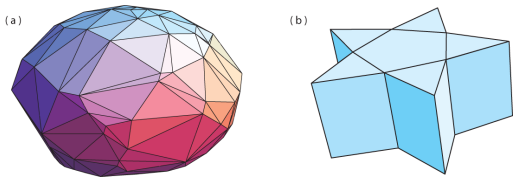
Computational Geometry

Polyhedra

Convexity

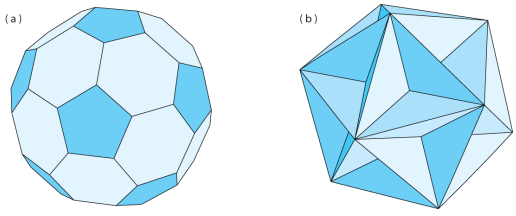


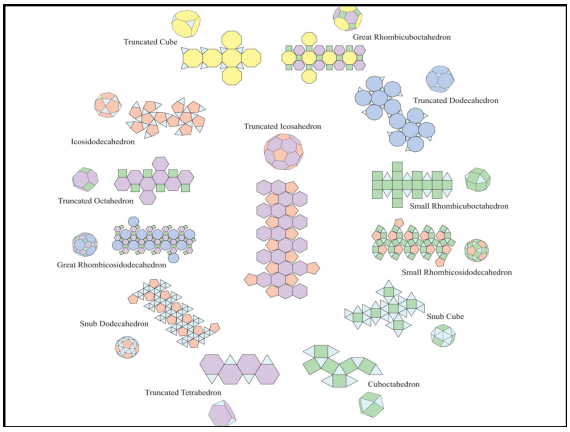
Platonic Solids

All Combinations of *k* and *m*

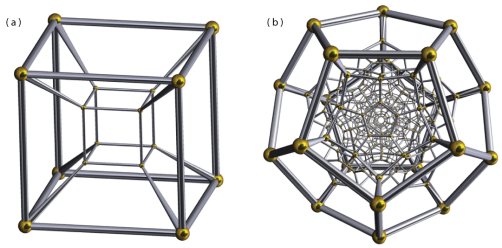
Name	*k*-gon	*m*	(*k*-2)(*m*-2)	V	E	F
Tetrahedron	3	3	1	4	6	4
Cube	4	3	2	8	12	6
Octahedron	3	4	2	6	12	8
Dodecahedron	5	3	3	20	30	12
Icosahedron	3	5	3	12	30	20

Loosely-Regular





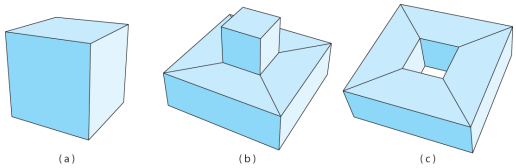
Higher Dimensions



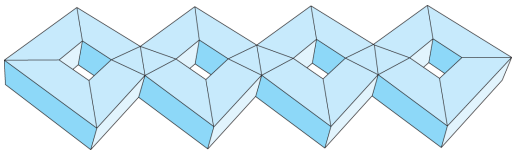
Polyhedron Definition

- Intersection condition: The intersection of any two faces of P is either empty, a single vertex or a single edge
- Local topology: Every point p on the surface of P has a neighborhood that is homeomorphic to an open disk
- Global topology: P is connected – a path exists on the surface between any two points.

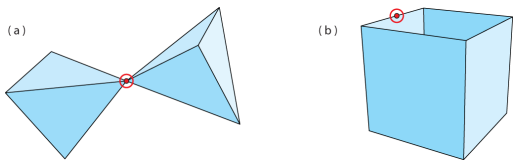
Polyhedra



A Polyhedron of Genus 4



Violation of Local Topology

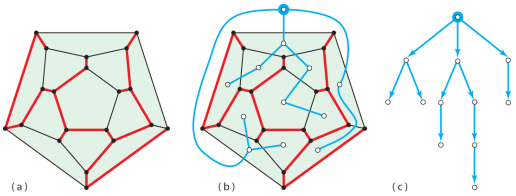


Euler's Formula Revisited

- For any polyhedron homeomorphic to a sphere with V vertices, E edges and F faces, V-E+F = 2.
- Proof
  - convert the 1-skeleton of the polyhedron to a planar graph G
  - select a spanning tree T of G
  - construct the dual graph G* of G
  - identify the complementary spanning tree T* in G*

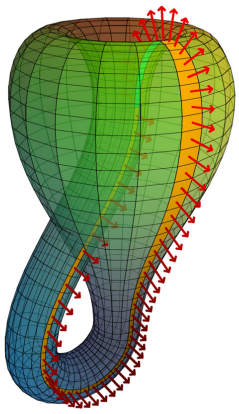
Euler's Formula Proof

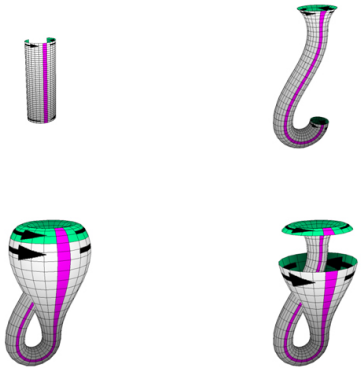
- A tree with *n* vertices has exactly *n*-1 edges.

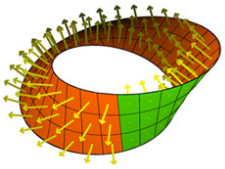


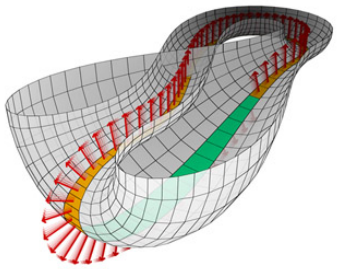
Generalization of Euler's Formula

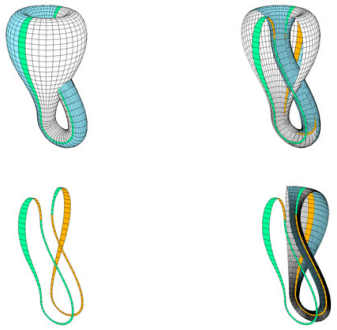
- For a triangulated surface *S* with *V* vertices, *E* edges and *F* faces, the Euler characteristics $\chi(S)$ of *S* is *V*-*E*+*F*
- $\chi(S) = 2\text{-}2g$, where *g* is the genus, if *S* is orientable.
- $\chi(S) = 2\text{-}k$, where *k* is the number of handles, if *S* is non-orientable.











Klein Bottle as Twisted Double Tube

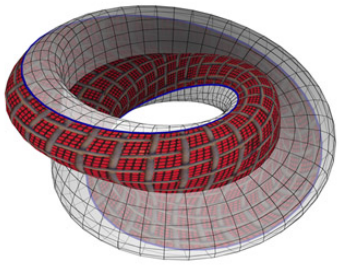
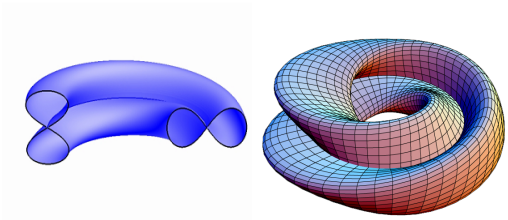
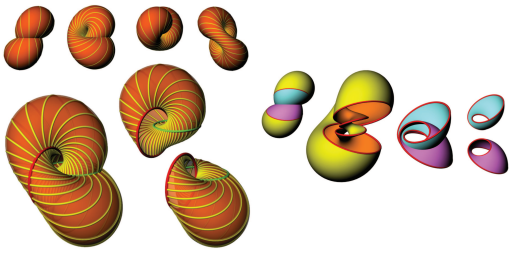


Figure-8 Immersion



As a One-Parameter Family of Circles



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