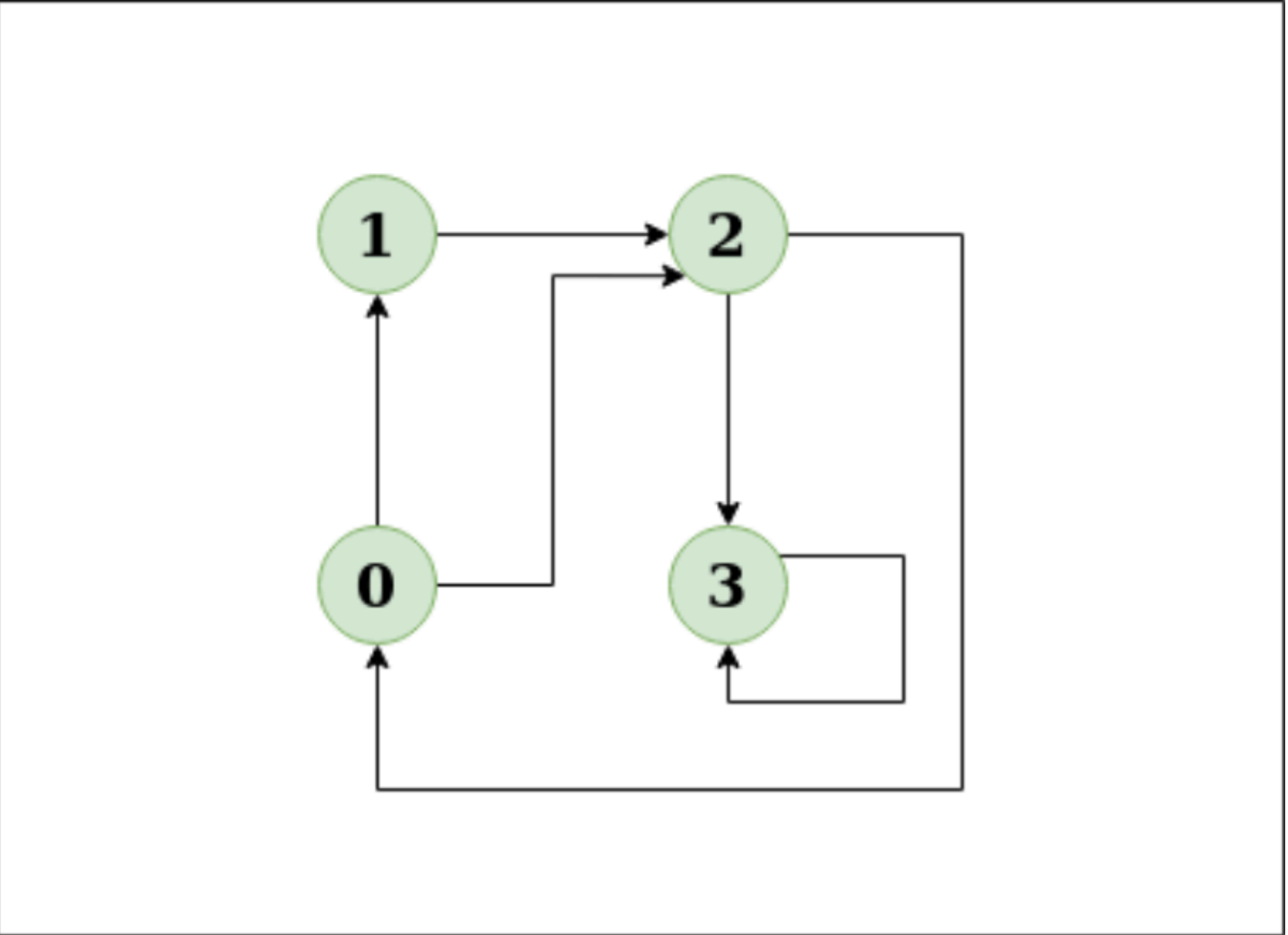
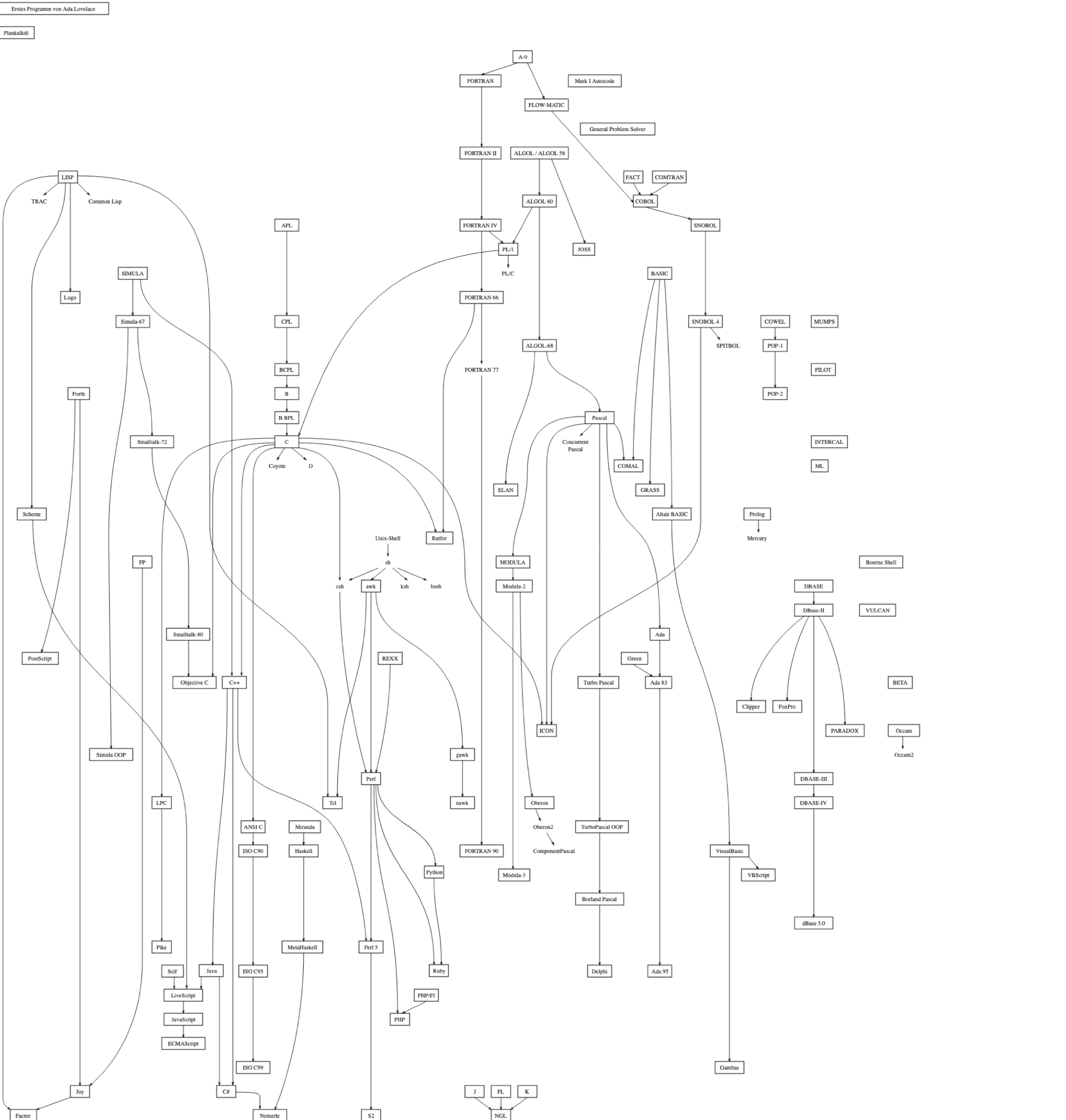


Does a graph have a cycle?

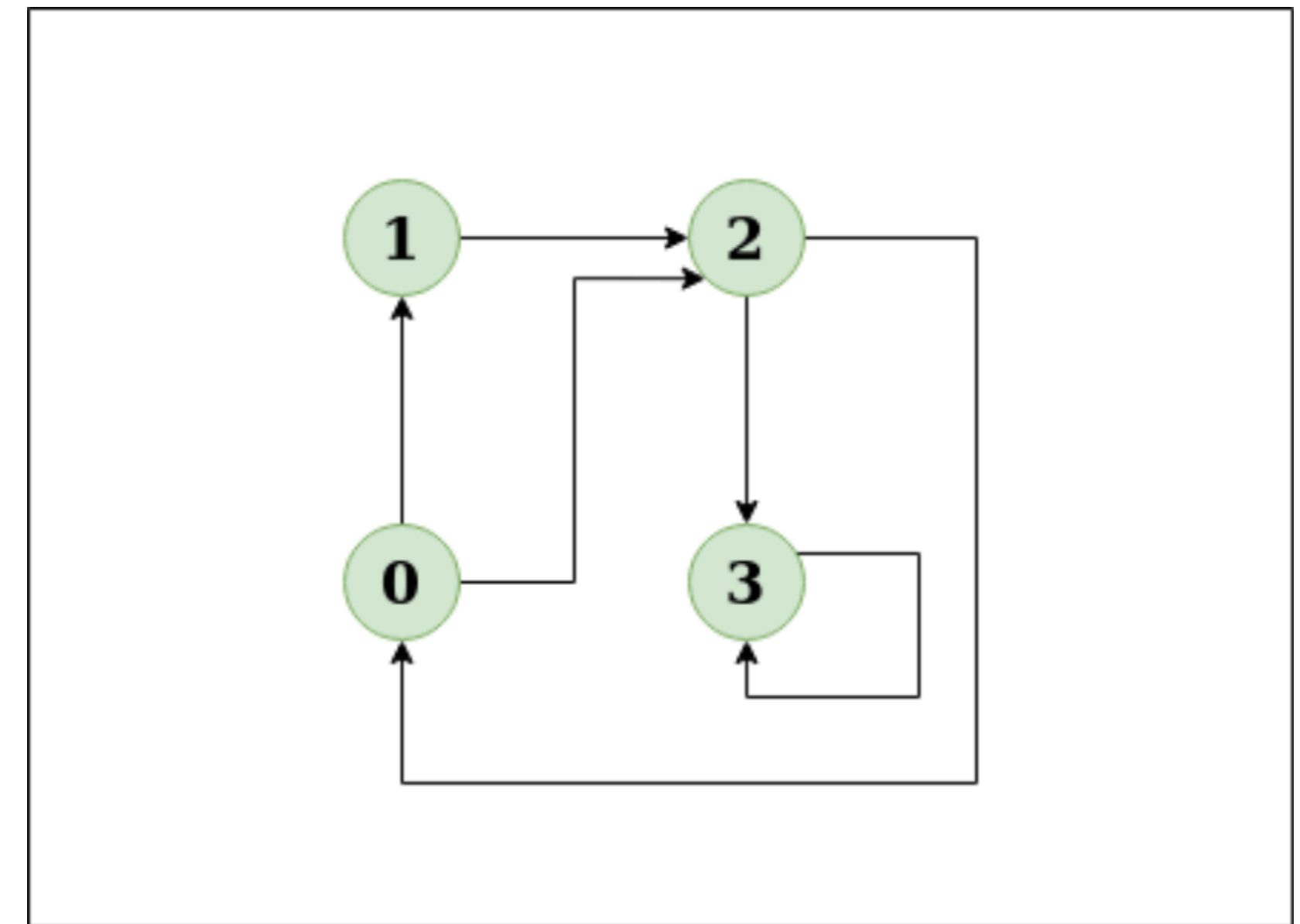
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Representing a graph

- Use an adjacency list



Node:

id: an integer

links: an array of integers

Graph:

g: an array of nodes

Position	Node
0	{id: 0, links: [1,2]}
1	{id: 1, links:[2]}
2	{id: 2, links:[0,3]}
3	{id: 3: links[3]}

Depth First Search

On an acyclic graph

- Recursion
- No base case!
- Time / space complexity

Given: g – a graph

n – an integer

(of some node in graph)

DFS(g, n):

for $link$ in $g[n].links$:

DFS($g, link$)

- Will this explore whole graph?
- How do we handle possibility of cycles?

Coloring

Extending Node

- white: unvisited
 - grey: being processed
 - black: visited
-
- Start by setting color of every node to white

Node:

id: an integer

links: an array of integers

color: white, black, or grey

Graph:

g: an array of nodes

DFS: with coloring

- Is this enough?
 - Detect a cycle if it exists?
 - Stops if there is a cycle?

Given: `g` – a graph
`n` – an integer
 (of some node in graph)
Return -- true if there is a cycle

```
DFS(g, n):  
    g[n].color=grey  
    for link in g[n].links:  
        if g[link].color==grey  
            return true  
        if DFS(g, link)==true  
            return true  
    g[n].color=black  
    return false
```

Final Algorithm

```
for i in 0..nodes.count
    g[i].color=white
for i in 0..nodes.count
    if g[i].color==white
        if DFSc(g,i)
            return true
return false
```

```
Given: g – a graph
       n – an integer
           (of some node in graph)
Return -- true if there is a cycle

DFSc(g, n):
    g[n].color=grey
    for link in g[n].links:
        if g[link].color==grey
            return true
        if DFSc(g, link)==true
            return true
    g[n].color=black
    return false
```

Aliases

assignment_num	count	graded
2	12	18
3	11	13
4	9	17
5	9	16
6	6	15
20	1	
21	1	

Graph Representations

Name			
Implementation (Java)			
O(space)			

Topological sort

- Realistically do not need, just need to find nodes with in-degree of zero
 - Complexity of finding this?
- Can we use my cycle colorer to determine path from Washington to DC

Finding Cycles and Topological Sort

Topo Sort Algorithm

Count in-degree of all nodes

Add all indegree=0 to Q

While Q

 remove n from Q

 for each e (edge from n)

 reduce indegree of node e.too

 if indegree of e.too == 0

 add e.too to Q

- for a graph G is topo sort unique
- For cycle finding do I need to do a full Topological Sort?
- if not, what do I need?

Graph Representations

Name			
Implementation (Java)			
O(space)			
Cost to find indegree: of each node of a given node			
Cost to find outdegree: of each node of a given node			

Final Algorithm With Topo Step 1

```
for i in 0..nodes.count
  g[i].color=white
for n in nodes with indegree==0
  if g[i].color==white
    if DFSc(g,i)
      return true
return false

Given: g - a graph
      n - an integer
      (of some node in graph)
Return -- true if there is a cycle

DFSc(g, n):
  g[n].color=grey
  for link in g[n].links:
    if g[link].color==grey
      return true
    if DFSc(g, link)==true
      return true
  g[n].color=black
  return false
```

Is Topo (step 1) worth doing?

- Time complexity??
- Likely real-world time??

Use Coloring for maze walking?

"Death of a salesman" problem is, at core, a maze

- Can I?
- Algorithm
 - change from -->
- Benefits?
- Costs?

```
Given: g - a graph
        n - an integer
           (of some node in graph)
Return -- true if there is a cycle
```

```
DFSc(g, n):
    g[n].color=grey
    for link in g[n].links:
        if g[link].color==grey
            return true
        if DFSc(g, link)==true
            return true
    g[n].color=black
    return false
```