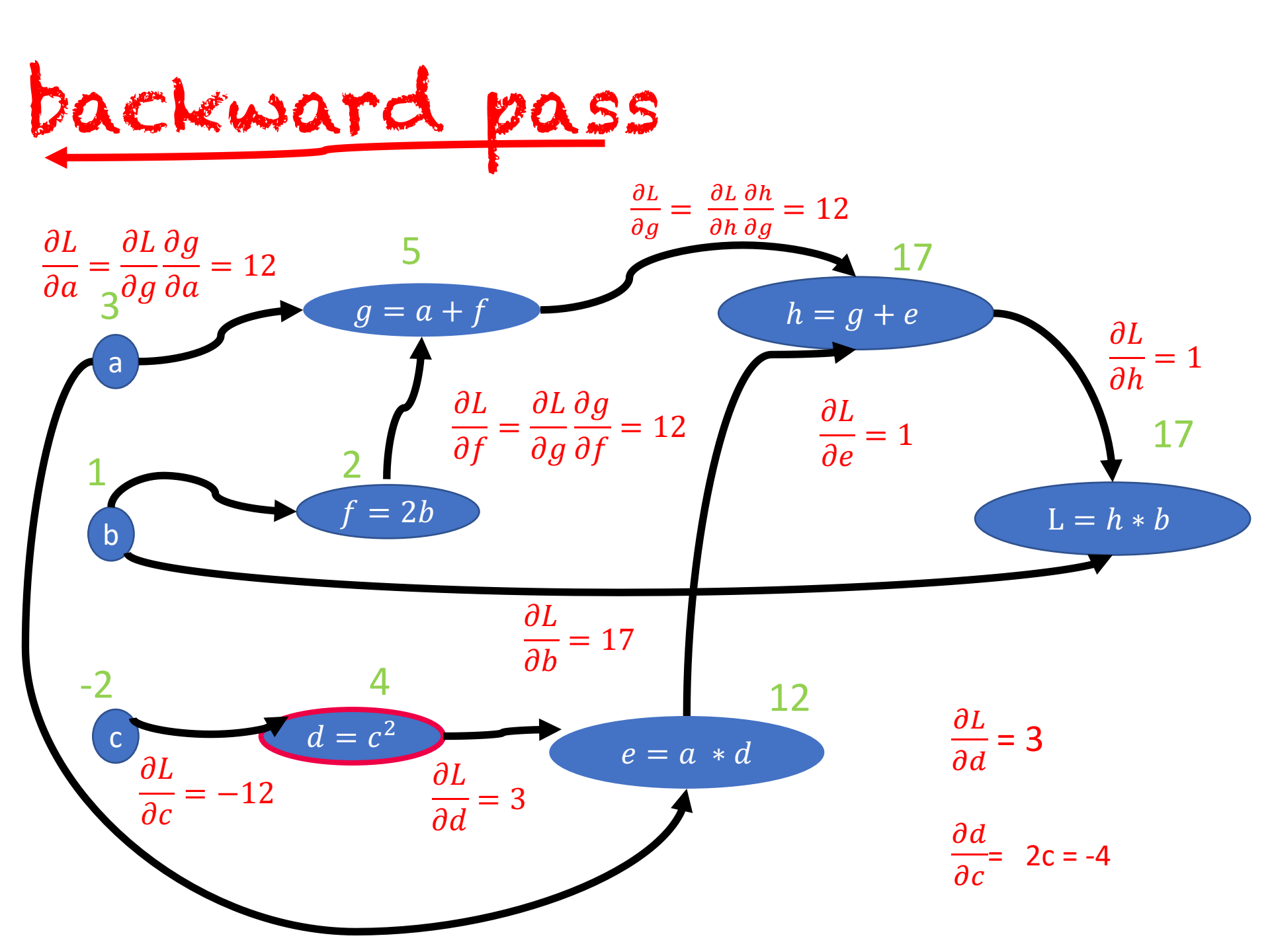
Issues:

- might move our weights too much
- result in Nan

Solution:

Clipping

- Maximum value for gradients
- Can be dynamic


backward pass
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Vanishing gradient

The gradient become 0

Issues:

wont be able to update weights (because 0 gets passed all the way back)

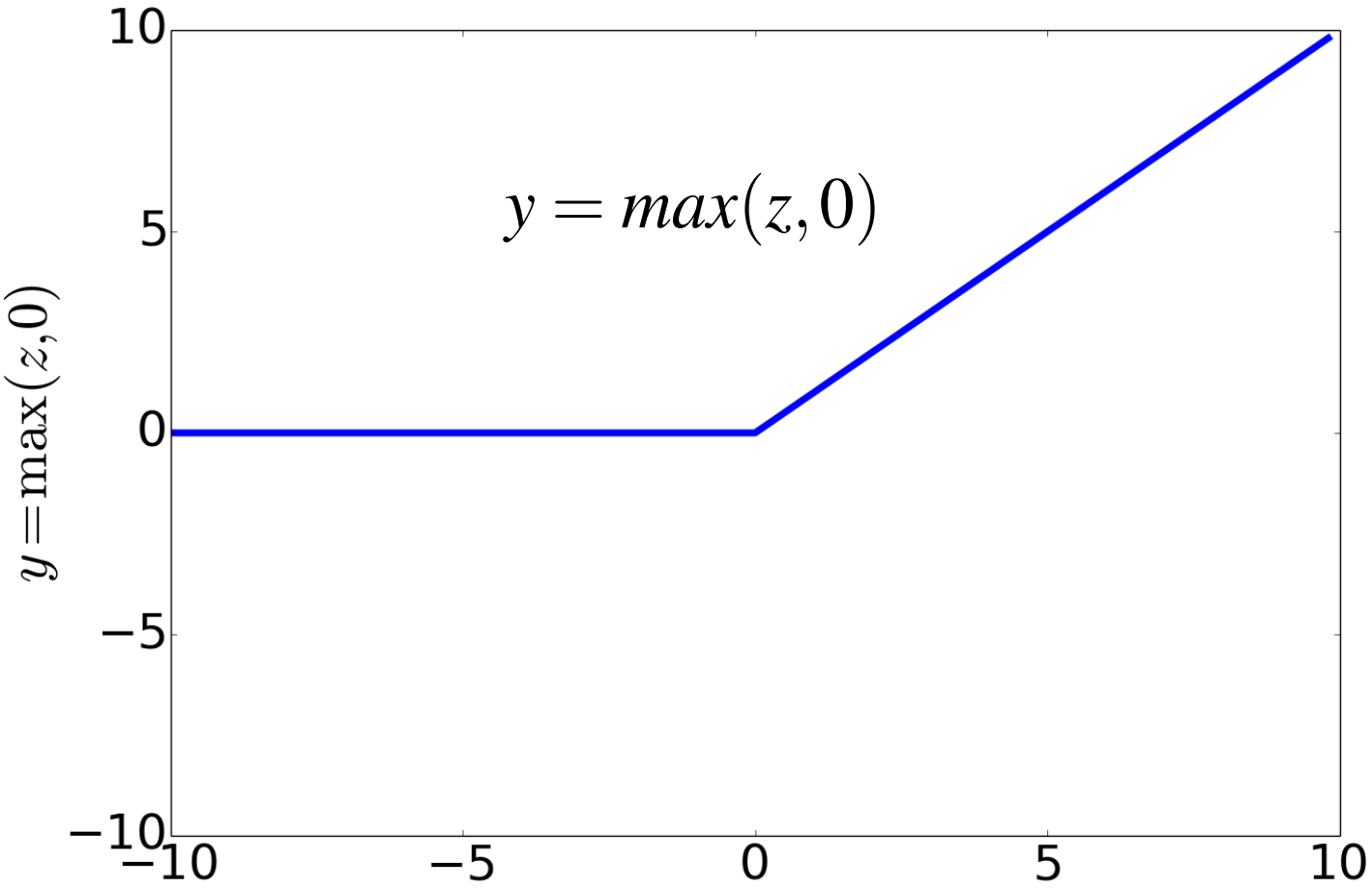
stuck in a local optima

Solution:

ReLU activation function

$z = \max(0, z)$

ReLU



One node view

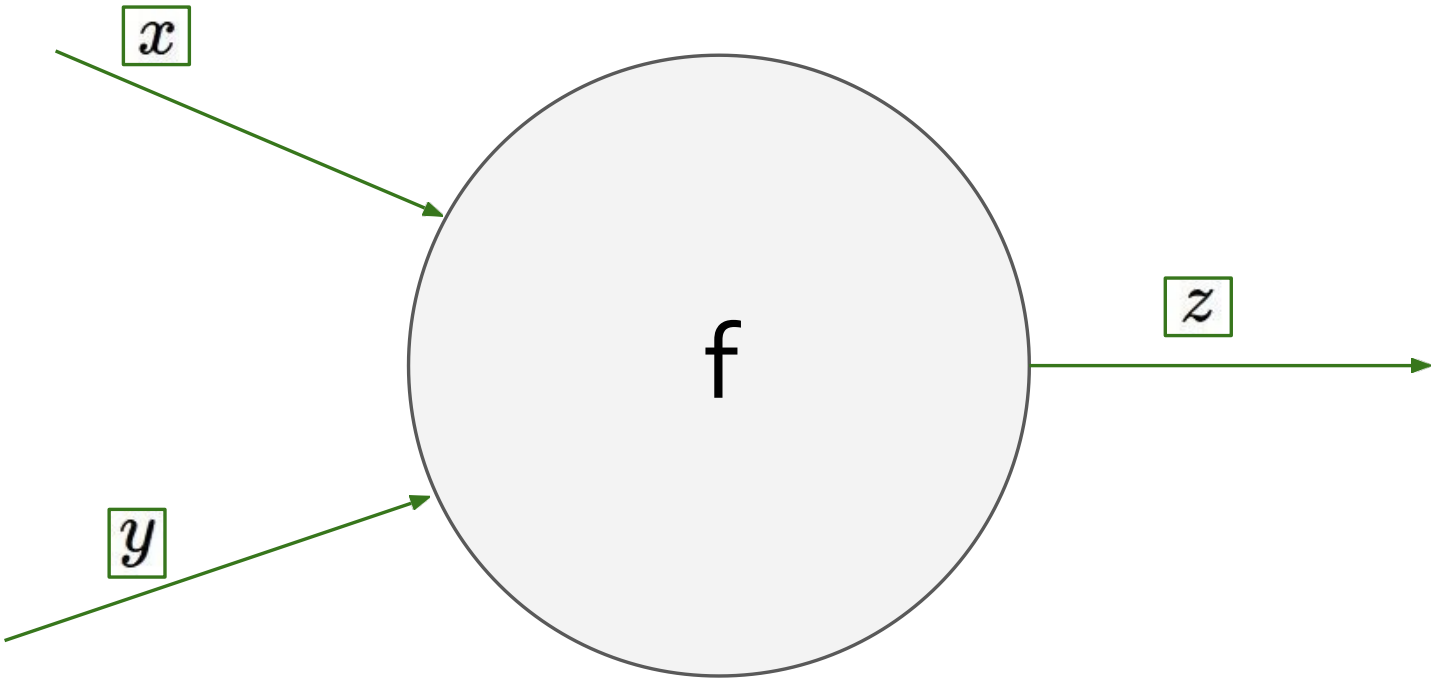


Figure from Andrej Karpathy

Dead neuron

In forward pass, output of a node w/ ReLU activation often will be 0

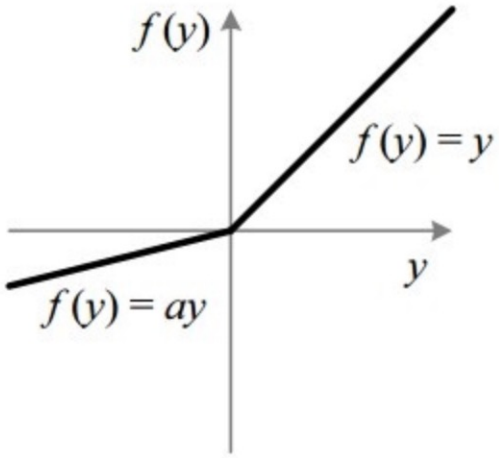
Issues:

wont pass information from one node to the next

lots of useless nodes

Solution:

Leaky ReLU activation function



Outline

Updating weights in LR

Computation Graph

Backpropagation

Issues when training NNs

Pytorch

Deep Averaging Neural Network

Pytorch

Torch: Facebook's deep learning framework

Originally written in Lua (C backend)

Optimized to run computations on GPU

Mature, industry-supported framework

Defining a model

```
import torch
from torch import nn


class LogisticRegression(nn.Module):
    def __init__(self, input_size, num_classes):
        super(LogisticRegression, self).__init__()
        self.linear = nn.Linear(input_size, num_classes)

    def forward(self, x):
        out = self.linear(x)
        return out
```

nn.Module

Base class for all neural network modules.

Creates a computation graph

Define the model in `__init__`

Specify how to make predictions in `forward`

- If only use built-in modules, no need to implement backprop

Defining a model

```
import torch
from torch import nn

class FNN(nn.Module):
    def __init__(self, input_size, hidden_size, num_classes):
        super(FNN, self).__init__()
        self.input_size = input_size
        self.l1 = nn.Linear(input_size, hidden_size)
        self.relu = nn.ReLU()
        self.l2 = nn.Linear(hidden_size, num_classes)

    def forward(self, x):
        out = self.l1(x)
        out = self.relu(out)
        out = self.l2(out)
        # no activation and no softmax at the end
        return out
```

Train a model

Define:

- Loss function
- Learning algorithm (e.g. SGD)
- Learning rate
- Number of epochs

```
num_epochs = 100
learning_rate = 0.003
optimizer = torch.optim.SGD(model.parameters(), lr=learning_rate)
loss_fn = nn.CrossEntropyLoss()
```

Train a model

In each iteration:

- Make a prediction
- Compute the loss
- Autograd (Automatic differentiation), backprop
- Update the weights

```
optimizer.zero_grad()
prediction = model(X[i])
loss_val = loss_fn(prediction, labels[0][i])
loss_val.backward()
optimizer.step()
```

Train a model

```
# Training the Model
for epoch in range(num_epochs):
  num_correct = 0
  for i in range(100):
    optimizer.zero_grad()
    prediction = model(X[i])
    loss_val = loss_fn(prediction, labels[0][i])
    loss_val.backward()
    optimizer.step()

  print(f"loss at epoch {epoch}: {loss_val}")
  print(f"accuracy at epoch {epoch}: {num_correct / 100}")
```

FFN's issues

Fixed input size

Solutions:

1. Create a fixed length representation
2. Recurrent Neural Networks