Matrix Algebra Basics

Based on slides by Pam Perlich (U.Utah)

Matrix

A matrix is any doubly subscripted array of elements arranged in rows and columns.

$$\mathbf{A} = \begin{bmatrix} a_{11} & ,\ldots, & a_{1n} \\ a_{21} & ,\ldots, & a_{2n} \\ \ldots & \ldots & \ldots\ \ldots \\ a_{m1} & ,\ldots, & a_{mn} \end{bmatrix} = \{A_{ij}\}$$

Row Vector

[1 x n] matrix

$$A\left[a_1 a_2, \ldots, a_n\right] = \left\{a_j\right\}$$

Column Vector

[m x 1] matrix

$$A = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_m \end{bmatrix} = \{a_i\}$$

Square Matrix

Same number of rows and columns

$$B = \begin{bmatrix} 5 & 4 & 7 \\ 3 & 6 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Identity Matrix

Square matrix with ones on the diagonal and zeros elsewhere.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transpose Matrix

Rows become columns and columns become rows

$$A' = \begin{bmatrix} a_{11} & a_{21} & ,\ldots, & a_{m1} \\ a_{12} & a_{22} & ,\ldots, & a_{m2} \\ \ldots & \ldots & \ldots & \ldots \quad \ldots \\ a_{1n} & a_{2n} & ,\ldots, & a_{mn} \end{bmatrix}$$

Matrix Addition and Subtraction

A new matrix **C** may be defined as the additive combination of matrices **A** and **B** where: **C = A + B**
is defined by:

$$\{C_{ij}\} = \{A_{ij}\} + \{B_{ij}\}$$

Note: all three matrices are of the same dimension

Addition

If $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and $$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

then $$C = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

Matrix Addition Example

$$A + B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix} = C$$

Matrix Subtraction

$\mathbf{C} = \mathbf{A} - \mathbf{B}$

Is defined by

$$\{C_{ij}\} = \{A_{ij}\} - \{B_{ij}\}$$

Matrix Multiplication

Matrices A and B have these dimensions:

[r x s] and [s x c]

Matrix Multiplication

Matrices A and B can be multiplied if:

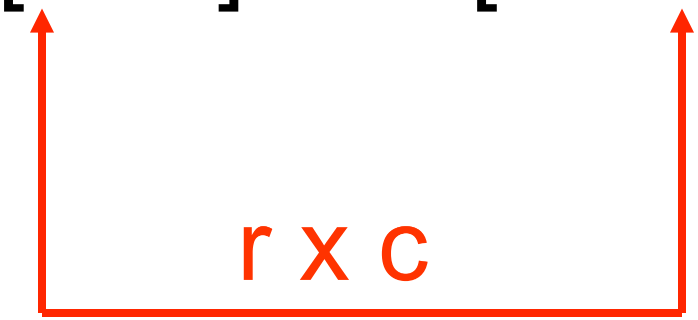
[r x s] and [s x c]

these must match

Matrix Multiplication

The resulting matrix will have the dimensions:

[r x s] and [s x c]



Computation: A x B = C

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad [2 \text{ x } 2]$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \quad [2 \text{ x } 3]$$

$$C = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \end{bmatrix}$$

[2 x 3]

Computation: A x B = C

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[3 x 2] [2 x 3]

A and B can be multiplied

$$C = \begin{bmatrix} 2*1+3*1=5 & 2*1+3*0=2 & 2*1+3*2=8 \\ 1*1+1*1=2 & 1*1+1*0=1 & 1*1+1*2=3 \\ 1*1+0*1=1 & 1*1+0*0=1 & 1*1+0*2=1 \end{bmatrix} = \begin{bmatrix} 5\,2\,8 \\ 2\,1\,3 \\ 1\,1\,1 \end{bmatrix}$$

[3 x 3]

Computation: A x B = C

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

[3 x 2] [2 x 3]

Result is 3 x 3

$$C = \begin{bmatrix} 2*1+3*1=5 & 2*1+3*0=2 & 2*1+3*2=8 \\ 1*1+1*1=2 & 1*1+1*0=1 & 1*1+1*2=3 \\ 1*1+0*1=1 & 1*1+0*0=1 & 1*1+0*2=1 \end{bmatrix} = \begin{bmatrix} 5\,2\,8 \\ 2\,1\,3 \\ 1\,1\,1 \end{bmatrix}$$

[3 x 3]

Matrix Inversion

$$B^{-1}B = BB^{-1} = I$$

Like a reciprocal
in scalar math

Like the number one
in scalar math