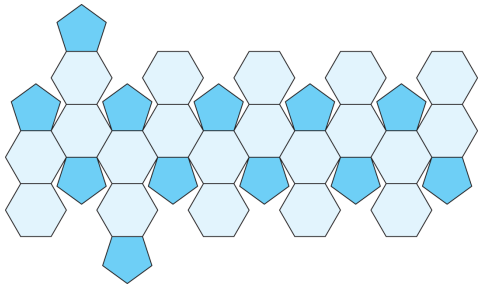
Computational Geometry

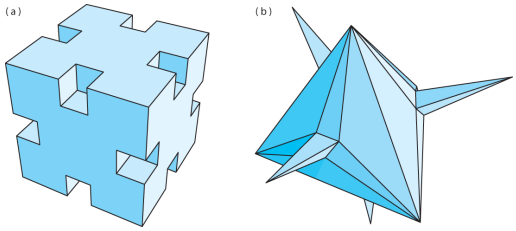
Shortest Paths

Geodesics

Net of a Polyhedron



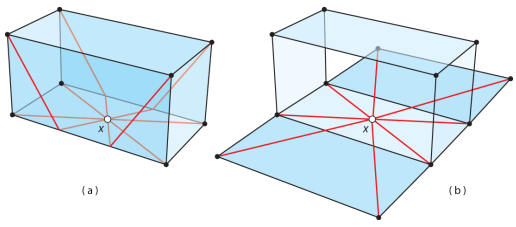
Polyhedra without Nets



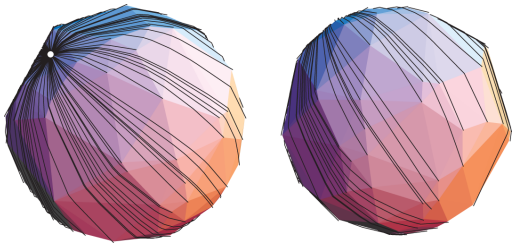
Shortest Path

- A shortest path on *P* between two points *x* and *y* on *P* is a curve connecting *x* and *y* whose length, measured on the surface, is shortest among all curves connecting those points on *P*.
  - Shortest paths do not self-intersect
  - Shortest paths do not pass through vertices
  - A shortest path passing through an edge will be unfolded straight.

2x1x1 Box

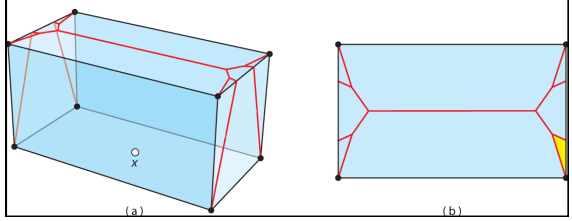


100 Vertices

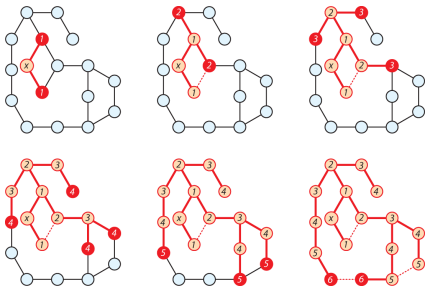


Cut Locus

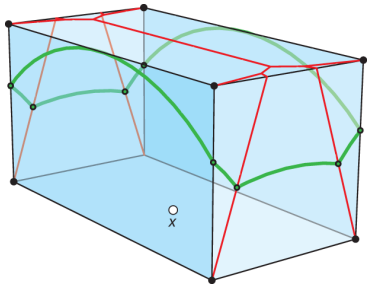
- The cut locus $C(x)$ of x is the closure of the set of all points y to which there is more than one shortest path from x



Dijkstra's



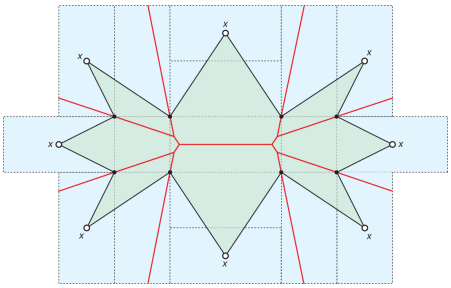
Continuous Frontier



Star Unfolding

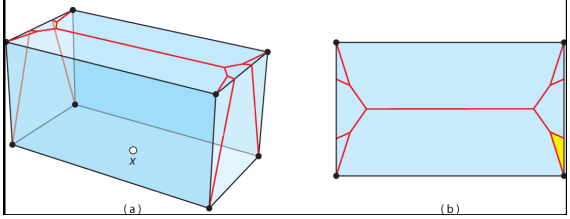
- The star unfolding of a convex polyhedron P fixes a generic point x on P, finds the shortest paths from x to every vertex of P, and simply cuts along these shortest paths.
- The star unfolding is a simple (nonoverlapping) polygon with $2n$ vertices. Furthermore, the Voronoi diagram of the n copies of the source x is the unfolding of the cut locus $C(x)$ on P.

Star Unfolding of 2x1x1 Box

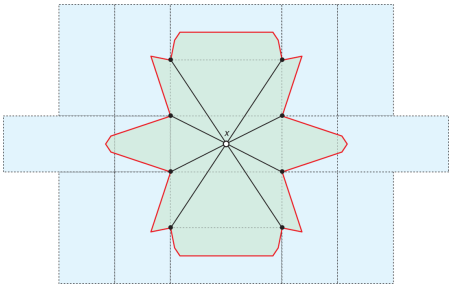


Source Unfolding

- The source unfolding of a convex polygon P is obtained by cutting along the cut locus $C(x)$ of a generic source point x.



Source Unfolding



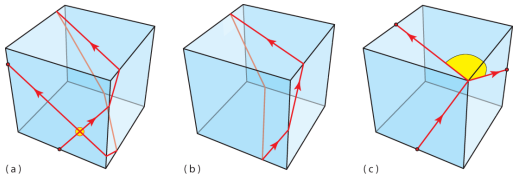
Open Questions

- Does every convex polyhedron have a net?
- Does every polyhedron have a general net?

Geodesic

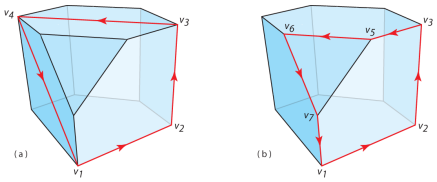
- A geodesic on a surface is a curve γ with the property that for any two sufficiently close points *x* and *y* on γ, the portion of γ between *x* and *y* is the shortest path on the surface connecting *x* and *y*.
- Every shortest path is a geodesic.
- Geodesics are locally shortest paths.

Geodesics



Quasigeodesics

- There are at least three simple closed geodesics on any surface homeomorphic to a sphere.



Curve Shortening