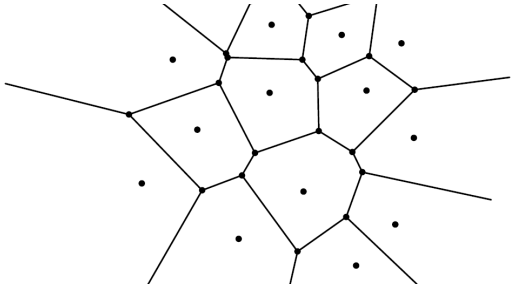
# Computational Geometry

Medial Axis

Straight Skeleton

## Voronoi



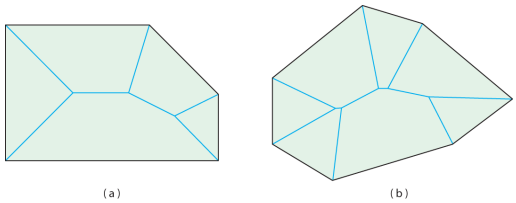
## Alternative Voronoi Definitions

- Vor(*S*) is the locus of centers of maximal empty circles – those whose interior contain no site of *S*.
- Vor(*S*) is the locus of points to which there are two or more nearest sites
- Vor(*S*) is the set of "quench points" if the plane is burned uniformly and simultaneously from all sites in *S*.

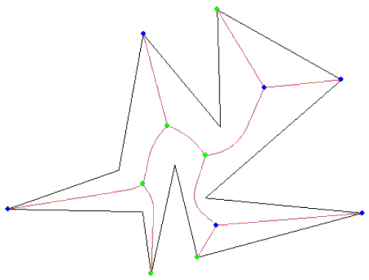
## Definition

- The *medial axis* M(*P*) of a polygon *P* is the closure of the set of points in *P* that have two or more closest points among the points on $\delta P$.
- This also known as the *cut locus* of $\delta P$ in Mathematics.

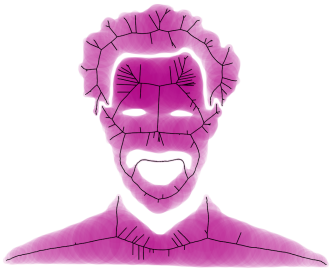
## Medial Axes



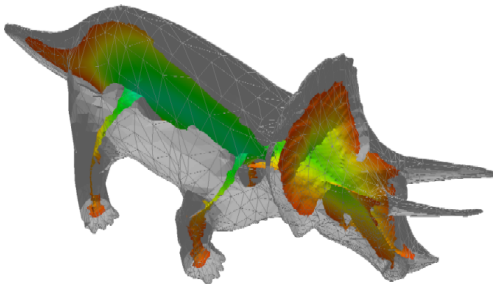

## Non-Convex



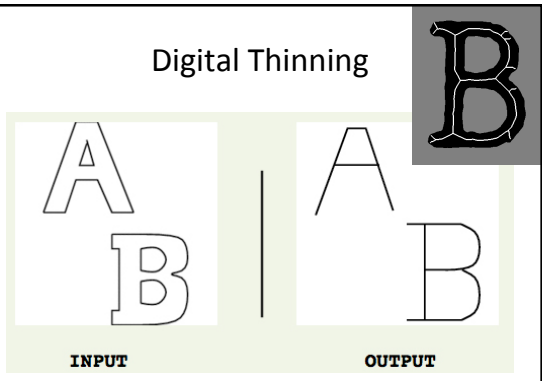
## Face



## 3D Distance Field



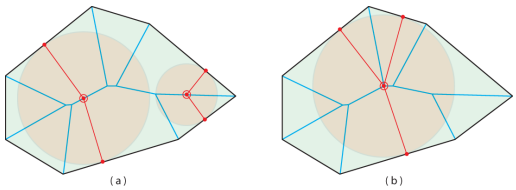
## Digital Thinning



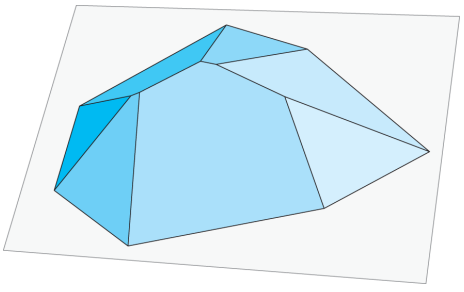

## Medial Axis of a Convex Polygon

- A geometric tree of straight segments whose leaves are the vertices of P.
- Points on the medial axis are centers of maximal circles that touch $\delta P$ in two or more points.
- Starting a fire $\delta P$ and burning into the interior would again result in the "quench points" being the medial axis.

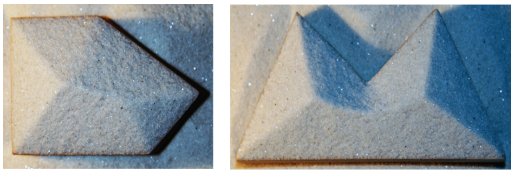
## Maximal Circles




## Medial Axis Polyhedron

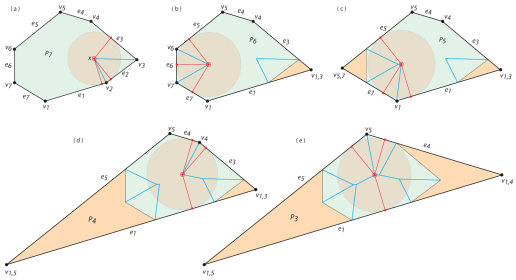


## Sand-constructed Physical Models



(a) (b)

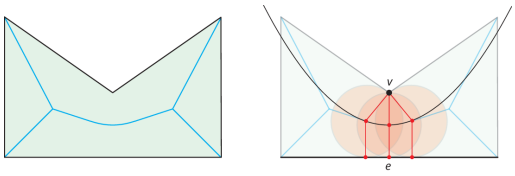
## Constructing the Medial Axis



## Time Complexity

- Find the first pair of intersecting bisectors by checking all pairs of adjacent vertices – O($n$)
- Recursion
- O($n^2$)
- O($n$log$n$) is possible with data structure (priority queue)
- O($n$) possible with considerably more work and cleverness

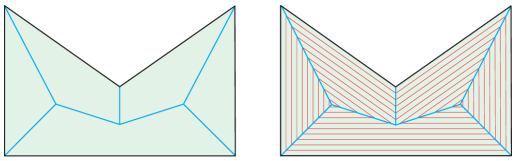
## Non-Convexity




## Straight Skeleton

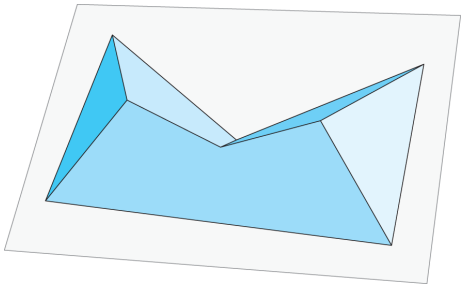
- shrink $\delta P$ via parallel transformation of all edges inward
- each vertex (including reflex) follows the angle bisector
- stop when
  - an edge is 0 length
  - a reflex vertex collides with an edge – pinch into two polygons and continue

## Straight Skeleton



(a) (b)

## Straight Skeleton Polyhedron



## Notes

- Voronoi definition does not hold
- Best algorithm runs in $O(n^{17/11})$
- Unsolved in 3D (2008)
- Unsolved for higher dimensions