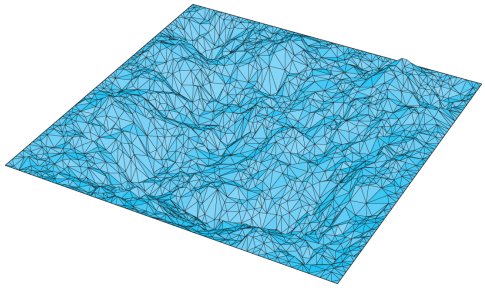
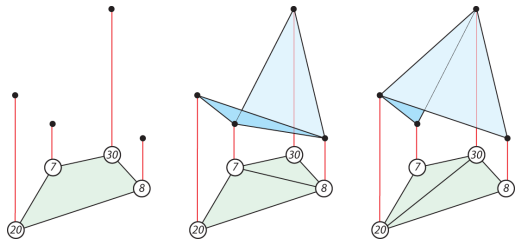
Computational Geometry

Delaunay and Other Special Triangulations

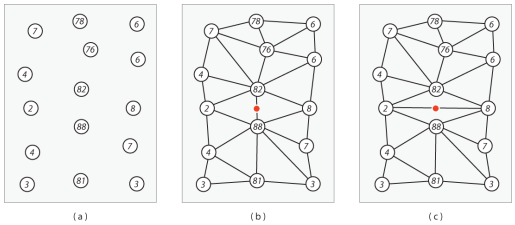
3D Terrain from Sampled Points



Lifting the Triangles



Skinny is Bad

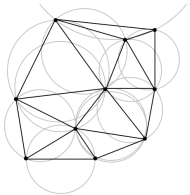


Angle Sequence

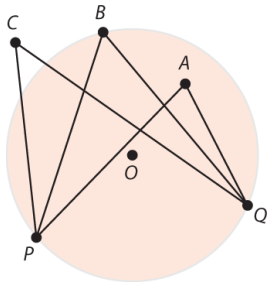
- Let T be a triangulation of a point set S, and suppose T has n triangles. The angle sequence $\{a_1, a_2, \ldots, a_n\}$ lists all $3n$ angles of T in sorted order.
- A triangulation T_1 is fatter than T_2 ($T_1 > T_2$) if the angle sequence of T_1 is lexicographically greater than T_2's.
  - {20°, 30°, 45°, 65°, 120°} > {20°, 30°, 45°, 60°, 120°}

Delaunay Triangulation

- For each convex quad in a triangulation T_1 with diagonal e, if a diagonal flip results in a triangulation T_2, s.t. $T_1 \geq T_2$, then e is legal.
- A Delaunay triangulation is a triangulation with all legal edges.

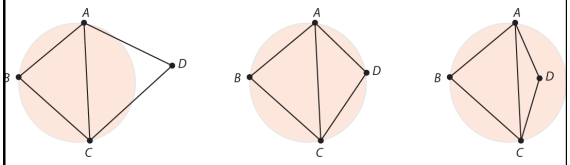


Thales' Theorem

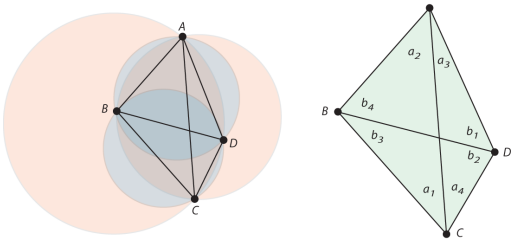


Thales' and Illegal Edges

- Let *e* be an edge of a triangulation, where *e* = *AC* belongs to the two triangles *ABC* and *ACD*. Then *e* is legal if *D* is outside of the circumcircle of *ABC* and illegal if *D* is inside.

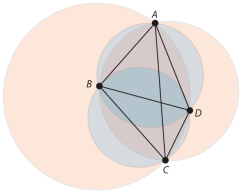


Proof

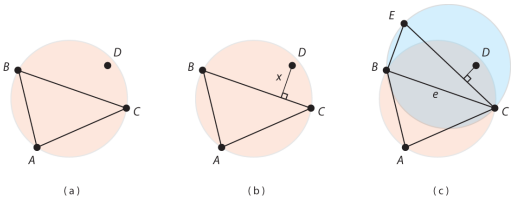


Empty Circle Property

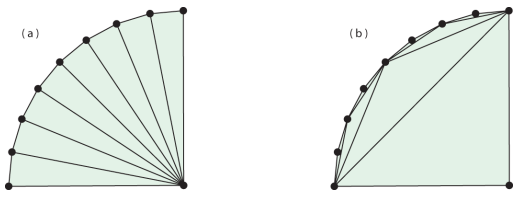
- Let *S* be a point set in general position. A triangulation *T* is Delaunay if and only if no point from *S* is in the interior of any circumcircle of any triangle of *T*.



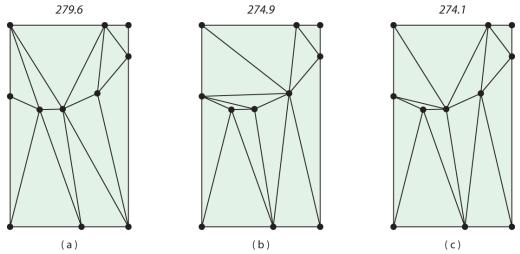
Proof



Delaunay is not MWT



Delaunay vs. Greedy vs. MWT



Theorem

- For point set *S*, a minimum spanning tree of *S* is a subset of the Delaunay triangulation of *S*.
- Proof by contradiction.

Compatible Triangulations