

Announcements (1/2)

- Office Hours:
- Thursdays 3-4:30pm
- HWOO late deadline tonight
- Reading01 late deadline tonight
- HW01 due Monday 01/30
- Based on Monday's lecture
- Reading02 released tonight, due Monday 01/30


## Announcements (2/2)

- Monday 01/30 lecture
- Lecture will start late, time tbd
- Will use lab time for lecture too


## Outline

- LMs: smoothing, perplexity, <s>
- Document Representations
- Document-Term Matrix
- BoW
- Linear Algebra:
- Vectors
- Vector similarity
- tf-idf


## The intuition of smoothing (from Dan Klein)

- When we have sparse statistics:
$P(w \mid$ denied the)
3 allegations
2 reports
1 claims
1 request
7 total

- Steal probability mass to generalize better
$\mathrm{P}(\mathrm{w} \mid$ denied the $)$
2.5 allegations
1.5 reports
0.5 claims
0.5 request
2 other
7 total



## Raw bigram probabilities

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 0.002 | 0.33 | 0 | 0.0036 | 0 | 0 | 0 | 0.00079 |
| want | 0.0022 | 0 | 0.66 | 0.0011 | 0.0065 | 0.0065 | 0.0054 | 0.0011 |
| to | 0.00083 | 0 | 0.0017 | 0.28 | 0.00083 | 0 | 0.0025 | 0.087 |
| eat | 0 | 0 | 0.0027 | 0 | 0.021 | 0.0027 | 0.056 | 0 |
| chinese | 0.0063 | 0 | 0 | 0 | 0 | 0.52 | 0.0063 | 0 |
| food | 0.014 | 0 | 0.014 | 0 | 0.00092 | 0.0037 | 0 | 0 |
| lunch | 0.0059 | 0 | 0 | 0 | 0 | 0.0029 | 0 | 0 |
| spend | 0.0036 | 0 | 0.0036 | 0 | 0 | 0 | 0 | 0 |

Laplacian bigram probabilities $P_{A d d-1}\left(w_{i} \mid w_{i-1}\right)=\frac{c\left(w_{i}, w_{i-1}\right)+1}{c\left(w_{i-1}\right)+V}$

|  | i | want | to | eat | chinese | food | lunch | spend |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i | 0.0015 | 0.21 | 0.00025 | 0.0025 | 0.00025 | 0.00025 | 0.00025 | 0.00075 |
| want | 0.0013 | 0.00042 | 0.26 | 0.00084 | 0.0029 | 0.0029 | 0.0025 | 0.00084 |
| to | 0.00078 | 0.00026 | 0.0013 | 0.18 | 0.00078 | 0.00026 | 0.0018 | 0.055 |
| eat | 0.00046 | 0.00046 | 0.0014 | 0.00046 | 0.0078 | 0.0014 | 0.02 | 0.00046 |
| chinese | 0.0012 | 0.00062 | 0.00062 | 0.00062 | 0.00062 | 0.052 | 0.0012 | 0.00062 |
| food | 0.0063 | 0.00039 | 0.0063 | 0.00039 | 0.00079 | 0.002 | 0.00039 | 0.00039 |
| lunch | 0.0017 | 0.00056 | 0.00056 | 0.00056 | 0.00056 | 0.0011 | 0.00056 | 0.00056 |
| spend | 0.0012 | 0.00058 | 0.0012 | 0.00058 | 0.00058 | 0.00058 | 0.00058 | 0.00058 |

Evaluating Language Models

## Perplexity

Perplexity $\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)=$

$$
\begin{aligned}
& =P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)^{\frac{1}{n}} \\
& =\sqrt[n]{\frac{1}{P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)}}
\end{aligned}
$$

$P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)$ depends on the LM we use

The lower the perplexity, the better the model

## Perplexity - implementation

Perplexity $\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)=$

$$
\begin{aligned}
& =P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)^{\frac{1}{n}} \\
& =\sqrt[n]{\frac{1}{P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)}} \\
& =e^{\frac{1}{n} \sum_{i=1}^{n}-\log P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)}
\end{aligned}
$$

def perplexity(self, sentence, method):
II II II
Compute
II II
return $2.0 * *$ ( $-1.0 *$ mean([method(context, word) for context, word in \} bigrams(self.tokenize_and_censor(sentence))]))
exponentiated average negative log-likelihood

## Perplexity

Perplexity $\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)=$

$$
=\sqrt[n]{\frac{1}{P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)}}
$$

The lower the perplexity, =>
the higher the probability =>
the model is less surprised by the text

This is based on $P(M \mid T)$, i.e. we fit the model based on the training data
"less surprised" - based on just the training data, how shocked is the model when it sees $w_{1}, w_{2}, w_{3}, \ldots, w_{n}$

## Perplexity

$$
=\sqrt[n]{\frac{1}{P\left(w_{1}, w_{2}, w_{3}, \ldots, w_{n}\right)}}
$$

What we generally use for word sequence is the entire sequence of words in some test set. Since this sequence will cross many sentence boundaries, we need to include the begin- and end-sentence markers in the probability computation. We also need to include the end-of-sentence marker (but not the beginning-of-sentence marker) in the total count of word tokens N .

## Training set: "a a b"

We add <s> and </s> to our example:
"<s> a a b </s>"

Unigram probabilities:

$$
P(<s>) \quad P(a) \quad P(b) \quad P(</ s>)
$$

## Training set: "a a b"

We add <s> and </s> to our example:
"<s> a a b </s>"

Unigram probabilities:

$$
\begin{array}{cccc}
P(<s>) & P(a) & P(b) & P(</ s>) \\
.2 & .4 & .2 & .2
\end{array}
$$

## Training set: "a a b"

We add <s> and </s> to our example:
"<s > <s> a a b </s>"

Bigram probabilities:

$$
\begin{array}{llll}
P(<s>\mid<s>) & P(a) \mid<s> & P(b \mid<s>) & P(</ s>\mid<s>) \\
P(<s>\mid a) & P(a \mid a) & P(b \mid a) & P(</ s>\mid a) \\
P(<s>\mid b) & P(a \mid b) & P(b \mid b) & P(</ s>\mid a b) \\
P(<s>\mid</ s>) & P(a) \mid</ s> & P(b \mid</ s>) & P(</ s>\mid</ s>)
\end{array}
$$

## Training set: "a a b"

We add $\langle s>$ and </s> to our example:
"<s > <s> a a b </s>"

Unigram probabilities now:

$$
P(<s>) \quad P(a) \quad P(b) \quad P(</ s>)
$$

## Training set: "a a b"

Dont treat $\mathrm{c}(\langle\mathrm{s}\rangle$ ) as a regular token
"<s> <s> a a b </s>"

Unigram probabilities:

$$
P(<s>)
$$

$$
P(a)
$$

$$
P(b)
$$

$$
P(</ s>)
$$

Now: .33
.33
.1667 .1667
Before: . 2
. 4
. 2
. 2

## Training set: "a a b"

## Dont treat $\mathrm{c}(\langle\mathrm{s}\rangle$ ) as a regular token

"<s> <s> a a b </s>"

Unigram probabilities:

$$
\begin{equation*}
P(<s\rangle) \tag{a}
\end{equation*}
$$

$$
\begin{equation*}
P(</ s>) \tag{b}
\end{equation*}
$$

Correct:
. 5
.25
.25

# Why not include <s> in our counts? 

Why do we include <s>?

Why do we include </s>?

Generating text perspective?

What happens if we use unigram to generate text?

## Outline

- LMs: smoothing, perplexity, <s>, MLE vs EM
- Document Representations
- Document-Term Matrix
- BoW
- Linear Algebra:
- Vectors
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## Maximum Likelihood Estimates

- The maximum likelihood estimate
- of some parameter of a model $M$ from a training set $T$
- maximizes the likelihood of the training set $T$ given the model $M$

Parameters are:
n-gram probabilities

## Approach 2 - Combine the 'grams

Context specific weights

## Jelinek-Mercer smoothing

 (1980)Lambdas are parameters too

$$
\begin{aligned}
\hat{P}\left(w_{n} \mid w_{n-2} w_{n-1}\right)= & \lambda_{1}\left(w_{n-2}^{n-1}\right) P\left(w_{n} \mid w_{n-2} w_{n-1}\right) \\
& +\lambda_{2}\left(w_{n-2}^{n-1}\right) P\left(w_{n} \mid w_{n-1}\right) \\
& +\lambda_{3}\left(w_{n-2}^{n-1}\right) P\left(w_{n}\right)
\end{aligned}
$$

## Approach 2 - Combine the 'grams

Context specific weights

Lambdas are parameters too


How do we decide what values our lambdas should be?

# Approach 2 - Combine the 'grams 

Context specific weights


Lambdas are parameters too

How do we decide what values our lambdas should be?

Split our data into train and evaluation sets try different lambdas, compare model's perplexity on the evaluation set

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## Recap so far

The first class was all about

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The first class was all about counting words
$2^{\text {nd }}$ class was about the power of counting words.

By counting words we can

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$2^{\text {nd }}$ class was about the power of counting words.

By counting words we can $\qquad$
learn about language
generate language
categorize language

## Documents \& Corpora

## Terminology - Corpus

- Corpus:
- A collection of documents
- Corpora - plural of corpus



## Terminology - Document

- Document:
- Often unit of text of interest (dependent on RQ)
- Often represents one data point
- Examples:
- Book
- Chapter
- News article
- Tweet
- Product Review
- ....


## How do we represent documents?

## Dictionaries of word counts



Often called Bag of Words

## Bag of Words - Start with document

Very good drama although it appeared to have a few blank areas leaving the viewers to fill in the action for themselves. I can imagine life being this way for someone who can neither read nor write. This film simply smacked of the real world: the wife who is suddenly the sole supporter, the live-in relatives and their quarrels, the troubled child who gets knocked up and then, typically, drops out of school, a jackass husband who takes the nest egg and buys beer with it. 2 thumbs up... very very very good movie.

## Bag of Words - Break document into words

Very good drama although it appeared to have a few blank areas leaving the viewers to fill in the action for themselves. I can imagine life being this way for someone who can neither read nor write. This film simply smacked of the real world: the wife who is suddenly the sole supporter, the live-in relatives and their quarrels, the troubled child who gets knocked up and then, typically, drops out of school, a jackass husband who takes the nest egg and buys beer with it. 2 thumbs up... very very very good movie.


## Bag of Words - compute word counts

Very good drama although it appeared to have a few blank areas leaving the viewers to fill in the action for themselves. I can imagine life being this way for someone who can neither read nor write. This film simply smacked of the real world: the wife who is suddenly the sole supporter, the live-in relatives and their quarrels, the troubled child who gets knocked up and then, typically, drops out of school, a jackass husband who takes the nest egg and buys beer with it. 2 thumbs up... very very very good movie.

('the', 8), (',', 5),
('very', 4),
('.', 4),
('who', 4),
('and', 3),
('good', 2),
('it', 2),
('to', 2),
('a', 2),
('for', 2),
('can', 2),
('this', 2),
('of', 2),
('drama', 1),
('although', 1),
('appeared', 1),
('have', 1),
('few', 1),
('blank', 1)

## Bag of Words

Very good drama although it appeared to have a few blank areas leaving the viewers to fill in the action for themselves. I can imagine life being this way for someone who can neither read nor write. This film simply smacked of the real world: the wife who is suddenly the sole supporter, the live-in relatives and their quarrels, the troubled child who gets knocked up and then, typically, drops out of school, a jackass husband who takes the nest egg and buys beer with it. 2 thumbs up... very very very good movie.

## Document vectors

## Document vectors

- Vector is just an array of numbers

- Index represents a word
- Value represents ....


## Document vectors

- Vector is just an array of numbers

- Index represents a word
- Value represents something about that word
- For now, unigrams


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- LMs: smoothing, perplexity, <s>
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## Vectors

Physics:
arrow pointing in space
it has a length, and a direction its pointing
CS:
ordered lists of numbers number of dimensions is size of the list
Math:
we can add them together
we can multiply them by a number

## Document as a Vector

Vocabulary is \{" g ", "t"\}


## Document as a Vector




## Document as a Vector




## Which two are the most similar?




## Vector similarity

Dot product of $\mathbf{a}$ and $\mathbf{b}$ :

$$
\boldsymbol{a} * \boldsymbol{b}=\sum_{i}^{n} a_{i} b_{i}
$$



$$
\begin{aligned}
& =\# g_{\text {blue }} * \# g_{\text {orange }}+\# t_{\text {blue }} * \# t_{\text {orange }} \\
& =5 * 3+1 * 3
\end{aligned}
$$

$15 * 3$
45

## Issues with dot product

$$
\boldsymbol{a} * \boldsymbol{b}=\sum_{i}^{n} a_{i} b_{i}
$$



## Issues with dot product

$\boldsymbol{a} * \boldsymbol{b}=\sum_{i}^{n} a_{i} b_{i}$ Which is most similar:
(brown, yellow), (brown, orange), or (yellow, orange)?


## Issues with dot product

$\sum^{n} \quad$ Which is most similar:
(brown, yellow), (brown, orange), or (yellow, orange)?


## Issues with dot product

$\boldsymbol{a} * \boldsymbol{b}=\sum_{i}^{n} a_{i} b_{i}$
More weight given to longer vectors (documents)


## Solution - normalize by length

 $\frac{a * b}{|a||b|}$

How to compute the length of a vector

How long is
gttttttttt

Option 1:
Just add the number of g's and the number of t's

In terms of documents, its just length of document

How to compute the length of a vector

What's the length of blue with option 1 ?

$$
-1+1=0
$$



How to compute the length of a vector

Option 2: add absolute values

$$
|-1|+|1|=2
$$



How to compute the length of a vector

Option 3: add squared values, then take square root
$-1^{2}+1^{2}=\sqrt{2}$


## Cosine similarity

$\frac{\boldsymbol{a} * \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}=\cos \theta$

Normalized dot product is the same as the cosine of the angle between the vectors

Cosine Distance/Similarity


## Document vectors



## Finding most similar documents?

CTA examples of finding most similar documents

What did we use to represent documents?

## Document Matrix



DIMENSIONS
Slide taken from Dirk Hovy

## Recap so far

The first class was all about counting words
$2^{\text {nd }}$ class was about the power of counting words.

By counting words we can $\qquad$
learn about language
generate language
categorize language
group documents

## What to count? How to count?

Next lecture
HWO2

