

# **Network Centrality**

Based on materials by Lada Adamic, UMichigan

# Network Centrality

Which nodes are most 'central' ?

Definition of 'central' varies by context/purpose.

Local measure:  
degree

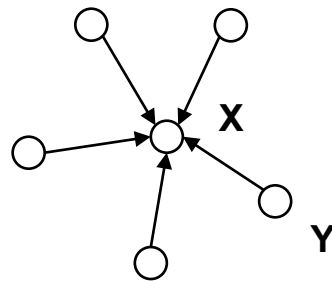
Relative to rest of network:  
closeness, betweenness,  
eigenvector (Bonacich power centrality)

How evenly is centrality distributed among nodes?  
centralization...

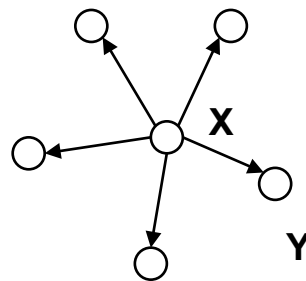
Applications:  
Friedkin: Interpersonal Influence in Groups  
Baker: The Social Organization of Conspiracy

# Centrality: Who's Important Based On Their Network Position

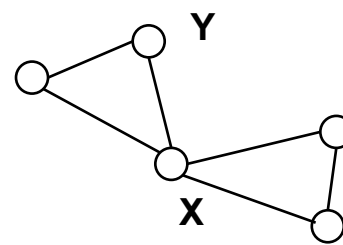
In each of the following networks, X has higher centrality than Y according to a particular measure



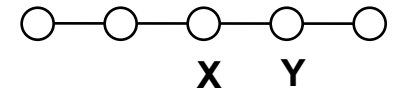
indegree



outdegree



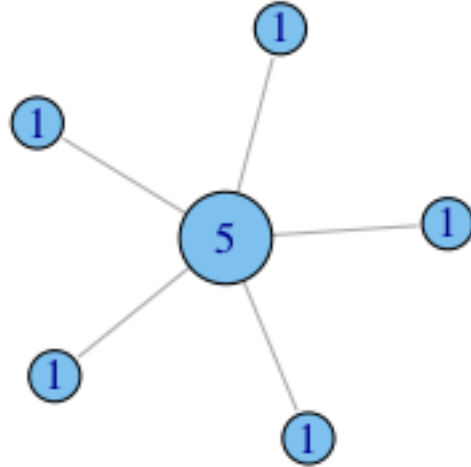
betweenness



closeness

## Degree Centrality (Undirected)

He or she who has many friends is most important.

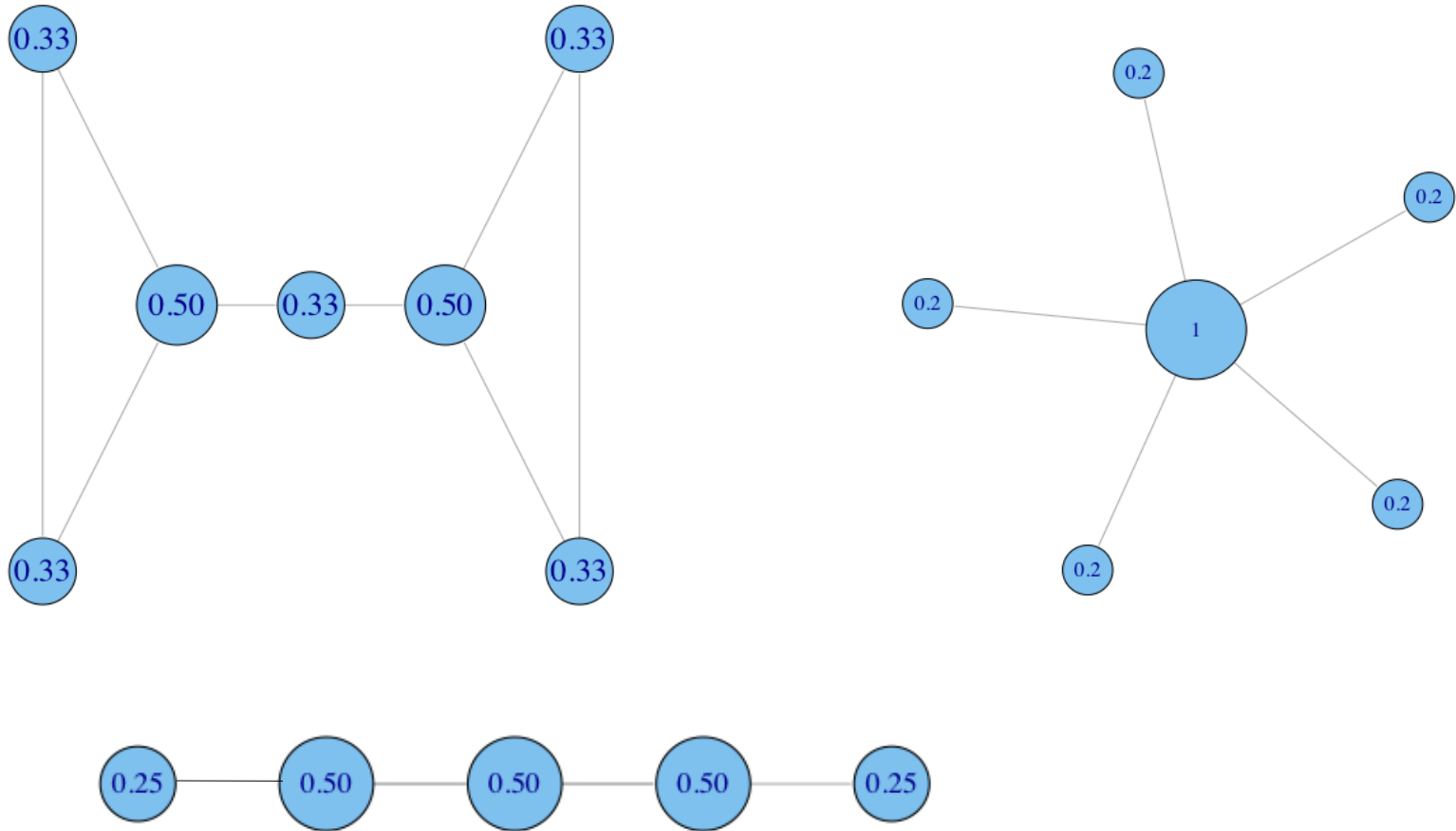


When is the number of connections the best centrality measure?

- people who will do favors for you
- people you can talk to / have coffee with

# Degree: Normalized Degree Centrality

divide by the max. possible, i.e. (N-1)



# Centralization: How Equal Are The Nodes?

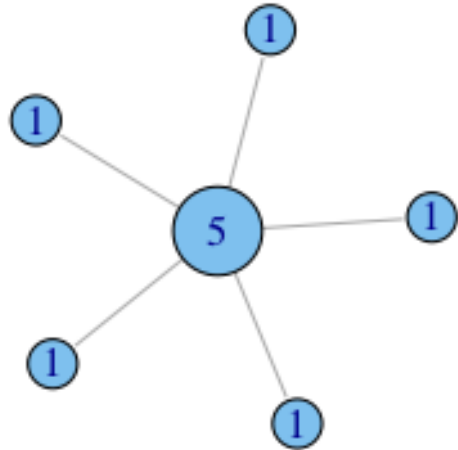
How much variation is there in the centrality scores among the nodes?

Freeman's general formula for centralization (can use other metrics, e.g. gini coefficient or standard deviation):

$$C_D = \frac{\sum_{i=1}^g [C_D(n^*) - C_D(i)]}{[(N-1)(N-2)]}$$

maximum value in the network

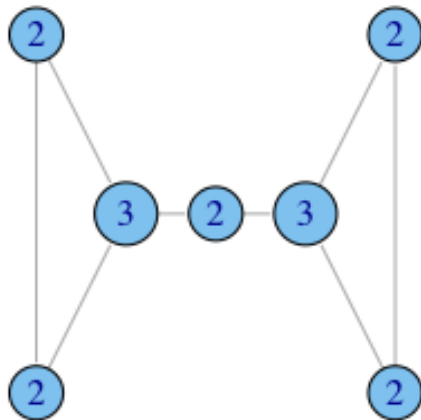
# Degree Centralization Examples



$$C_D = 1.0$$



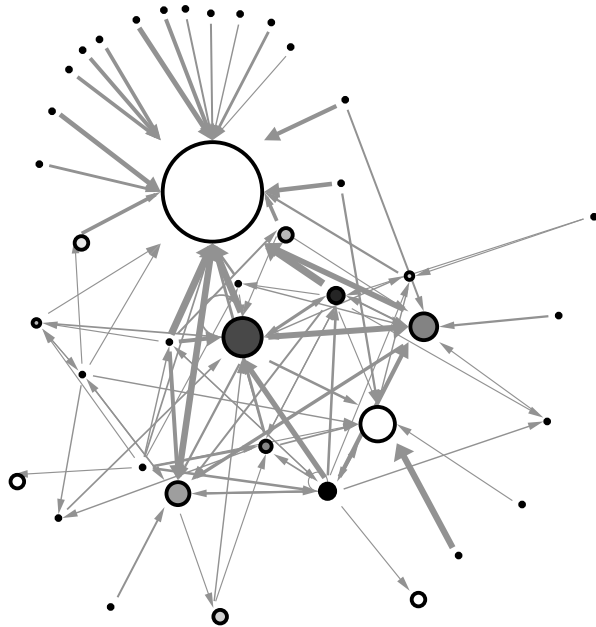
$$C_D = 0.167$$



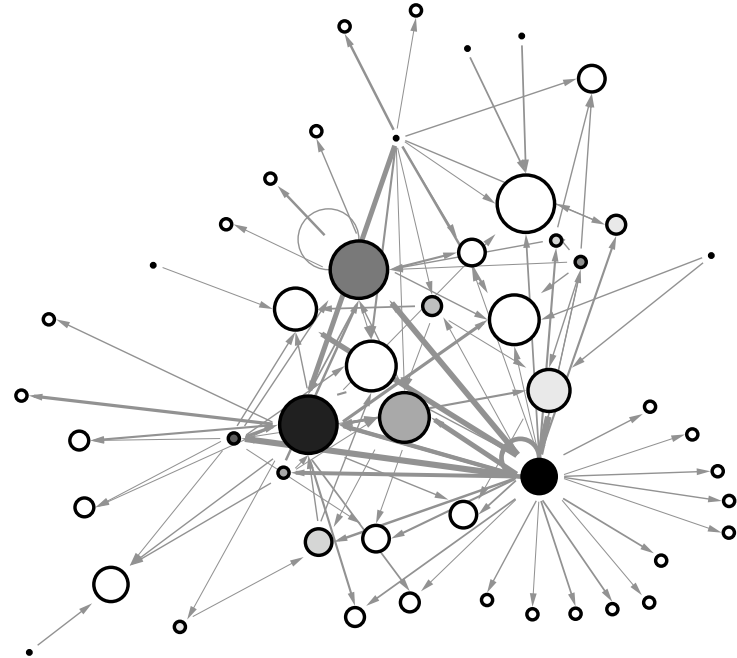
$$C_D = 0.167$$

# Degree Centralization Examples

example financial trading networks



high centralization: one node trading with many others

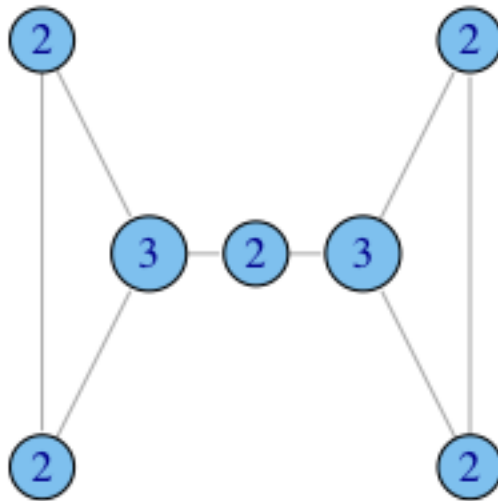


low centralization: trades are more evenly distributed



## When Degree Isn't Everything

In what ways does degree fail to capture centrality in the following graphs?

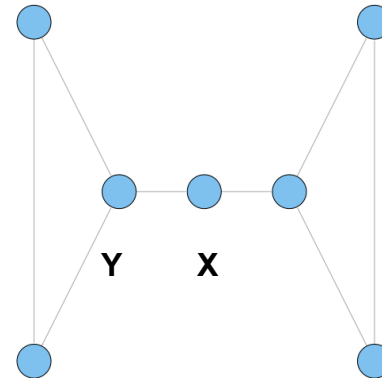
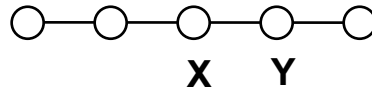
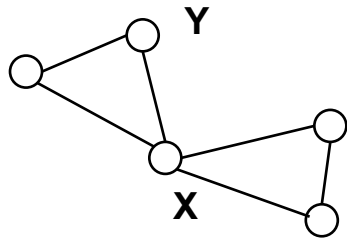


# In What Contexts May Degree Be Insufficient To Describe Centrality?

- ability to broker between groups
- likelihood that information originating anywhere in the network reaches you...

# Betweenness: Another Centrality Measure

- Intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?
- Who has higher betweenness, X or Y?



## Betweenness Centrality: Definition

$$C_B(i) = \sum_{j < k} g_{jk}(i) / g_{jk}$$

Where  $g_{jk}$  = the number of geodesics connecting  $jk$ , and  $g_{jk}(i)$  = the number of geodesics that actor  $i$  is on.

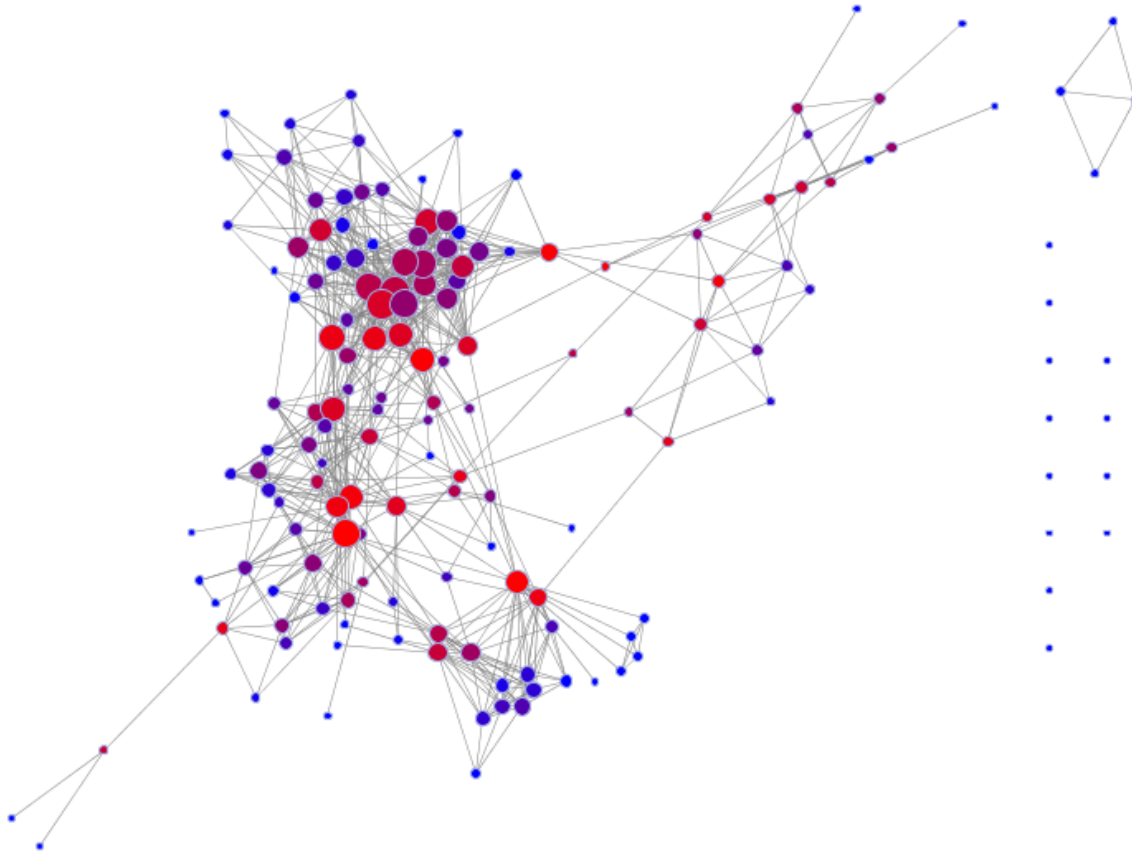
Usually normalized by:

$$C'_B(i) = C_B(i) / [(n-1)(n-2)/2]$$

number of pairs of vertices  
excluding the vertex itself

# Example

Example facebook network: nodes are sized by degree, and colored by betweenness.

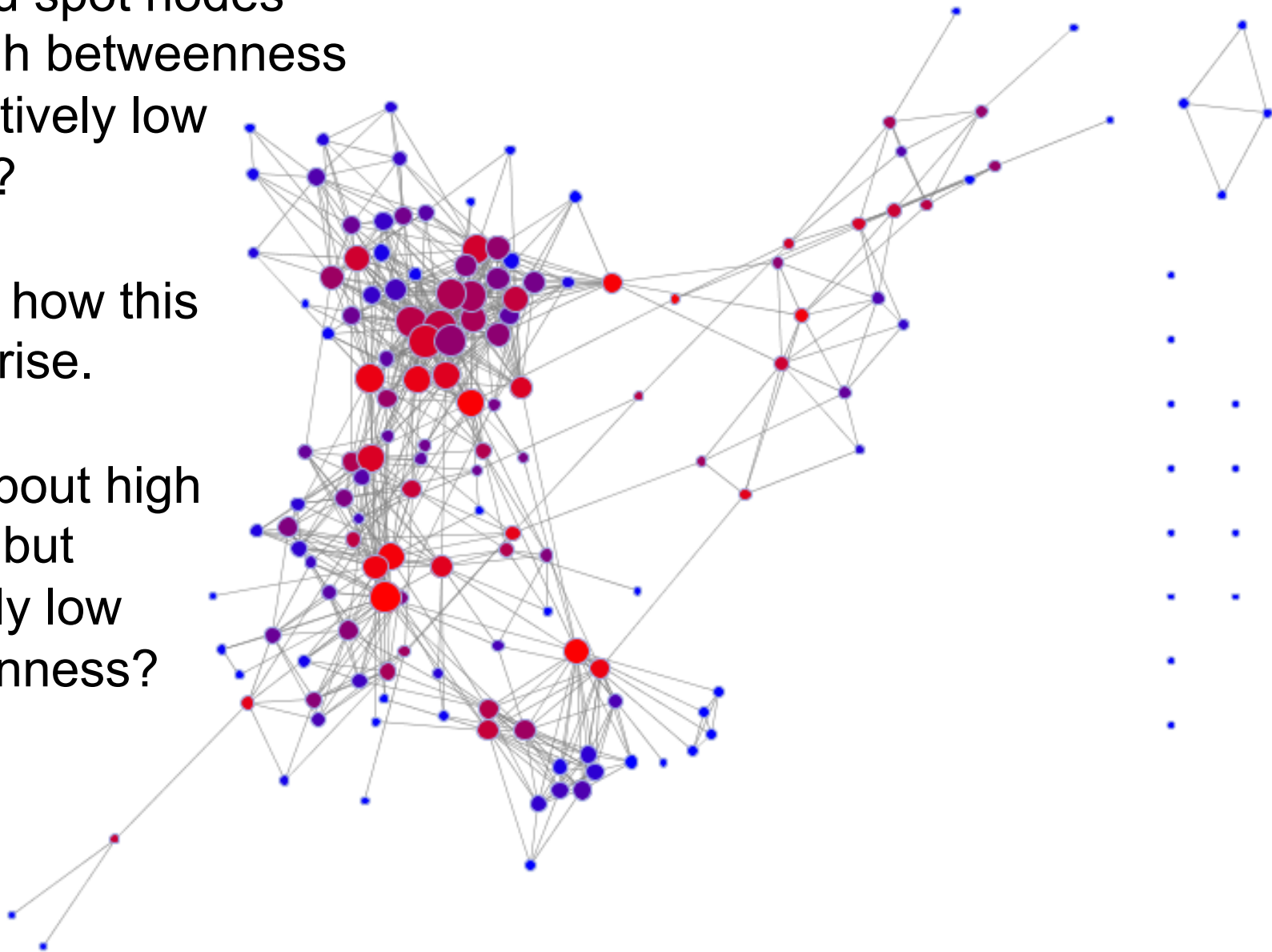


## Betweenness Example (Continued)

Can you spot nodes with high betweenness but relatively low degree?

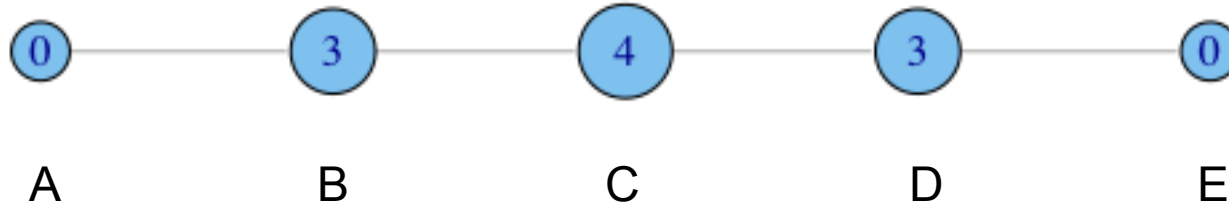
Explain how this might arise.

What about high degree but relatively low betweenness?



# Betweenness On Toy Networks

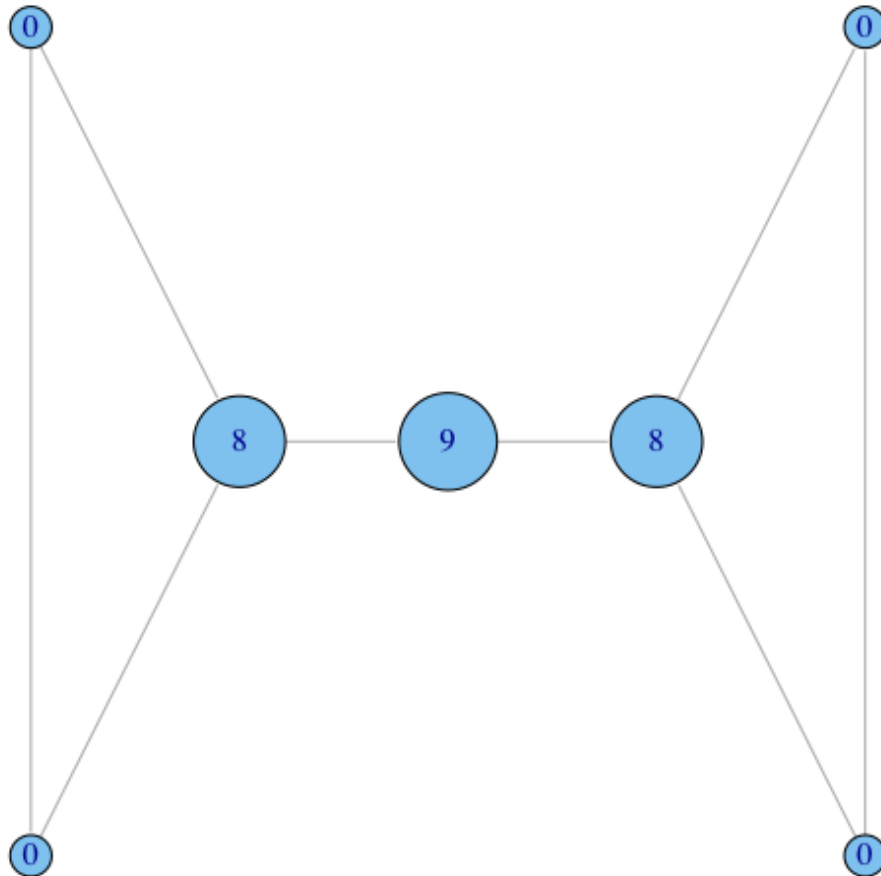
- non-normalized version:



- A lies between no two other vertices
  - B lies between A and 3 other vertices: C, D, and E
  - C lies between 4 pairs of vertices (A,D),(A,E),(B,D),(B,E)
- note that there are no alternate paths for these pairs to take, so C gets full credit

# Betweenness On Toy Networks

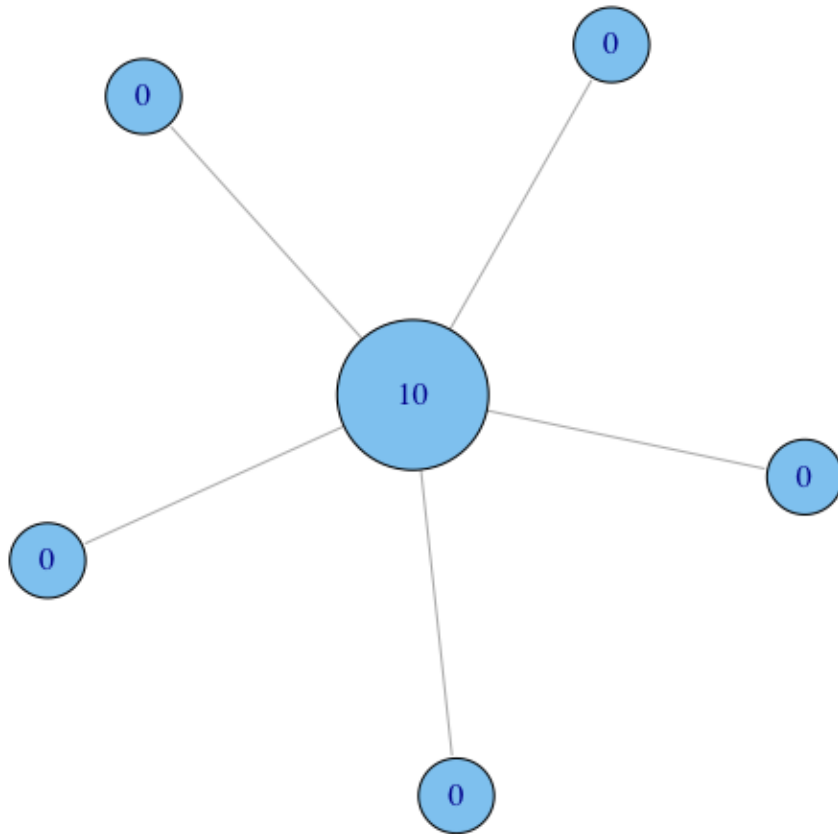
- non-normalized version:





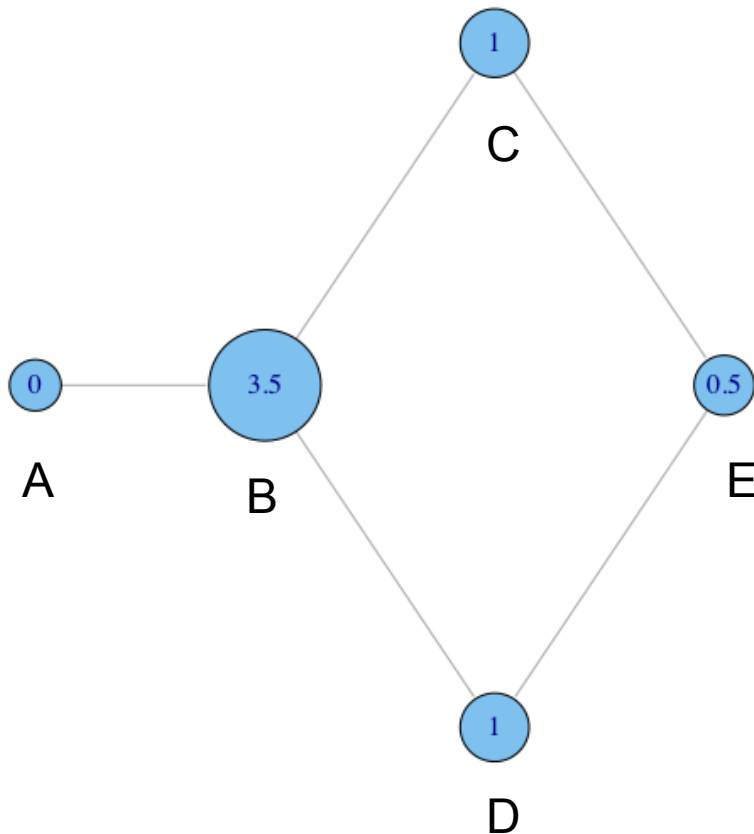
# Betweenness On Toy Networks

- non-normalized version:



# Betweenness On Toy Networks

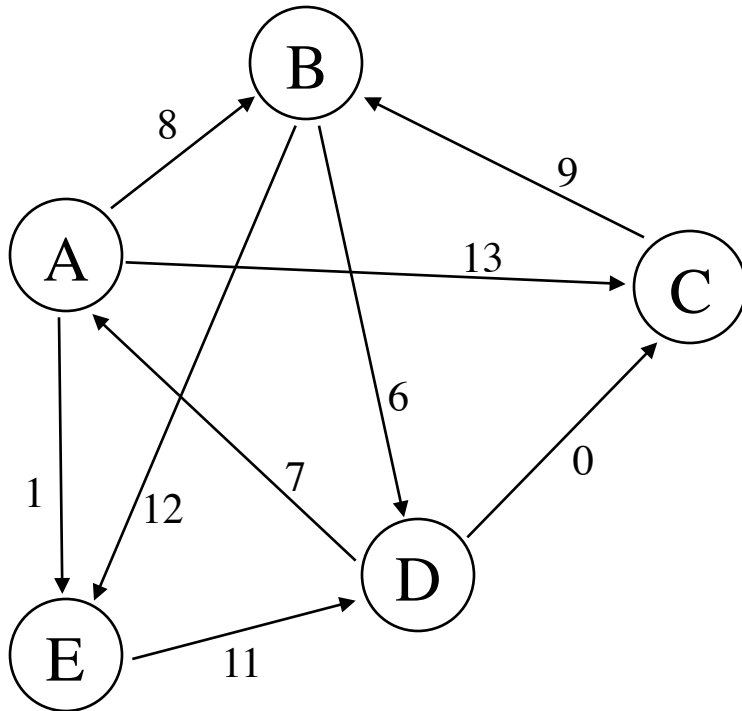
- non-normalized version:



- why do C and D each have betweenness 1?
- They are both on shortest paths for pairs (A,E), and (B,E), and so must share credit:
  - $\frac{1}{2} + \frac{1}{2} = 1$
- Can you figure out why B has betweenness 3.5 while E has betweenness 0.5?

# All-pairs shortest paths...

“Floyd-Warshall algorithm”



Matrix representation

TO

FROM

$$D^0 = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{pmatrix} 0 & 8 & 13 & - & 1 \\ - & 0 & - & 6 & 12 \\ - & 9 & 0 & - & - \\ 7 & - & 0 & 0 & - \\ - & - & - & 11 & 0 \end{pmatrix} \end{matrix}$$

# All-pairs shortest paths...

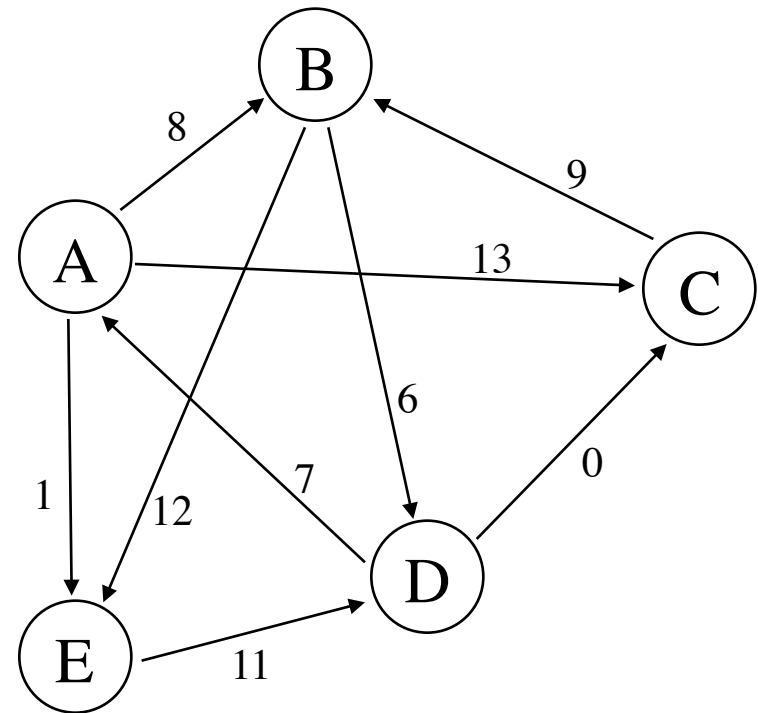
$$D^0 = (d_{ij}^0)$$

$$\begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \begin{pmatrix} 0 & 8 & 13 & - & 1 \\ - & 0 & - & 6 & 12 \\ - & 9 & 0 & - & - \\ 7 & - & 0 & 0 & - \\ - & - & - & 11 & 0 \end{pmatrix}$$

$d_{ij}^k$  = shortest distance from  $i$  to  $j$   
through  $\{1, \dots, k\}$

$$D^1 = (d_{ij}^1)$$

$$\begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \begin{pmatrix} 0 & 8 & 13 & - & 1 \\ - & 0 & - & 6 & 12 \\ - & 9 & 0 & - & - \\ 7 & \boxed{15} & 0 & 0 & \boxed{8} \\ - & - & - & 11 & 0 \end{pmatrix}$$



# All-pairs shortest paths...

$$D^2 = (d_{ij}^2)$$

|   |   |    |    |    |    |
|---|---|----|----|----|----|
| A | 0 | 8  | 13 | 14 | 1  |
| B | - | 0  | -  | 6  | 12 |
| C | - | 9  | 0  | 15 | 21 |
| D | 7 | 15 | 0  | 0  | 8  |
| E | - | -  | -  | 11 | 0  |

$$D^4 = (d_{ij}^4)$$

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| A | 0  | 8  | 13 | 14 | 1  |
| B | 13 | 0  | 6  | 6  | 12 |
| C | 22 | 9  | 0  | 15 | 21 |
| D | 7  | 9  | 0  | 0  | 8  |
| E | 18 | 20 | 11 | 11 | 0  |

$$D^3 = (d_{ij}^3)$$

|   |   |   |    |    |    |
|---|---|---|----|----|----|
| A | 0 | 8 | 13 | 14 | 1  |
| B | - | 0 | -  | 6  | 12 |
| C | - | 9 | 0  | 15 | 21 |
| D | 7 | 9 | 0  | 0  | 8  |
| E | - | - | -  | 11 | 0  |

$$D^5 = (d_{ij}^5)$$

|   |    |    |    |    |    |
|---|----|----|----|----|----|
| A | 0  | 8  | 12 | 12 | 1  |
| B | 13 | 0  | 6  | 6  | 12 |
| C | 22 | 9  | 0  | 15 | 21 |
| D | 7  | 9  | 0  | 0  | 8  |
| E | 18 | 20 | 11 | 11 | 0  |

to store the path, another matrix can track the last intermediate vertex

# Floyd-Warshall Pseudocode

Input:  $D^0 = (d_{ij}^0)$  (the initial edge-cost matrix)

Output:  $D^n = (d_{ij}^n)$  (the final path-cost matrix)

for  $k = 1$  to  $n$  // intermediate vertices considered

for  $i = 1$  to  $n$  // the “from” vertex

for  $j = 1$  to  $n$  // the “to” vertex

$$d_{ij}^k = \min\{ d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1} \}$$

best, ignoring vertex  $k$

best, including vertex  $k$

## **Closeness: Another Centrality Measure**

- What if it's not so important to have many direct friends?
- Or be “between” others
- But one still wants to be in the “middle” of things, not too far from the center

## Closeness Centrality: Definition

Closeness is based on the length of the average shortest path between a vertex and all vertices in the graph

Closeness Centrality:

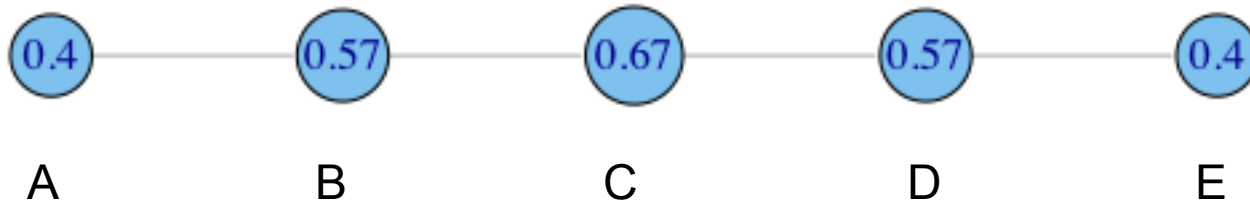
$$C_c(i) = \left[ \sum_{j=1}^N d(i, j) \right]^{-1}$$

Normalized Closeness Centrality

$$C'_c(i) = (C_c(i)) / (N - 1)$$

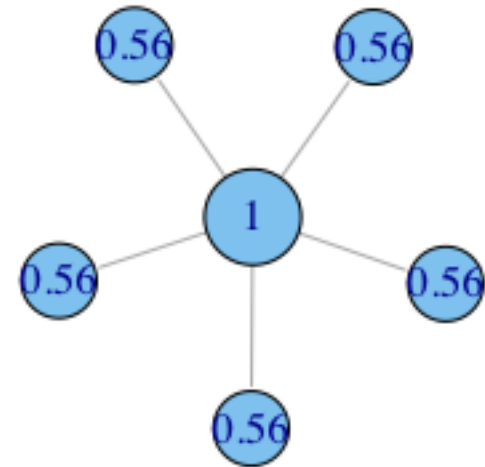
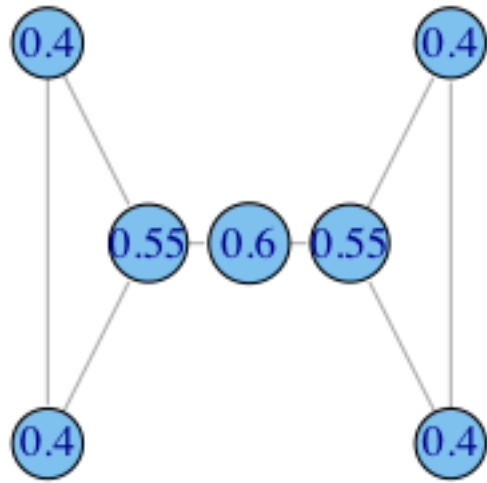


## Closeness Centrality: Toy Example

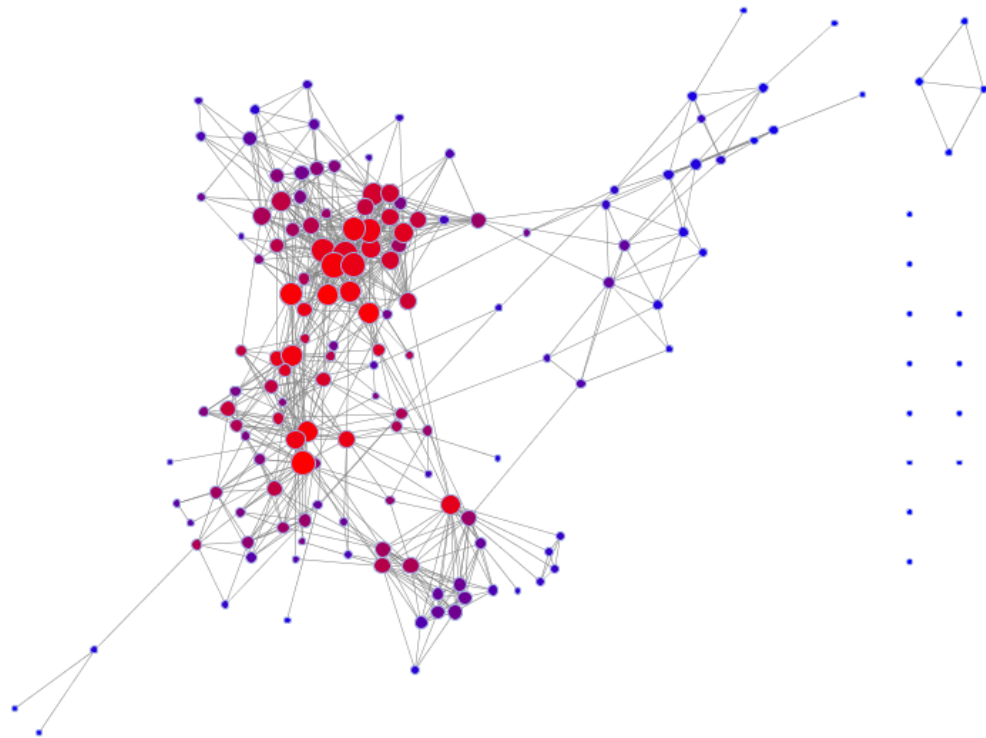


$$C'_c(A) = \left[ \frac{\sum_{j=1}^N d(A, j)}{N-1} \right]^{-1} = \left[ \frac{1+2+3+4}{4} \right]^{-1} = \left[ \frac{10}{4} \right]^{-1} = 0.4$$

# Closeness Centrality: More Toy Examples



# How Closely Do Degree And Betweenness Correspond To Closeness?



- **degree** (number of connections) denoted by size
- **closeness** (length of shortest path to all others) denoted by color

# Centrality: Check Your Understanding

- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

|                  | Low Degree | Low Closeness | Low Betweenness |
|------------------|------------|---------------|-----------------|
| High Degree      |            |               |                 |
| High Closeness   |            |               |                 |
| High Betweenness |            |               |                 |

# Centrality: Check Your Understanding

- generally different centrality metrics will be positively correlated
- when they are not, there is likely something interesting about the network
- suggest possible topologies and node positions to fit each square

|                  | Low Degree                                  | Low Closeness  | Low Betweenness   |
|------------------|---|--|---|
| High Degree      |   | Embedded in cluster that is far from the rest of the network   | Ego's connections are redundant - communication bypasses him/her                        |
| High Closeness   | Key player tied to important/active players |  | Probably multiple paths in the network, ego is near many people, but so are many others |
| High Betweenness | Ego's few ties are crucial for network flow | Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others. |   |

## Extending Betweenness Centrality To Directed Networks

- We now consider the fraction of all directed paths between any two vertices that pass through a node

betweenness of vertex  $i$

paths between  $j$  and  $k$  that pass through  $i$

$$C_B(i) = \sum_{j,k} g_{jk}(i) / g_{jk}$$

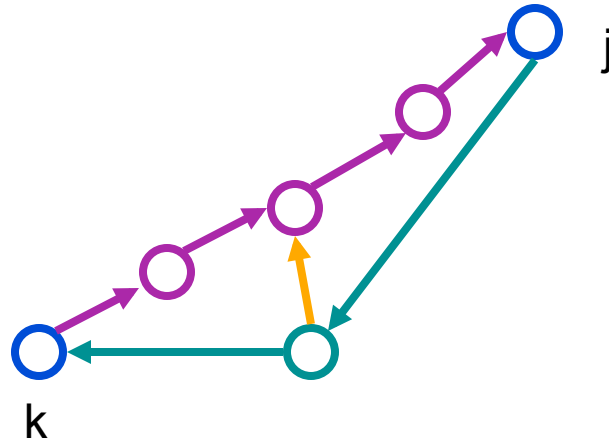
all paths between  $j$  and  $k$

- Only modification: when normalizing, we have  $(N-1)*(N-2)$  instead of  $(N-1)*(N-2)/2$ , because we have twice as many ordered pairs as unordered pairs

$$C'_B(i) = C_B(i) / [(N-1)(N-2)]$$

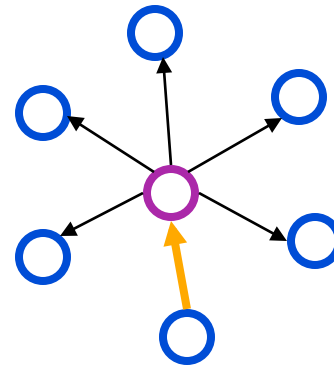
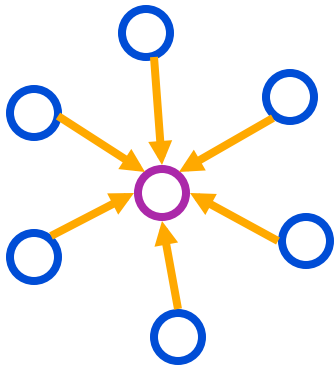
## Directed Geodesics

- A node does not necessarily lie on a geodesic from  $j$  to  $k$  if it lies on a geodesic from  $k$  to  $j$



# Extensions Of Undirected Degree Centrality - Prestige

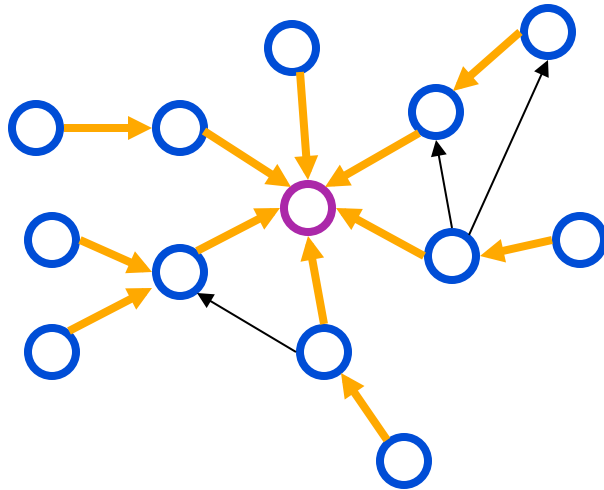
- degree centrality
  - indegree centrality
    - a paper that is cited by many others has high prestige
    - a person nominated by many others for a reward has high prestige





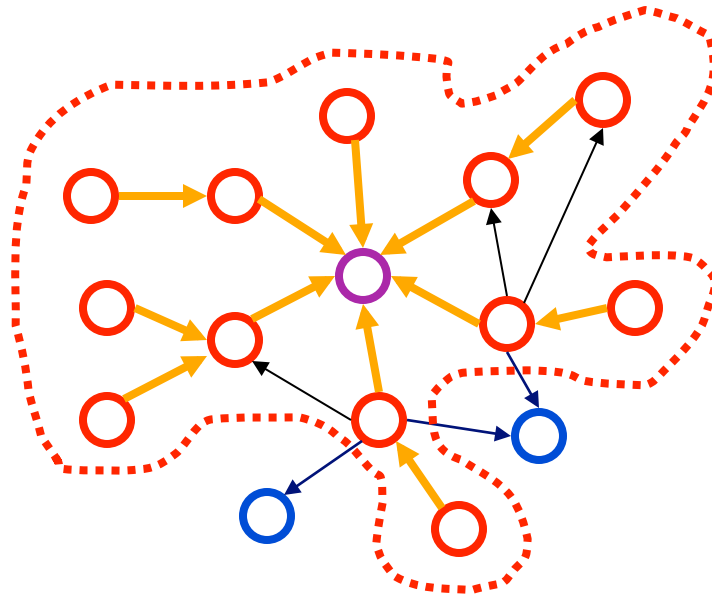
# Extensions Of Undirected Closeness Centrality

- closeness centrality usually implies
  - all paths should lead to you  
and unusually not:
  - paths should lead from you to everywhere else
- usually consider only vertices from which the node  $i$  in question can be reached



# Influence Range

- The influence range of  $i$  is the set of vertices who are reachable from the node  $i$



# Wrap Up

## Centrality

- many measures: degree, betweenness, closeness, ...
- may be unevenly distributed
  - measure via centralization
- extensions to directed networks:
  - prestige
    - influence
  - PageRank

# **Additional Material**

(Not covered in class)

# Bonachich Power Centrality: When Your Centrality Depends On Your Neighbors' Centrality

An eigenvector measure:

$$C(\alpha, \beta) = \alpha(I - \beta R)^{-1} R \mathbf{1}$$

- $\alpha$  is a scaling vector, which is set to normalize the score.
- $\beta$  reflects the extent to which you *weight* the centrality of people ego is tied to.
- $\mathbf{R}$  is the adjacency matrix (can be valued)
- $\mathbf{I}$  is the identity matrix (1s down the diagonal)
- $\mathbf{1}$  is a matrix of all ones.

## Bonacich Power Centrality: $\beta$

The magnitude of  $\beta$  reflects the radius of power. Small values of  $\beta$  weight local structure, larger values weight global structure.

If  $\beta > 0$ , ego has higher centrality when tied to people who are central.

If  $\beta < 0$ , then ego has higher centrality when tied to people who are not central.

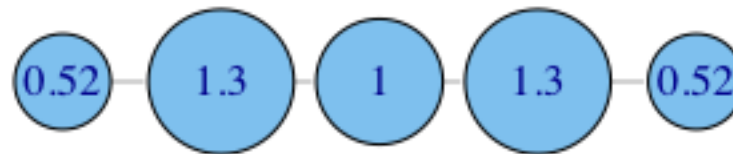
With  $\beta = 0$ , you get degree centrality.

# Bonacich Power Centrality: Examples

$\beta = .25$



$\beta = -.25$



Why does the middle node have lower centrality than its neighbors when  $\beta$  is negative?

# Centrality When Edges Are Directed

## Review: Examples Of Directed Networks

- WWW
- food webs
- population dynamics
- influence
- hereditary
- citation
- transcription regulation networks
- neural networks

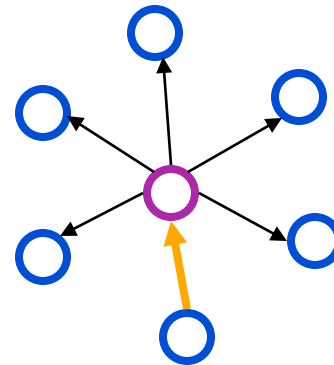
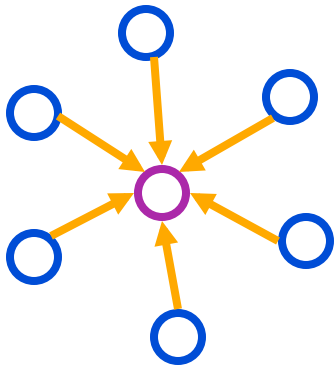


# Prestige In Directed Social Networks

- when 'prestige' may be the right word
  - admiration
  - influence
  - gift-giving
  - trust
- directionality especially important in instances where ties may not be reciprocated (e.g. dining partners choice network)
- when 'prestige' may not be the right word
  - gives advice to (can reverse direction)
  - gives orders to (- " -)
  - lends money to (- " -)
  - dislikes
  - distrusts

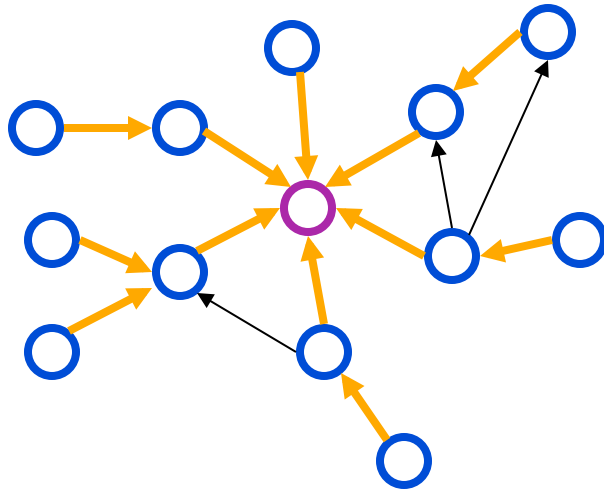
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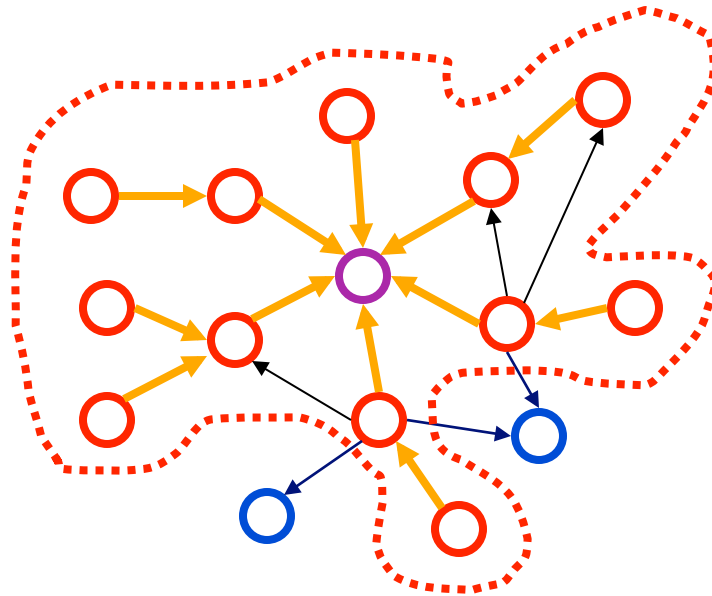
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# Prestige in Pajek

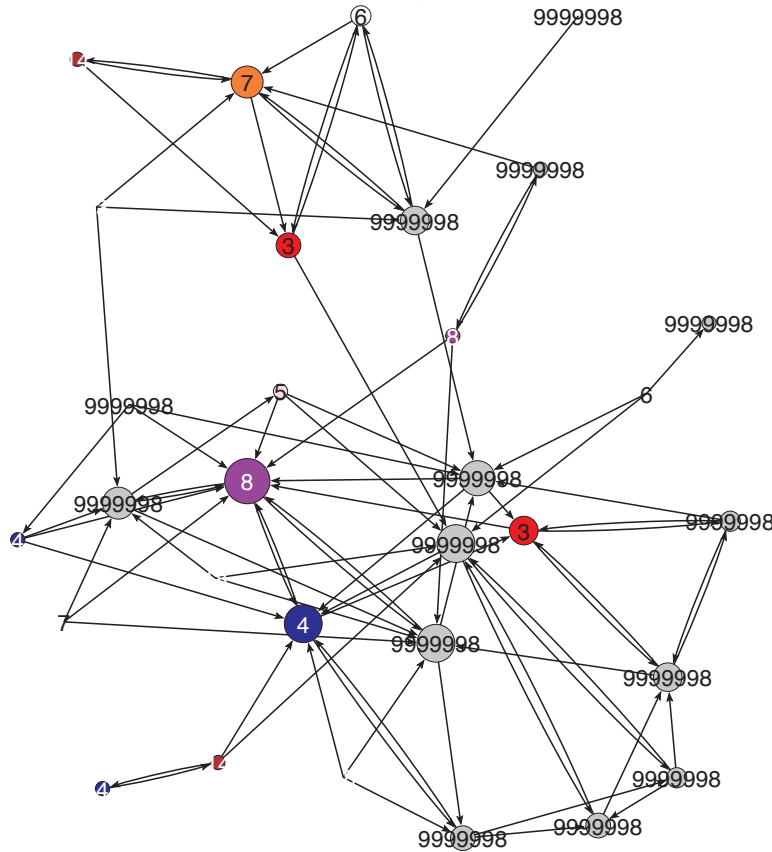
- Calculating the indegree prestige
  - Net>Partition>Degree>Input
  - to view, select File>Partition>Edit
  - if you need to reverse the direction of each tie first (e.g. lends money to -> borrows from):  
Net>Transform>Transpose
- Influence range (a.k.a. input domain)
  - Net>k-Neighbours>Input
    - enter the number of the vertex, and 0 to consider all vertices that eventually lead to your chosen vertex
    - to find out the size of the input domain, select Info>Partition
  - Calculate the size of the input domains for all vertices
    - Net>Partitions>Domain>Input
  - Can also limit to only neighbors within some distance

## Proximity Prestige In Pajek

- Direct nominations (choices) should count more than indirect ones
- Nominations from second degree neighbors should count more than third degree ones
- So consider proximity prestige

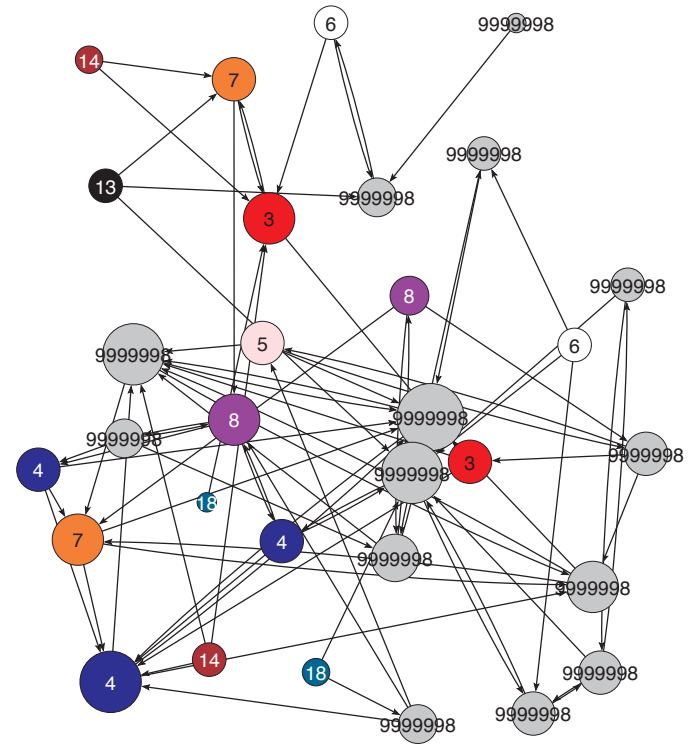
$$C_p(n_i) = \frac{\text{fraction of all vertices that are in } i' \text{ s input domain}}{\text{average distance from } i \text{ to vertex in input domain}}$$

# Prestige vs. Centrality In Diffusion



physician discussion network

nodes are sized by indegree



physician friendship network

nodes are sized by degree

# Friedkin: Structural Bases Of Influence

- Interested in identifying the structural bases of power. In addition to resources, he identifies:
  - Cohesion
  - Similarity
  - Centralitywhich are thought to affect interpersonal visibility & salience



# Friedkin: Structural Bases Of Influence

## Centrality

Central actors are likely more influential. They have greater access to information and can communicate their opinions to others more efficiently. Research shows they are also more likely to use the communication channels than are periphery actors.

# Friedkin: Structural Bases Of Influence

## Structural Similarity

- Two people may not be directly connected, but occupy a similar position in the structure. As such, they have similar interests in outcomes that relate to positions in the structure.
- Similarity must be conditioned on visibility. P must know that O is in the same position, which means that the effect of similarity might be conditional on communication frequency.

# Friedkin: Structural Bases Of Influence

## Cohesion

- Members of a cohesive group are likely to be aware of each others opinions, because information diffuses quickly within the group.
- Groups encourage (through balance) reciprocity and compromise. This likely increases the salience of opinions of other group members, over non-group members.

# Friedkin: Structural Bases Of Influence

Substantive questions: Influence in establishing school performance criteria.

- Data on 23 teachers
- Collected in 2 waves
- Dyads are the unit of analysis (P--> O): want to measure the extent of influence of one actor on another.
- Each teacher identified how much an influence others were on their opinion about school performance criteria.
  
- Cohesion = probability of a flow of events (communication) between them, within 3 steps.
- Similarity = pairwise measure of equivalence (profile correlations)
- Centrality = TEC (power centrality)

# Friedkin: Structural Bases Of Influence

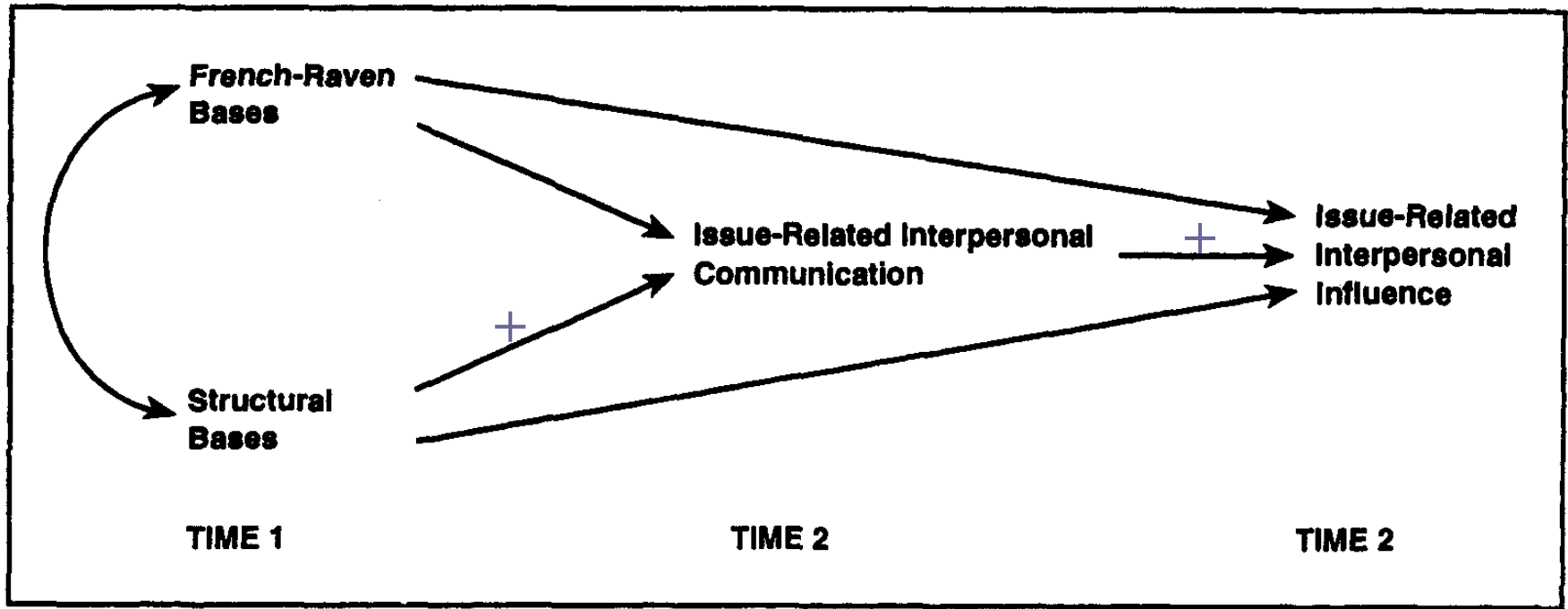


Figure 1. Model of Relationship Between Bases of Interpersonal Power, Communication, and Influence

Interpersonal communication matters, and communication is what matters most for interpersonal influence.

# Baker & Faulkner: Social organization of conspiracy

Questions: How are relations organized to facilitate illegal behavior?

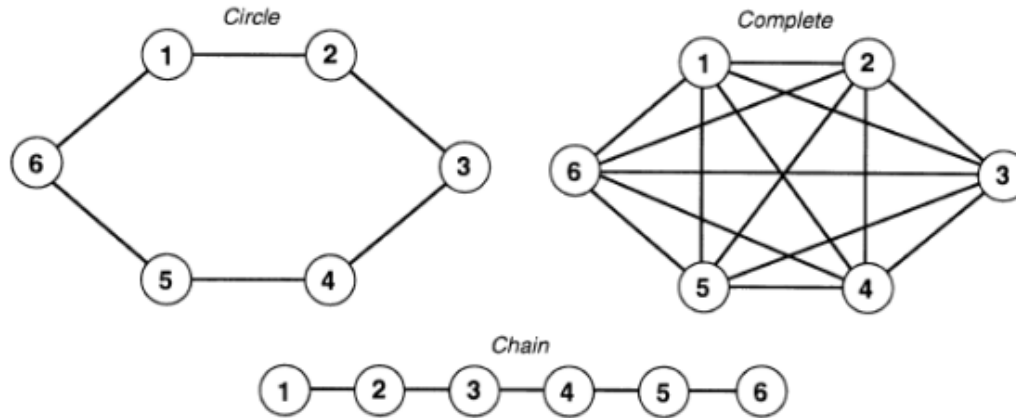
Pattern of communication maximizes concealment, and predicts the criminal verdict.

Inter-organizational cooperation is common, but too much 'cooperation' can thwart market competition, leading to (illegal) market failure.

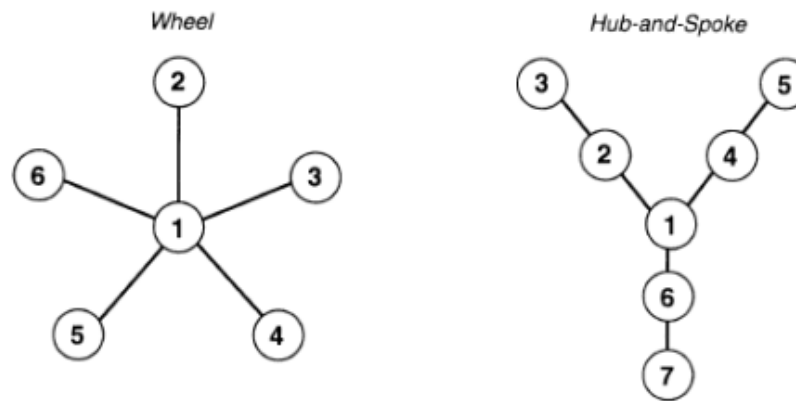
Illegal networks differ from legal networks, in that they must conceal their activity from outside agents. A "Secret society" should be organized to (a) remain concealed and (b) if discovered make it difficult to identify who is involved in the activity

The need for secrecy should lead conspirators to conceal their activities by creating *sparse* and *decentralized* networks.

### Decentralized Networks



### Centralized Networks



The Social Organization of Conspiracy: Illegal Networks in the Heavy Electrical Equipment Industry, Wayne E. Baker, Robert R. Faulkner. *American Sociological Review*, Vol. 58, No. 6 (Dec., 1993), pp. 837-860. Published by: American Sociological Association, <http://www.jstor.org/stable/2095954>.

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| Organization Objective | Information-Processing Requirement |                        |
|------------------------|------------------------------------|------------------------|
|                        | High                               | Low                    |
| Concealment            | Centralized networks               | Decentralized networks |
| Coordination           | Decentralized networks             | Centralized networks   |

Figure 1. Concealment Versus Coordination: Theoretical Expectations **and experimental results**



# **Baker & Faulkner: Social organization of conspiracy**

center: good for reaping the benefits  
periphery: good for remaining concealed

They examine the effect of Degree, Betweenness and Closeness centrality on the criminal outcomes, based on reconstruction of the communication networks involved.

At the **organizational level**,

low information-processing conspiracies are decentralized

high information processing load leads to centralization

At the **individual level**, degree centrality (net of other factors) predicts verdict.

# Wrap Up

## ■ Centrality

- many measures: degree, betweenness, closeness, Bonacich
- may be unevenly distributed
  - measure via centralization
- extensions to directed networks:
  - prestige
    - input domain...
    - PageRank (down the road...)
- consequences:
  - interpersonal influence (Friedkin)
  - benefits & risks (Baker & Faulkner)