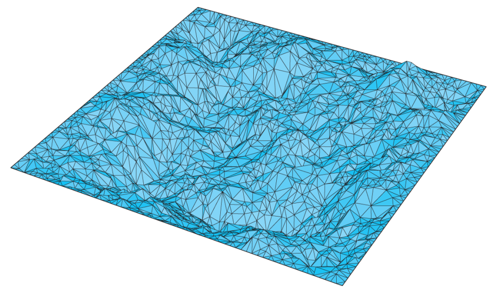
Computational Geometry

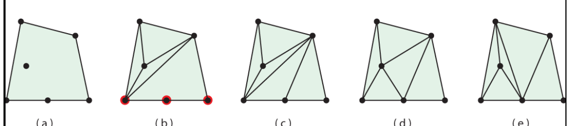
Triangulation

Terrain Reconstruction from Sampled Heights

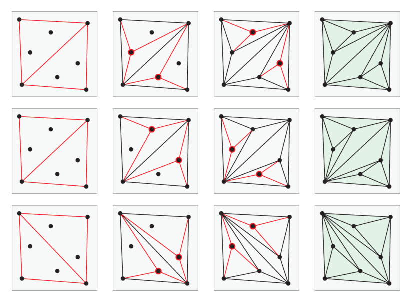


Definition

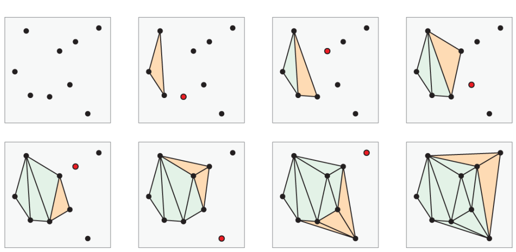
- A triangulation of a planar point set *S* is a subdivision of the plane determined by a *maximal* set of noncrossing edges whose vertex set is *S*.



Triangle Splitting

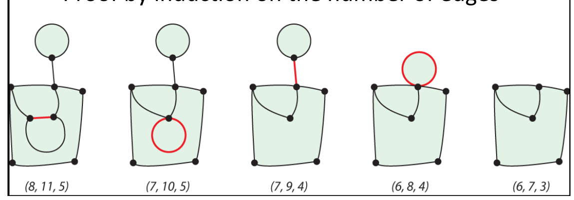


Incremental



Euler's Formula

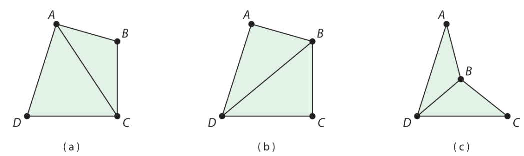
- Let G be a connected planar graph with V vertices, E edges and F faces, then V-E+F = 2
- The outer face is unbounded
- Proof by induction on the number of edges



Theorem

- Let S be a point set with h points on the hull and k in the interior. If all points are in general position, then any triangulation of S has exactly $2k+h-2$ triangles and $3k+2h-1$ edges.

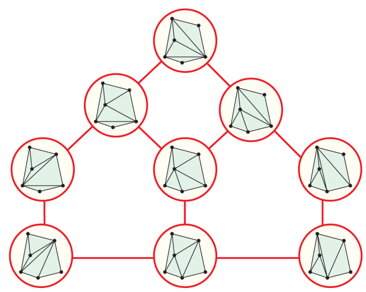
Edge Flip



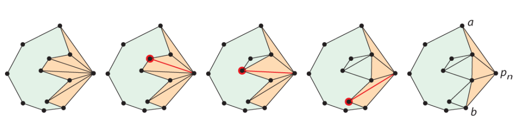
Definition

- For a point set S, a flip graph of S is a graph whose nodes are the set of triangulations of S. Two nodes T_1 and T_2 are connected by an edge if one diagonal of T_1 can be flipped to obtain T_2.

Flip Graph



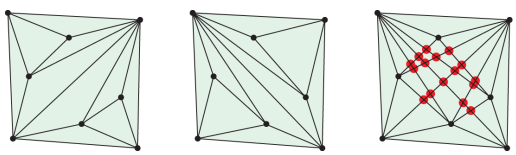
Flipping a Star



Theorem

- The flip graph of any planar point set is connected.
- The *diameter* of a graph is the longest path between any two nodes of the graph, in number of edges.
- For a planar point set S of n points, the diameter of its flip graph is at most $(n-2)(n-3)$

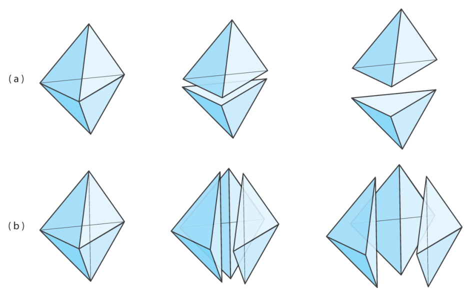
Tighter Bound



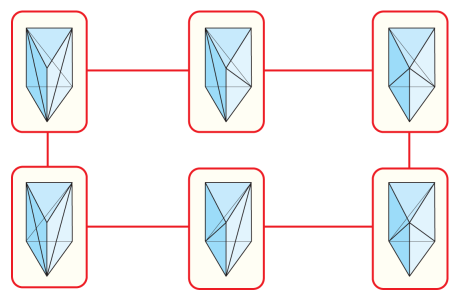
3D

- Let S be a point set in R^3 in general position, with k points in the interior and h on the hull. Then there exists a tetrahedralization of S with at most $3k+2h-7$ tetrahedra.

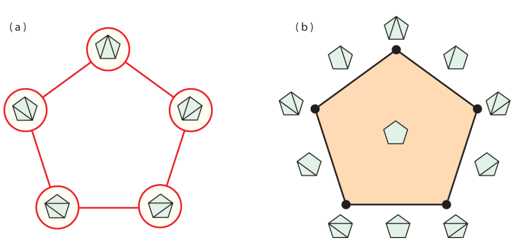
Face Flip



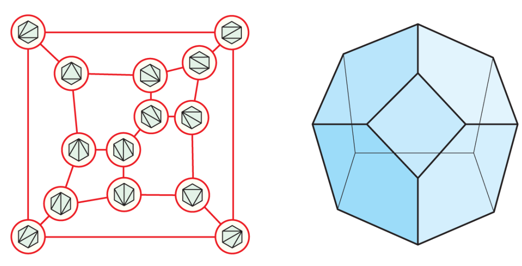
3D Flip Graph



Diagonalization of a Pentagon and the 2D Associahedron



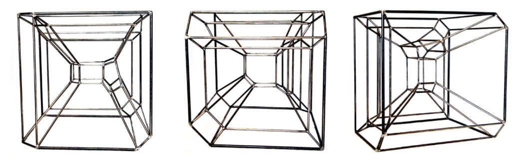
Hexagon Flip Graph and the 3D Associahedron



Associahedra in Higher Dimensions

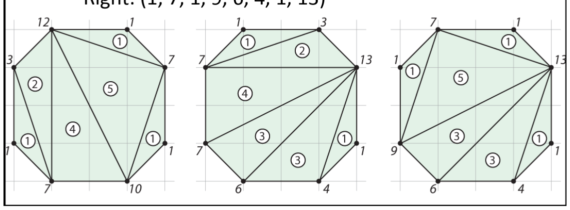
- There exists a convex *n*-dimensional polytope called the associahedron whose vertices and edges form the flip graph of a convex (*n*+3)-sided polygon. The *k*-dimensional faces of this polytope are in one-to-one correspondence with the diagonalization of the polygon using exactly *n*-*k* diagonals.

4D Associahedron Representing the Flip Graph of a Heptagon



Sum of Area of Triangles around a Vertex in a Triangulation

- Area Vectors
  - Left: (1, 12, 3, 1, 7, 10, 1, 7)
  - Middle: (3, 1, 7, 7, 6, 4, 1, 13)
  - Right: (1, 7, 1, 9, 6, 4, 1, 13)



Theorem

- If *P* is a convex polygon with *n* vertices, the convex hull of the area vectors of all triangulations of *P* is combinatorially equivalent to the associahedron of dimension *n*-3.