CS 260 – Foundations of Data Science

Lecture 4 – Linear models, vectors, and matrices

09/10/2026

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Announcements

HW2 due Tuesday night

Complete pre-course survey

Office Hours:

Tuesdays: 4pm-4:30pm

Thursdays 2-3:30pm, Park 200D

Outline

Recap *simple* (i.e. p=1) linear regression

Introduction to applied linear algebra

Multiple linear regression

Analytic solution to multiple linear regression

Simple linear regression

Model: $h_{\vec{w}}(x) = w_0 + w_1 x = \hat{y}$

Cost function:

$$J(w_0, w_1) = \frac{1}{2}\sum_{i=1}^{n}(y_i - w_0 - w_1 x_i)^2$$

$$\hat{w}_1 = \frac{\mathrm{Cov}(\boldsymbol{x}, \boldsymbol{y})}{\mathrm{Var}(\boldsymbol{x})} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \hat{w}_0 = \bar{y} - \hat{w}_1\bar{x}$$

Handout 4

Goals of fitting a linear model

1) Which of the features/explanatory variables/predictors (x) are associated with the response variable (y)?

2) What is the relationship between x and y?

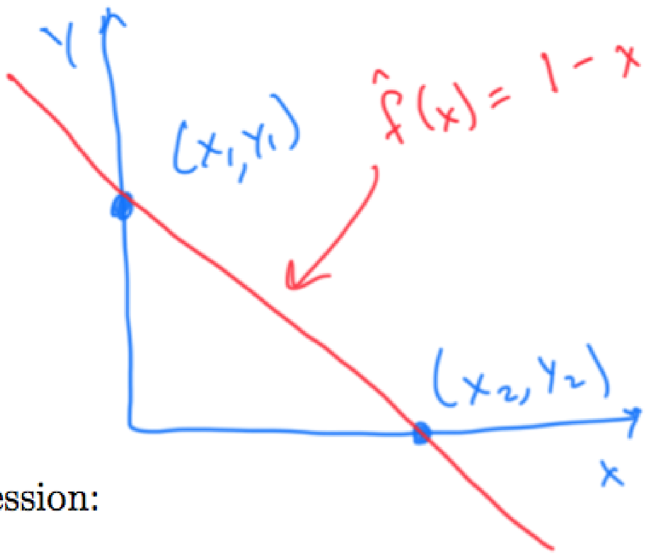
3) Can we (accurately) predict y given a new x?

4) Is a linear model enough?

Handout 4

Let $n = 2$ and $p = 1$, with the following data (we will omit the first column of 1's in simple linear regression):

$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$



(a) Plot these two points – what should $\hat{w}_0$ and $\hat{w}_1$ be?

$\hat{w}_0 = 1$

$\hat{w}_1 = -1$

(b) This week we derived the solution for simple linear regression:

note: $\bar{x} = \frac{1}{2}$, $\bar{y} = \frac{1}{2}$

$$\hat{w}_1 = \frac{\text{Cov}(\boldsymbol{x}, \boldsymbol{y})}{\text{Var}(\boldsymbol{x})} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad \hat{w}_0 = \bar{y} - \hat{w}_1\bar{x}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. Use these equations to compute $\hat{w}_0$ and $\hat{w}_1$ and verify your answer to (a).

$$\hat{w}_1 = \frac{\cancel{\tfrac{1}{2}}\left[(1-\tfrac{1}{2})(0-\tfrac{1}{2}) + (0-\tfrac{1}{2})(1-\tfrac{1}{2})\right]}{\cancel{\tfrac{1}{2}}\left[(1-\tfrac{1}{2})^2 + (0-\tfrac{1}{2})^2\right]}$$

$$= \frac{-\frac{1}{4} - \frac{1}{4}}{\frac{1}{4} + \frac{1}{4}} \Rightarrow \boxed{\hat{w}_1 = -1}$$

$\Rightarrow \hat{w}_0 = \frac{1}{2} - (-1)\frac{1}{2}$

$\Rightarrow \boxed{\hat{w}_0 = 1}$

Maybe a linear model is not enough

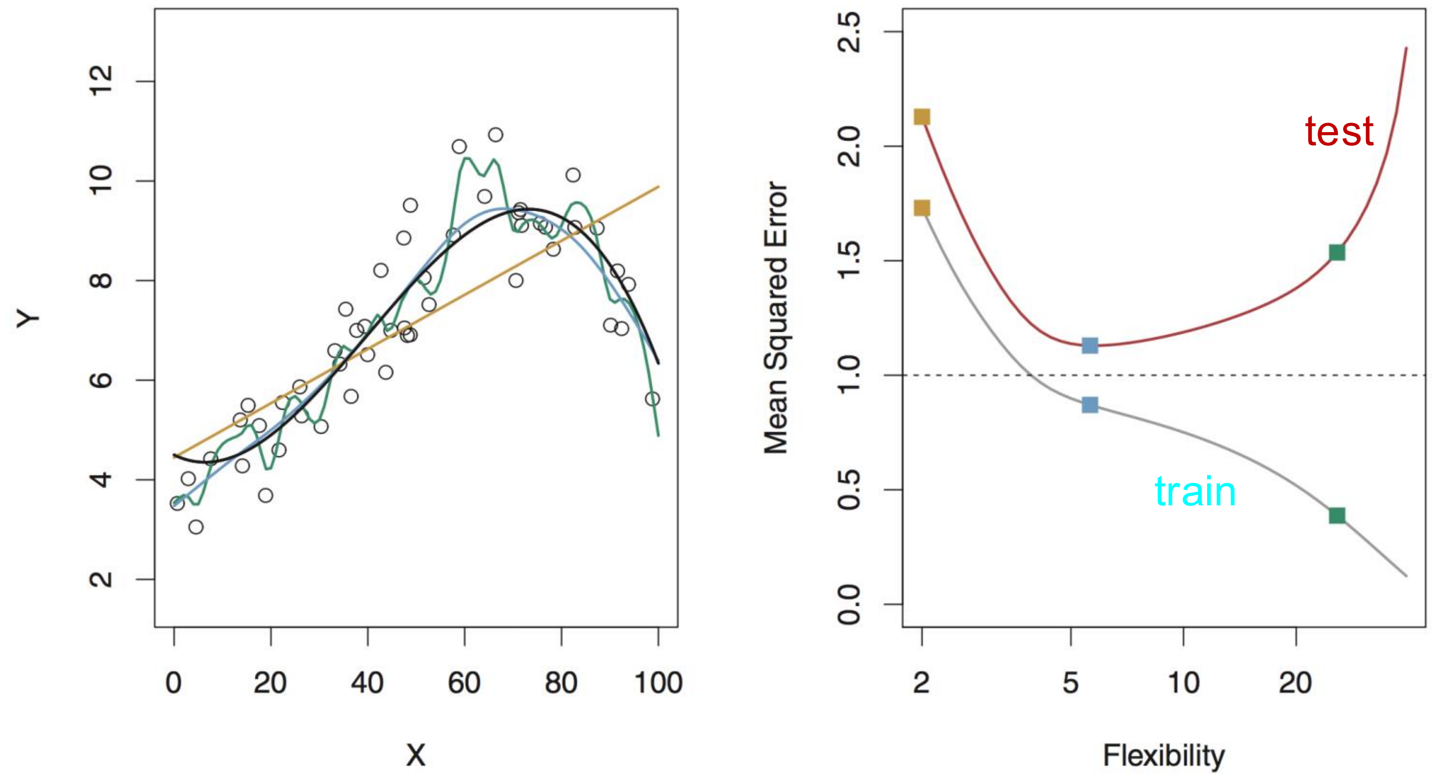


FIGURE 2.9. Left: *Data simulated from f, shown in black. Three estimates of f are shown: the linear regression line (orange curve), and two smoothing spline fits (blue and green curves).* Right: *Training MSE (grey curve), test MSE (red curve), and minimum possible test MSE over all methods (dashed line). Squares represent the training and test MSEs for the three fits shown in the left-hand panel.*

ISL, Fig 2.9

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Vectors

Vector magnitude

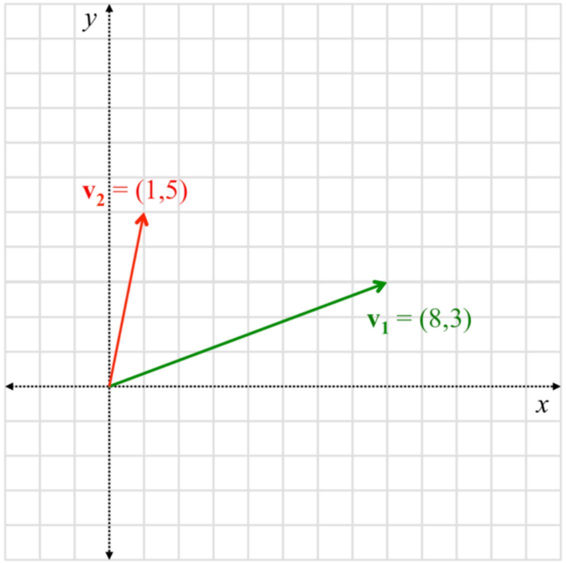
$$\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{then} \quad |\mathbf{v}| = \sqrt{x^2 + y^2}.$$

Different ways to write a vector

$$\mathbf{v} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{T}}$$

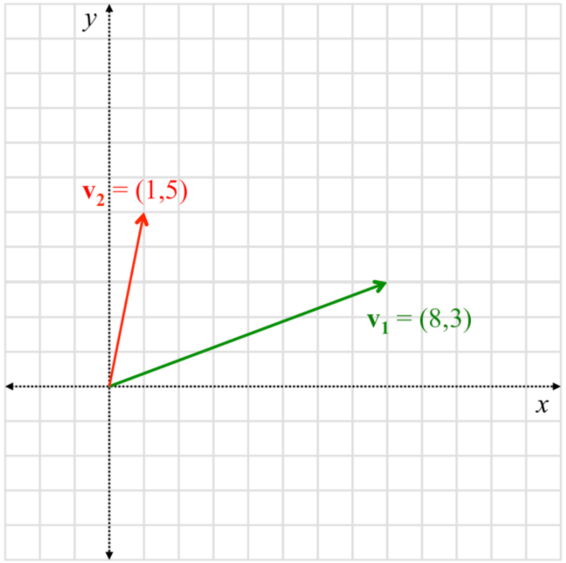
Vector Addition

$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$



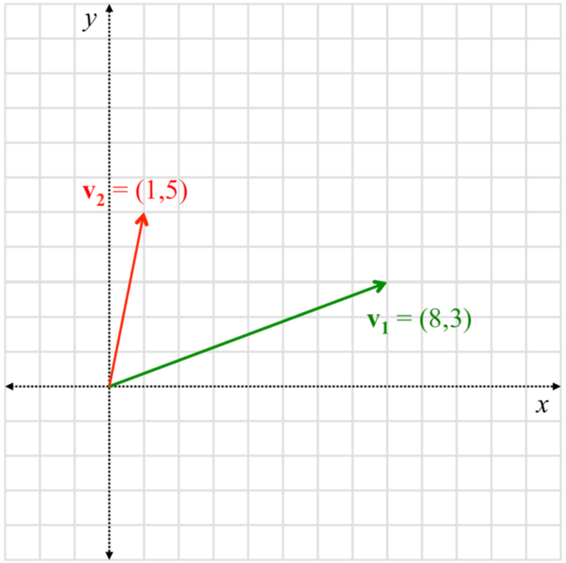
Vector Addition

$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8+1 \\ 3+5 \end{bmatrix}$$



Vector Addition

$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8+1 \\ 3+5 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$



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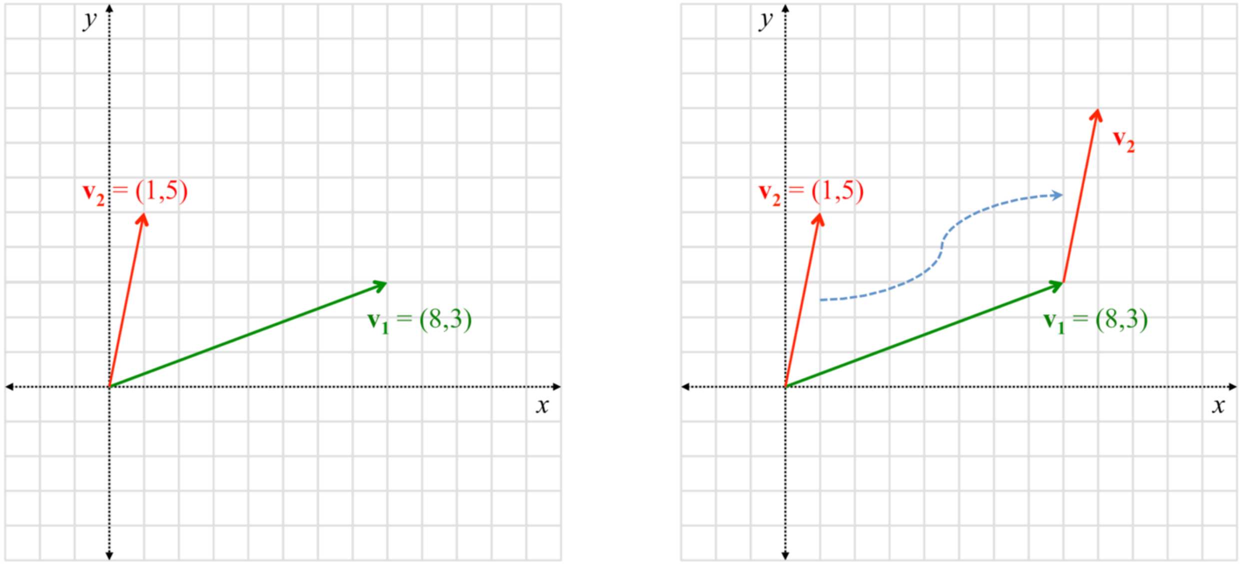
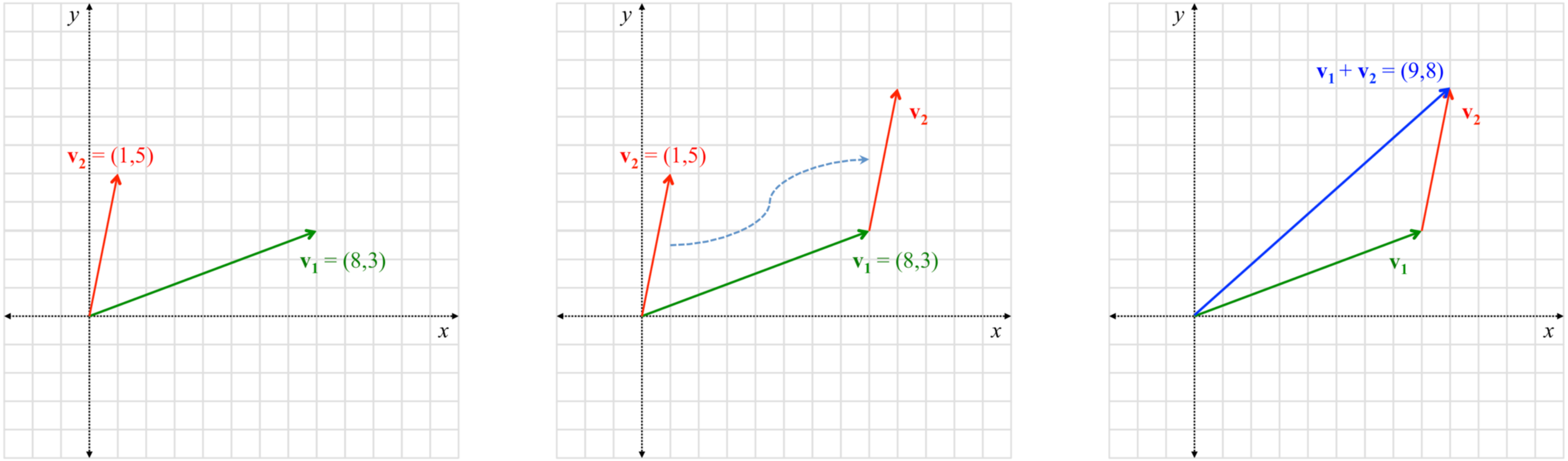
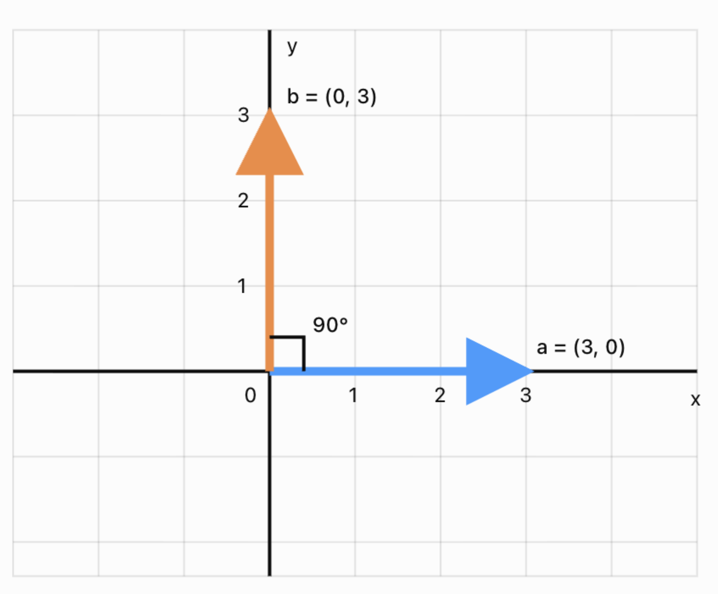


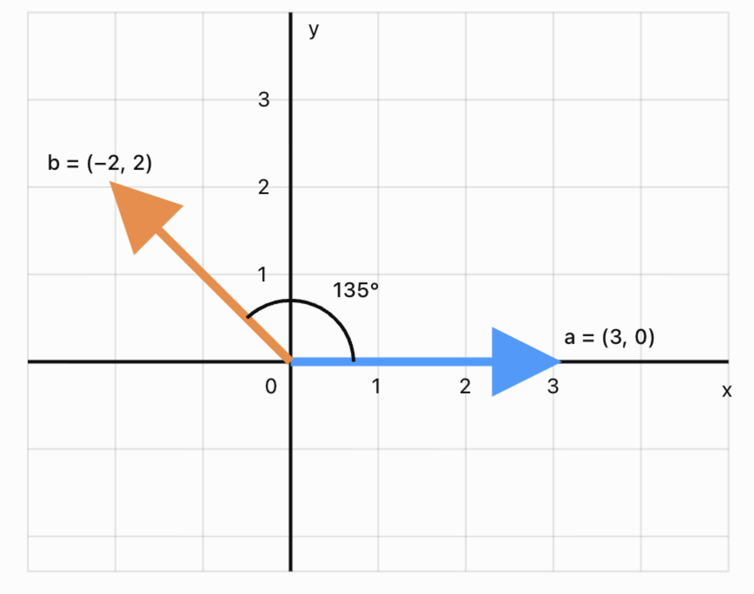
Vector Addition

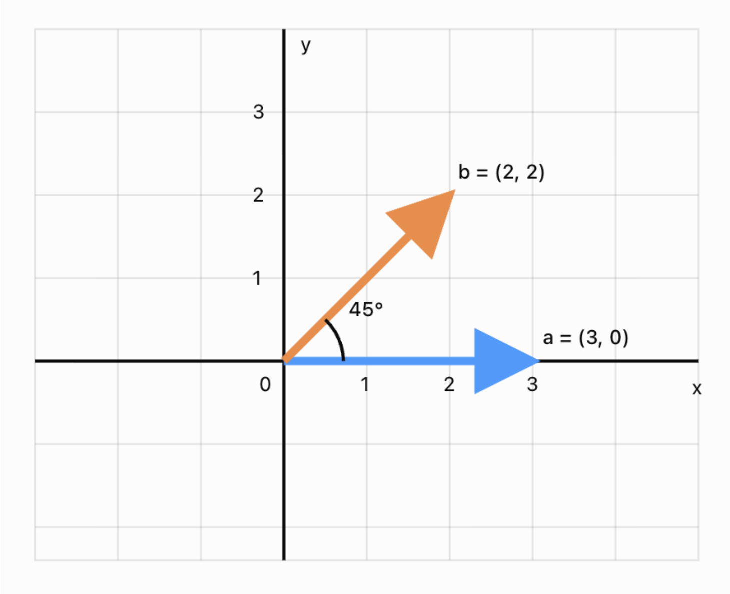
$$\mathbf{v}_1 + \mathbf{v}_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8+1 \\ 3+5 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$



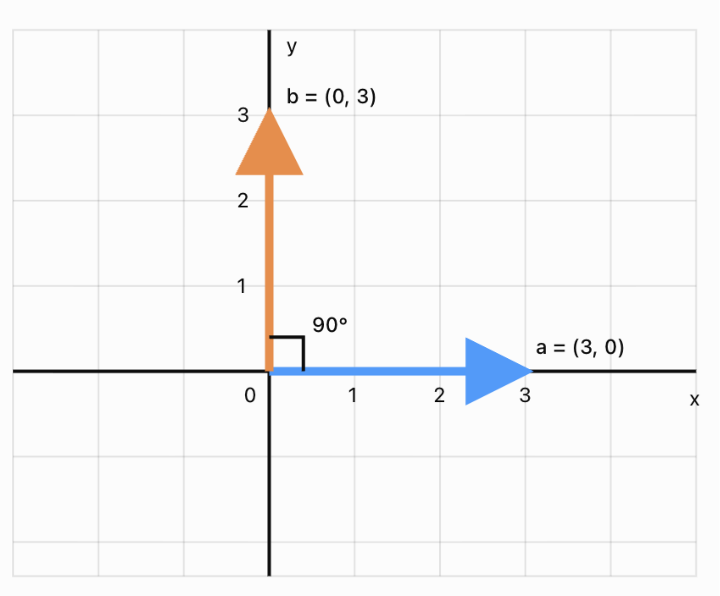
How aligned are these vectors



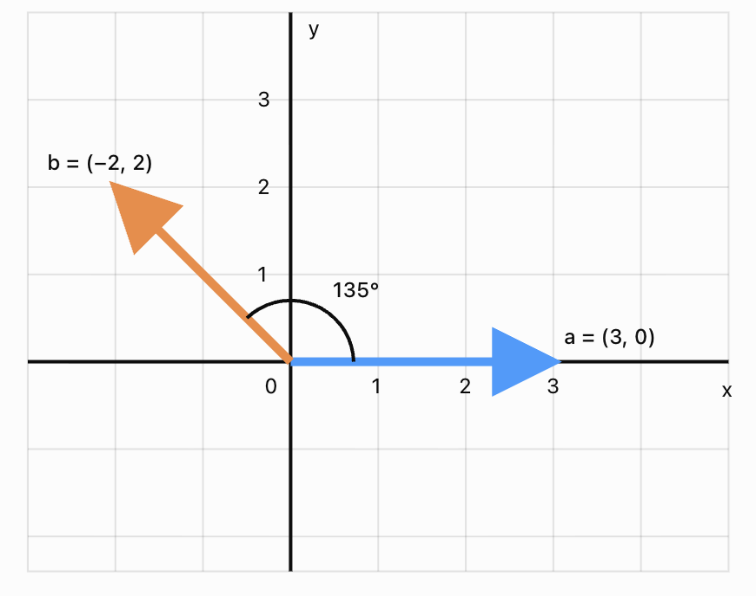




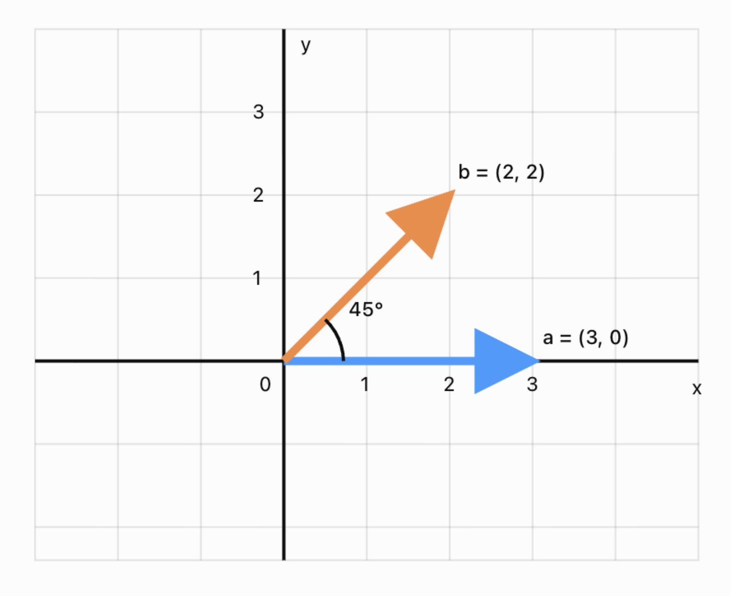
How aligned are these vectors



No alignment

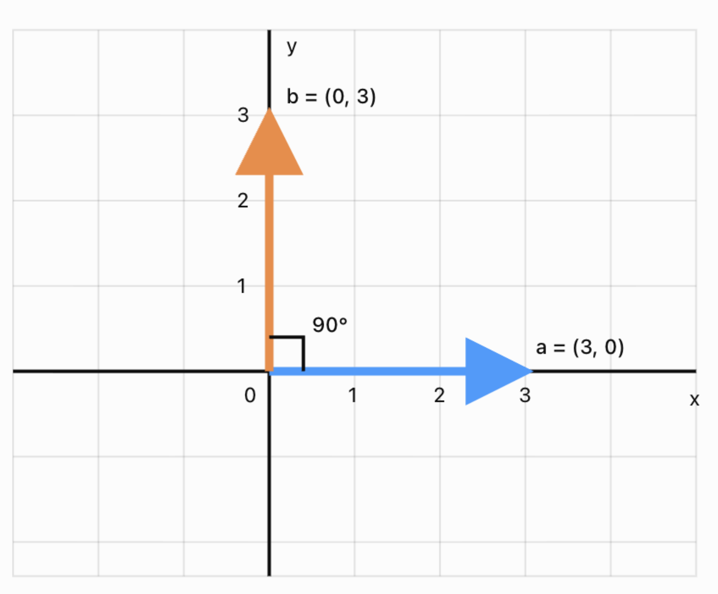


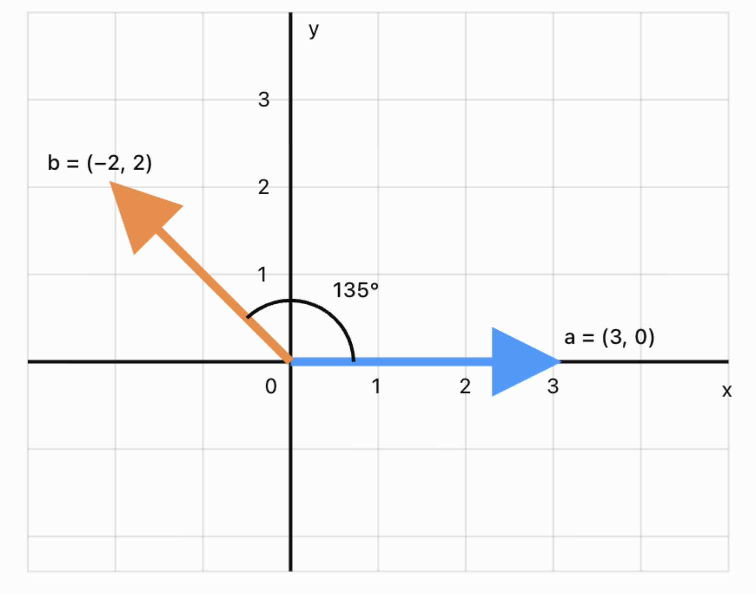
Pointing apart

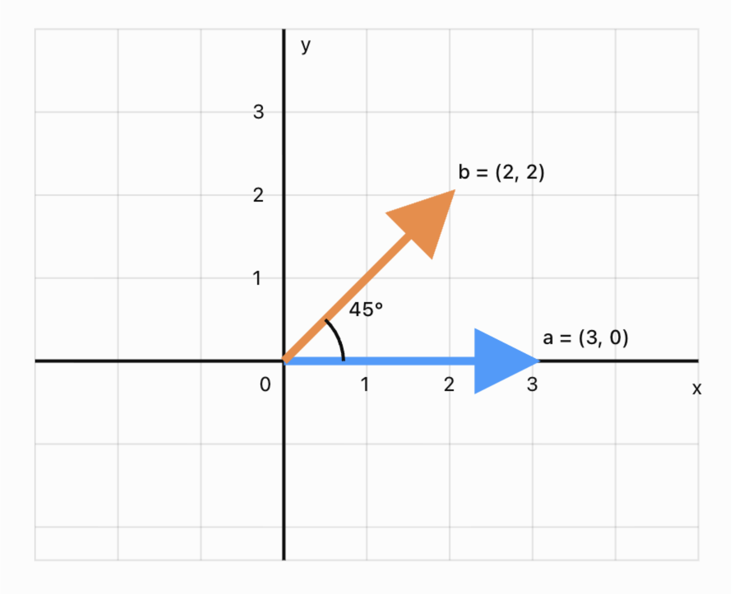


Pointing together

How aligned are these vectors







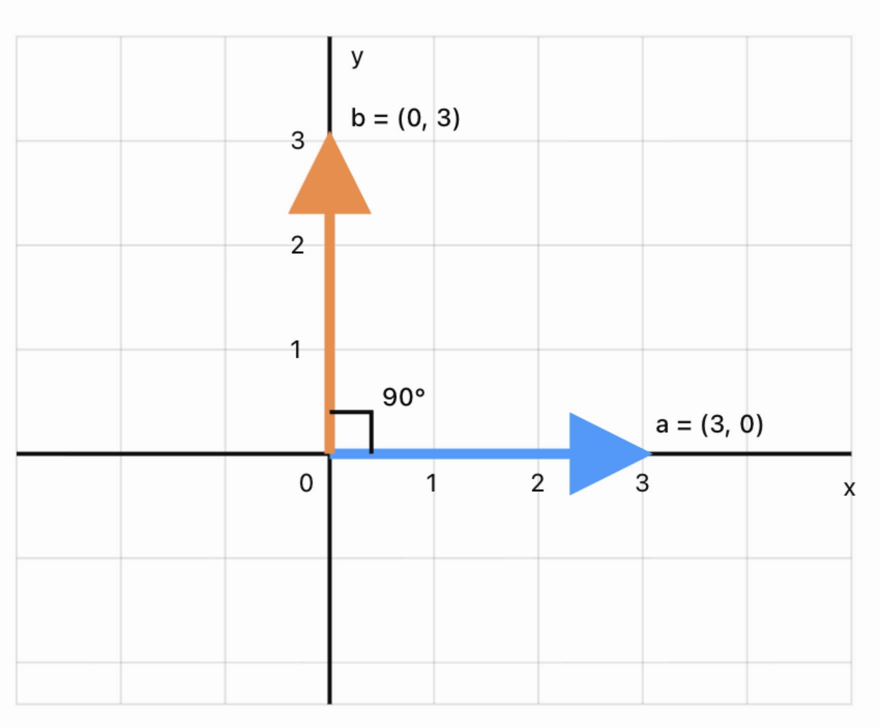
No alignment
0

Pointing apart
< 0

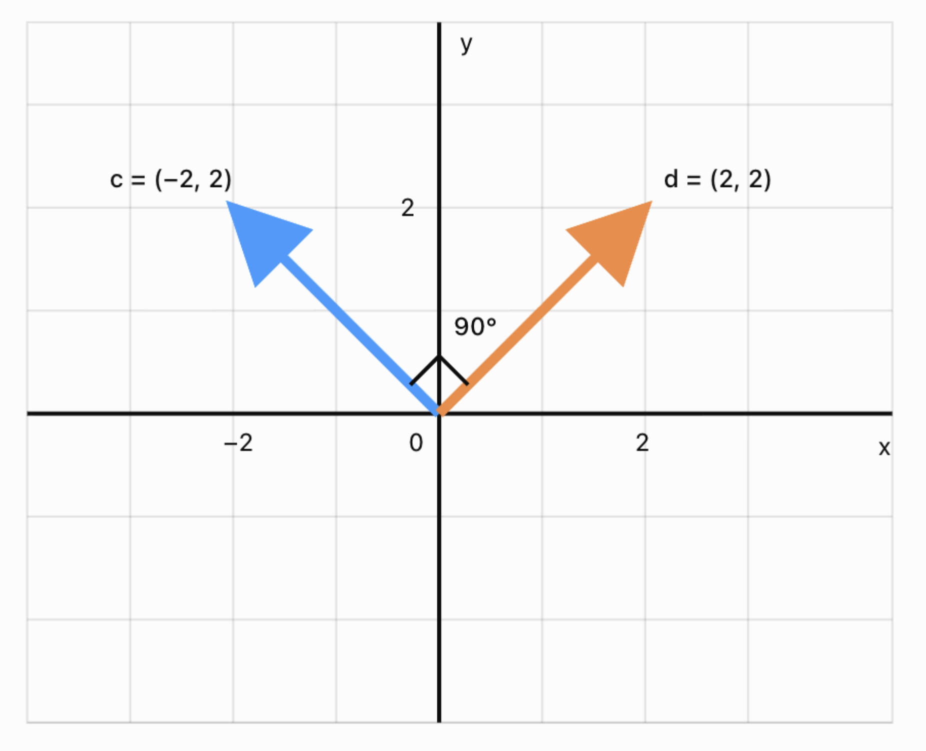
Pointing together
> 0

How aligned are these vectors

$\vec{a} = [3, 0], \vec{b} = [0, 3]$



$\vec{c} = [-2, 2], \vec{d} = [2, 2]$



No alignment - Should be 0

How aligned are these vectors

$\vec{\boldsymbol{a}} = [\mathbf{3}, \mathbf{0}], \vec{\boldsymbol{b}} = [\mathbf{0}, \mathbf{3}]$ $\vec{\boldsymbol{c}} = [-\mathbf{2}, \mathbf{2}], \vec{\boldsymbol{d}} = [\mathbf{2}, \mathbf{2}]$

What operation f can we perform s.t. $f\left(\vec{a}, \vec{b}\right) = f\left(\vec{c}, \vec{b}\right) = 0$?

Vector Dot Product

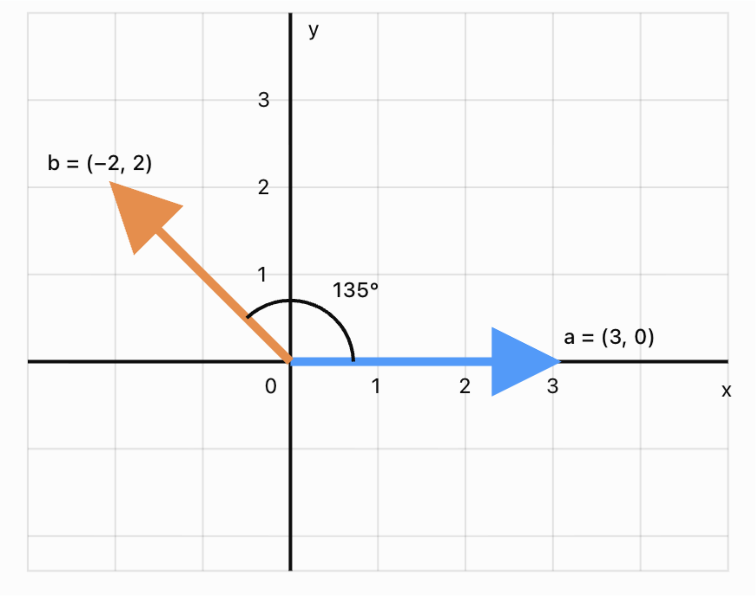
$$\vec{\boldsymbol{a}} = [\mathbf{3}, \mathbf{0}], \vec{\boldsymbol{b}} = [\mathbf{0}, \mathbf{3}] \qquad \vec{\boldsymbol{c}} = [-\mathbf{2}, \mathbf{2}], \vec{\boldsymbol{d}} = [\mathbf{2}, \mathbf{2}]$$

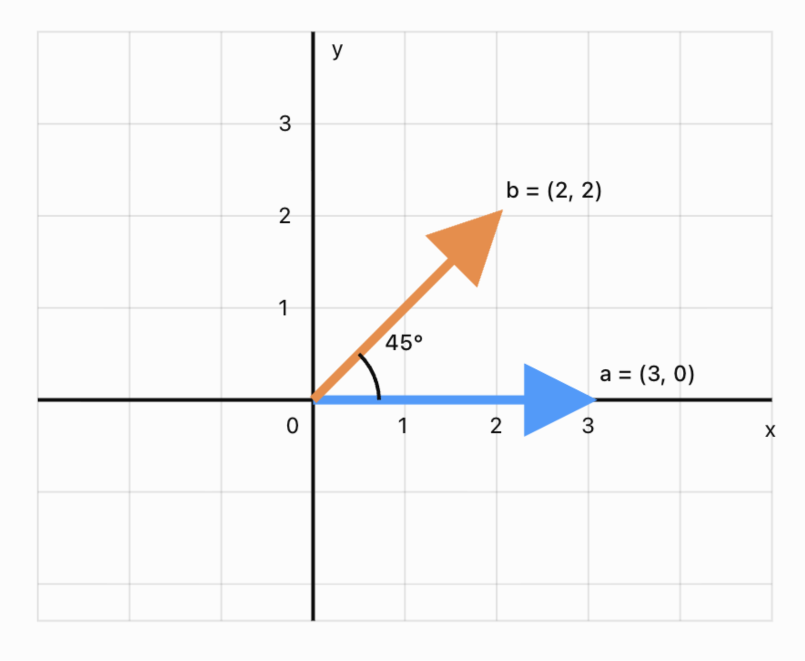
What operation f can we perform s.t. $f\left(\vec{a}, \vec{b}\right) = f\left(\vec{c}, \vec{b}\right) = 0$?

$$\vec{\boldsymbol{a}} \cdot \vec{\boldsymbol{b}} = \sum_{i}^{n} a_i * b_i$$

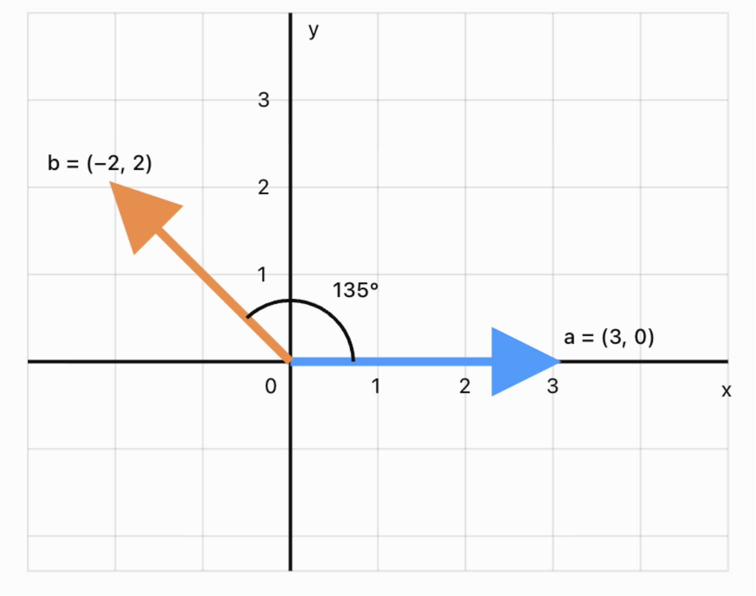
If $f\left(\vec{a}, \vec{b}\right) = 0$, then $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$ are ***<u>orthogonal</u>***

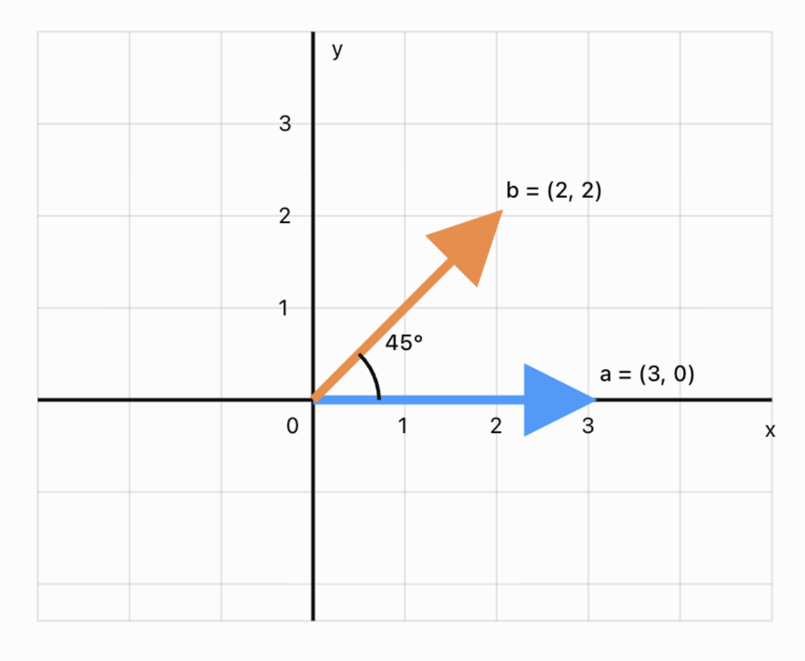
Vector Dot Product





Vector Dot Product





$$(-2 \ * 3) + (2 \ * 3) = -6$$

$$(2 \ * 3) + (2 \ * 3) = 6$$

Matrices

$$A = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ | & | & | \end{bmatrix}$$

$$A = \begin{bmatrix} — & \mathbf{a}_1 & — \\ — & \mathbf{a}_2 & — \\ — & \mathbf{a}_3 & — \end{bmatrix}$$

Matrices

Matrix addition (must be exactly the same dimension!)

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix}$$

Matrices

Matrix addition (must be exactly the same dimension!)

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Matrix Multiplication

Inner dimensions must match

If A.shape = (m, n) and B.shape = (n, p), then
AB.shape = (m,p)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & \\ & \end{bmatrix}$$

dot product of row 1 and col 1

Matrix Multiplication

Inner dimensions must match

If A.shape = (m, n) and B.shape = (n, p), then AB.shape = (m,p)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}=\begin{bmatrix} ae+bg & \\ & \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}=\begin{bmatrix} ae+bg & af+bh \\ & \end{bmatrix}$$

dot product of row 1 and col 2

Matrix Multiplication

Inner dimensions must match

If A.shape = (m, n) and B.shape = (n, p), then AB.shape = (m,p)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & \\ & \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ & \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Matrix Multiplication

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 2 \\ -4 & 1 \end{bmatrix}.$$

$$\mathbf{AB} = \begin{bmatrix} & \\ & \end{bmatrix} \quad \text{and} \quad \mathbf{BA} = \begin{bmatrix} & \\ & \end{bmatrix}.$$

Identity Matrix

Identity function: a function that returns its input unchanged

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad I = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Find ***I*** s.t.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Identity Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity Matrix

$$\begin{bmatrix}1 & 2 & 3\\0 & 1 & 4\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \text{first column of } A,$$

Identity Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \text{first column of } A,$$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \text{second column of } A$$

Identity Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \text{first column of } A,$$

$$A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \text{second column of } A$$

$$A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \text{third column of } A$$

Matrix Multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ | & | & | \end{bmatrix}$$

$$1\mathbf{a}_1 + 0\mathbf{a}_2 + 1\mathbf{a}_3$$

$$= 1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} | & | & | \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ | & | & | \end{bmatrix}$$

$$0\mathbf{a}_1 + 1\mathbf{a}_2 + 1\mathbf{a}_3$$

$$= 0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 5 \\ 1 \end{bmatrix}$$

Matrix Multiplication

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} | & | \\ 1\mathbf{a}_1 + 0\mathbf{a}_2 + 1\mathbf{a}_3 & 0\mathbf{a}_1 + 1\mathbf{a}_2 + 1\mathbf{a}_3 \\ | & | \end{bmatrix} = \boxed{\begin{bmatrix} 4 & 5 \\ 4 & 5 \\ 1 & 1 \end{bmatrix}}$$

Matrix Multiplication & Inverse

$$\mathbf{A}\mathbf{x} = \mathbf{y}$$

Matrix **A** is function that maps/transforms x into y

$$f(x) = y$$

Inverse function: a function that reverses what the original function does

$$f^{-1}(y) = x.$$

Matrix Inverse

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{I}$$

Matrix Inverse

$$\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{I}$$

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \boldsymbol{A}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix Transpose

$$\boldsymbol{A} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad \boldsymbol{A}^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

Useful note: $(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$

Outline

Recap *simple* (i.e. p=1) linear regression

Introduction to applied linear algebra

***Multiple* linear regression**

Analytic solution to multiple linear regression

HW 3: USA Housing data

Avg. Area Income	Avg. Area House Age	Avg. Area Number of Rooms	Avg. Area Number of Bedrooms	Area Population	Price
79545.45857	5.682861322	7.009188143	4.09	23086.8005	1059033.558
79248.64245	6.002899808	6.730821019	3.09	40173.07217	1505890.915
61287.06718	5.86588984	8.51272743	5.13	36882.1594	1058987.988
63345.24005	7.188236095	5.586728665	3.26	34310.24283	1260616.807
59982.19723	5.040554523	7.839387785	4.23	26354.10947	630943.4893
80175.75416	4.988407758	6.104512439	4.04	26748.42842	1068138.074
64698.46343	6.025335907	8.147759585	3.41	60828.24909	1502055.817
78394.33928	6.989779748	6.620477995	2.42	36516.35897	1573936.564
59927.66081	5.36212557	6.393120981	2.3	29387.396	798869.5328
81885.92718	4.42367179	8.167688003	6.1	40149.96575	1545154.813
80527.47208	8.093512681	5.0427468	4.1	47224.35984	1707045.722
50593.6955	4.496512793	7.467627404	4.49	34343.99189	663732.3969
39033.80924	7.671755373	7.250029317	3.1	39220.36147	1042814.098
73163.66344	6.919534825	5.993187901	2.27	32326.12314	1291331.518
69391.38018	5.344776177	8.406417715	4.37	35521.29403	1402818.21
73091.86675	5.443156467	8.517512711	4.01	23929.52405	1306674.66
79706.96306	5.067889591	8.219771123	3.12	39717.81358	1556786.6

x

y

Multiple Linear Regression

$$\hat{y} = h_{\vec{w}}(\vec{x})$$
$$= w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_p x_p = \vec{w} \cdot \vec{x}$$

"fake" ones

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \cdots & x_{1p} \\ 1 & & & & & \\ \vdots & & & & & \\ 1 & & & & & \end{bmatrix}$$

$n \times (p+1)$

- Goal: minimize cost function

$$J(\vec{w}) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \vec{w} \cdot \vec{x_i})^2$$

Multiple Linear Regression

$$\hat{y} = h_{\vec{w}}(X) \\ = \vec{w}X$$

- Goal: minimize cost function

$$J(\vec{w}) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (\vec{y} - \vec{w}X)^2$$

Outline

Recap *simple* (i.e. p=1) linear regression

Introduction to applied linear algebra

Multiple linear regression

Analytic solution to multiple linear regression

Analytic solution to multiple linear regression

SSE

Take the derivative

Set to 0

Divide by -2

Distribute

Rearrange

Multiply by inverse

Analytic solution to multiple linear regression

SSE

Take the derivative

Set to 0

Divide by -2

Distribute

Rearrange

Multiply by inverse

$$\boxed{\hat{\mathbf{w}} = (X^T X)^{-1} X^T \mathbf{y}}.$$

Variance of X

Covariance of X and $\vec{y}$

Analytic solution to multiple linear regression

$$J(\vec{w}) = \frac{1}{2}\sum_{i=1}^{n}(y_i - \vec{w}\cdot\vec{x_i})^2 = \frac{1}{2}(\vec{y} - X\vec{w})\cdot(\vec{y} - X\vec{w})$$

$$= \frac{1}{2}(\vec{y}\cdot\vec{y} - 2\vec{y}\cdot X\vec{w} + X\vec{w}\cdot X\vec{w})$$

- Take derivative and set to $\vec{0}$

$$\frac{\partial J}{\partial \vec{w}} = -X^T\vec{y} + (X^TX)\vec{w} = \vec{0}$$

$$\leftrightarrow (X^TX)\vec{w} = X^T\vec{y}$$

$$\leftrightarrow (X^TX)^{-1}(X^TX)\vec{w} = (X^TX)^{-1}X^T\vec{y}$$

$$\leftrightarrow \vec{w} = \underbrace{(X^TX)}_{\text{Variance of X}}{}^{-1}\underbrace{X^T\vec{y}}_{\text{Covariance of X and }\vec{y}}$$

(Keep this formula and its interpretation in mind!)

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Handout 4

2. $X = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

"fake" ones

3. $\vec{w} = (X^T X)^{-1} X^T \vec{y}$

$$= \left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{2-1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Same as before!