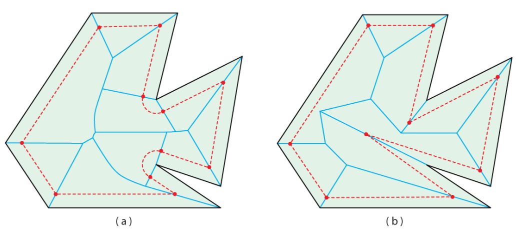
Computational Geometry

Minkowski Sums

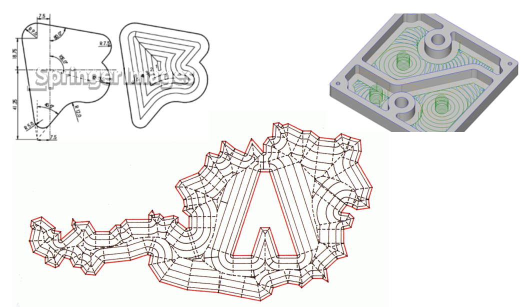
Offset Curve

- Given a smooth curve *C*, the offset curve is the locus of points offset by a constant distance *r* along the curve normal.
- The offset curve can also be defined as the envelope of a family of disks of radius *r* whose centers lie on *C*.

Medial Axis, Straight Skeleton and the Offset Curve



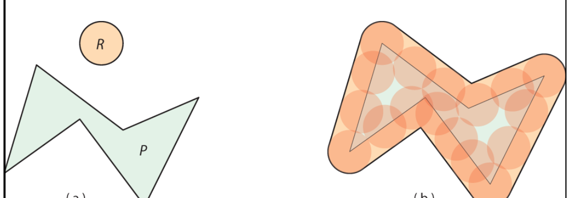
Pocket Machining



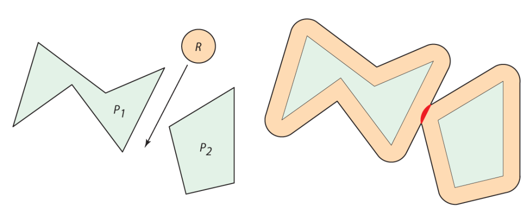
Minkowski Sum

- The Minkowski sum of two sets *A* and *B*

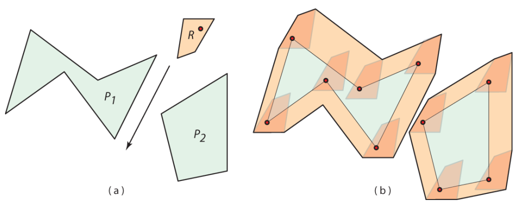
  $A \oplus B = \{x+y \mid x \in A, y \in B\}$



Motion Planning



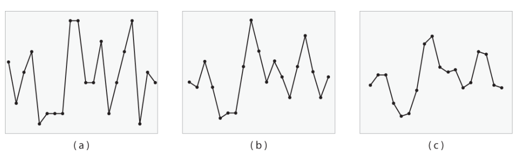
Convex Polygon



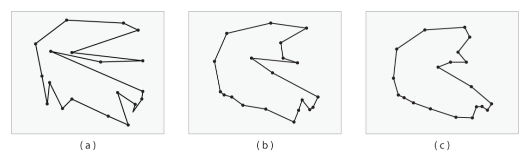
Computational Geometry

Curve Shortening

Midpoint Transformation



On a Closed Curve



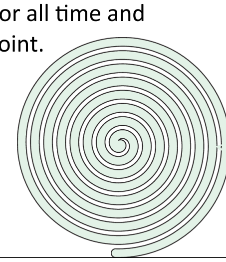
Curve Evolution

- Let $C(s) = (x(s), y(s))$ be a smooth closed curve parameterized by arc length s.
- Add a time variable t, defining a curve $C(s, t)$. The curve evolves with t according to the differential equation:

  $\partial C/\partial t = \partial^2 C/\partial s^2 = K\eta$

Curve Shortening Theorem

- Every smooth, simple closed curve C evolves under the flow defined by the equation so that it remains simple for all time and converges to a round point.
- Convexifies without self-intersection
- twisted curve
- spiral
- flower

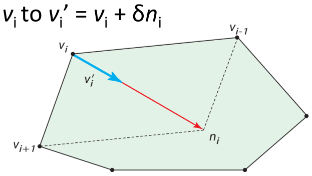


Discrete Flow

- Replace n-sided polygon P with its midpoint transformation
- Approximate normal at v_i as

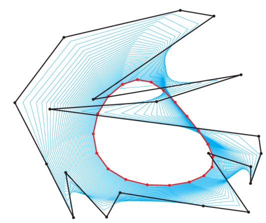
$$n_i = (v_{i+1} - v_i) + (v_{i-1} - v_i)$$

- Move vertex v_i to $v_i' = v_i + \delta n_i$

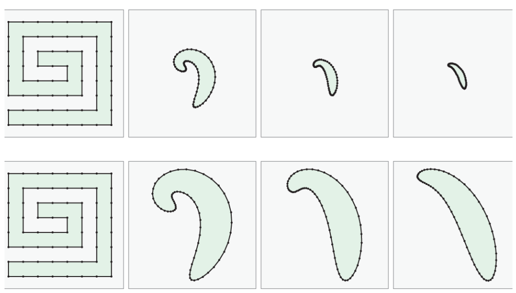


Discrete Curve Shortening

- Every simple polygon evolves under the flow so that it converges to a point whose shape is asymptotically an affine transformation of a regular polygon.



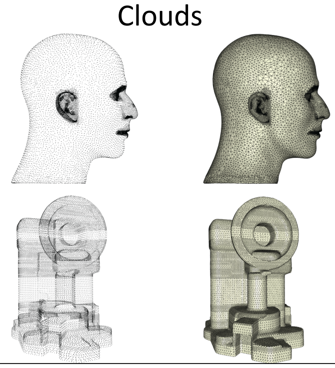
64-gon with 100 Iterations



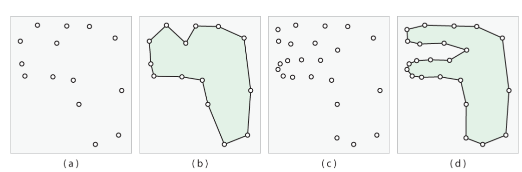
Computational Geometry

Curve Reconstruction

Surface Reconstruction from Point Clouds

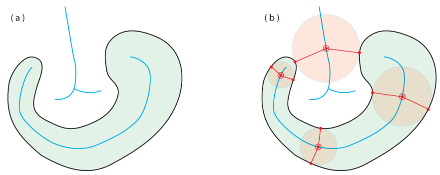


Curve Reconstruction



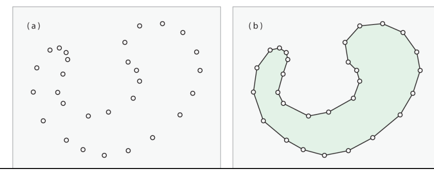
Local Feature Size

- Let *C* be a smooth closed curve and let *x* be a point on *C*. The local feature size ρ(*x*) of *x* is the shortest distance from *x* to the medial axis of *C*.



ε-sample

- Let 0<ε<1. A set *S* of points sampled from *C* is an ε-sample if each point *x* in *C* has point *p* in *S* such that |*x*-*p*| ≤ ερ(*x*).
- Forces sample to be dense in complicated sections of *C*.



the CRUST Algorithm

- The correct edges are a subset of Del(S).
- For sufficiently small ε
  - The Voronoi vertices V of Vor(S) lie near M(C)
  - Any circumscribing circle of an incorrect edge of Del(S) crosses the medial axis M(C)
  - An incorrect edge e of Del(S) cannot also appear in Del(S ∪ V) because a circumscribing circle for e contains a vertex in V.
  - Each correct edge of Del(S) also appears in Del(S ∪ V).

CRUST