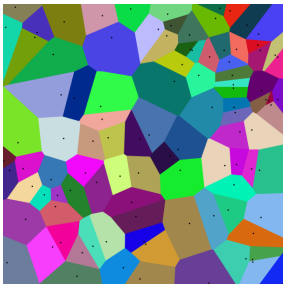
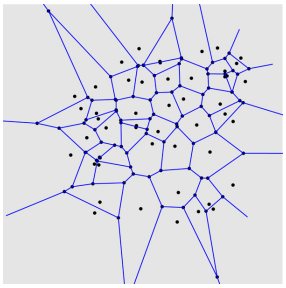
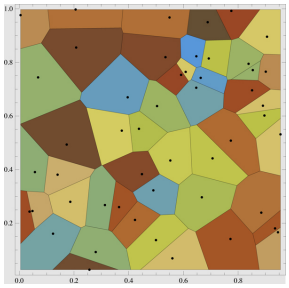
Computational Geometry

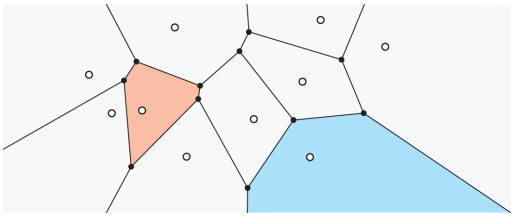
Voronoi Diagrams







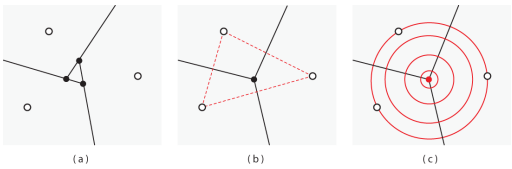
Voronoi Diagram of 10 Sites



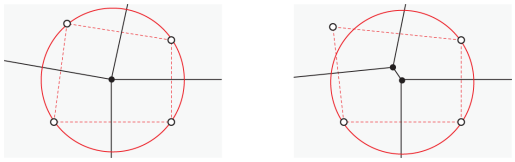
Definition

- The Voronoi region *Vor*(*p*) of a site *p* in *S* is
  $Vor(p) = \{x \in R^2 \mid |x-p| \leq |x-q|, \forall q \in S\}$,
  where $|x-y|$ denotes the distance between two points *x* and *y*.
- The Voronoi region of *p* consists of all points that are at least as close to *p* as any other sites in *S*.
- Vor(p) is the intersection of all halfplanes H(p, q), where q is any other sites in S.

3 Sites



4 or More Sites

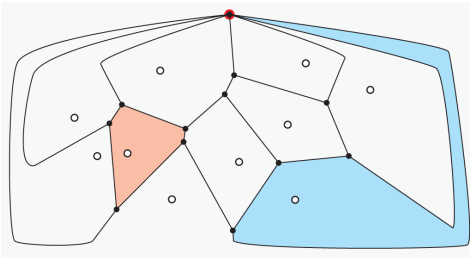


Theorem

- Let *S* be a point set with Voronoi diagram *Vor*(*S*). A point *v* is a Voronoi vertex of *Vor*(*S*) if and only if there exists an empty circumcircle centered at *v* of three or more sites.
- Let *e* be a connected subset of the bisector between sites *p* and *q* of *S*. *e* is a Voronoi edge of *Vor*(*S*) if and only if for each point *x* in *e*, the circle centered at *x* through *p* and *q* is empty in its interior and boundary.

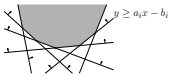
Combinatorics of Voronoi

- *Vor*(*S*) has at most 2*n*-5 Voronoi vertices and 3*n*-6 edges, when *S* has *n* ≥ 3 sites.

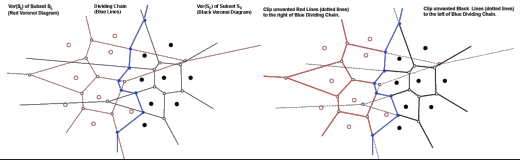


Voronoi Algorithms

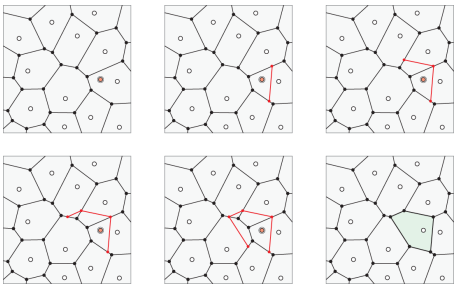
- Naïve Voronoi
  - Intersection of n-1 half planes.
  - Complexity?



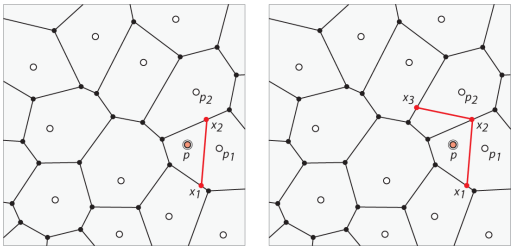
- Divide-and-Conquer (Shamos & Hoey 1975)
  - *O*(*n*log(*n*))



Incremental



Incremental - Details



Incremental

- Youtube video
- Applet

Fortune's

- Applet (University of Copenhagen)
- YouTube video

Dynamic Voronoi

- Particles
- Swarm
- Personal space