

Eigenvectors and Eigenvalues*(find and work with a partner)*

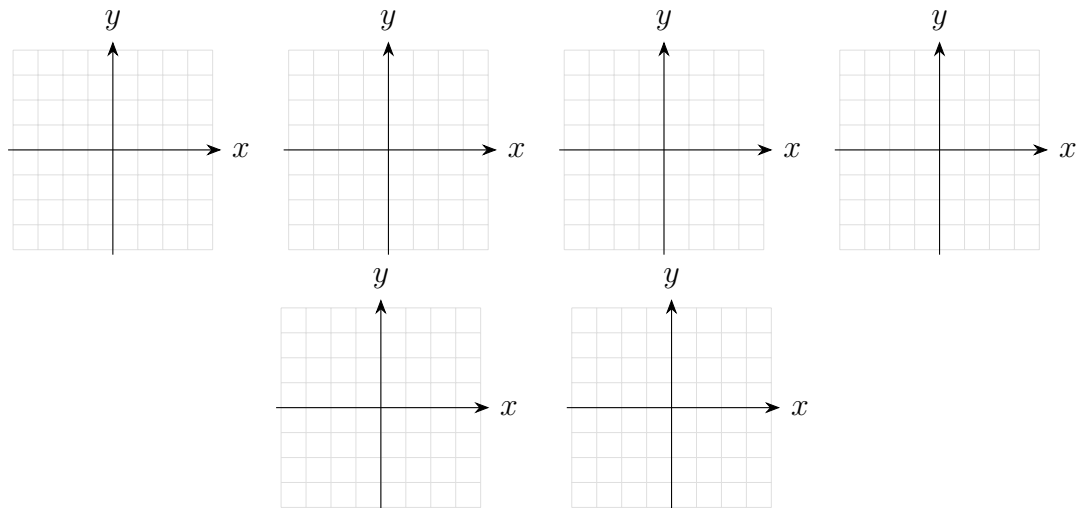
1. **What happens to a vector?** Consider the matrix

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$$

For each vector $\mathbf{v}$, compute $A\mathbf{v}$.

	$\mathbf{v}$	$A\mathbf{v}$
$\mathbf{v}_1$	$(1, 0)$	
$\mathbf{v}_2$	$(0, 1)$	
$\mathbf{v}_3$	$(1, 1)$	
$\mathbf{v}_4$	$(2, 0)$	
$\mathbf{v}_5$	$(1, -1)$	

- (a) Sketch the original and transformed vectors. Label your arrows.



- (b) Which vectors remain on the same *line through the origin*? Which change direction?

- (c) By what factor are the vectors on those unchanged lines scaled?

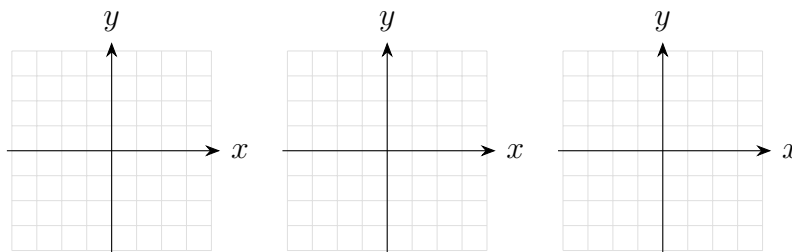
2. **Finding special directions.** Now consider

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

(a) Calculate $B\mathbf{v}$ for each of the following:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(b) Which of these vectors remain on the same line through the origin?



(c) For each one, find a number λ such that $B\mathbf{v} = \lambda\mathbf{v}$.

(d) Find another nonzero vector satisfying this equation. How many might there be?

Eigenvectors: Naming the Pattern

A nonzero vector $\mathbf{v}$ is an **eigenvector** of a matrix M if

$$M\mathbf{v} = \lambda\mathbf{v}$$

for some number λ , its **eigenvalue**. The transformed vector stays on the same line through the origin (or becomes zero). It may stretch, shrink, or reverse direction.

3. **What does the eigenvalue mean?** For each transformation and vector, determine its eigenvalue or write “not an eigenvector.”

Transformation	Vector	Transformed Vector	Eigenvalue (or not)
$(x, y) \mapsto (4x, 2y)$	$(1, 0)$		
$(x, y) \mapsto (4x, 2y)$	$(1, 1)$		
$(x, y) \mapsto (-x, -y)$	$(1, 2)$		
$(x, y) \mapsto (x, 0)$	$(0, 1)$		
$(x, y) \mapsto (3x, 3y)$	$(2, 1)$		

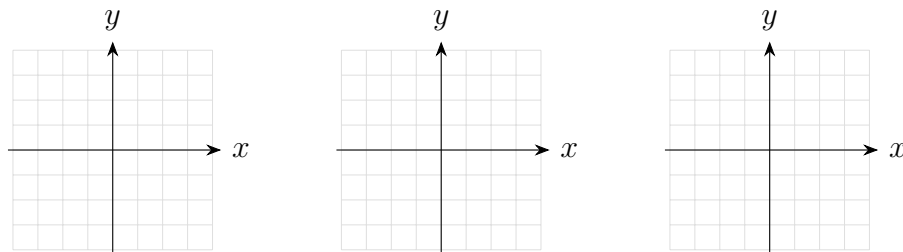
- (a) What does a **negative** eigenvalue mean geometrically?
- (b) What does an eigenvalue of **zero** mean?
- (c) Can two different vectors have the same eigenvalue? Give an example.

4. **Does every matrix have real eigenvectors?** Consider a 90-degree counterclockwise rotation:

$$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

(a) Compute $R(1, 0)$, $R(0, 1)$, and $R(1, 1)$.

(b) Sketch the three transformations on the axes.



(c) Can you find *any* nonzero real vector that stays on its original line? Explain.

(d) Does every 2×2 real matrix have a real eigenvector?

Bonus: Find a 2×2 matrix for which every nonzero vector is an eigenvector.