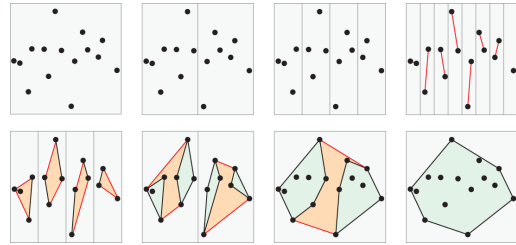


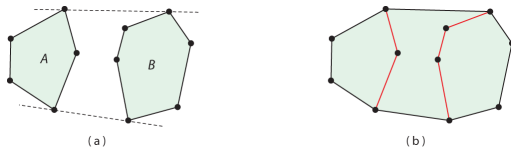
## Computational Geometry

### Convex Hull

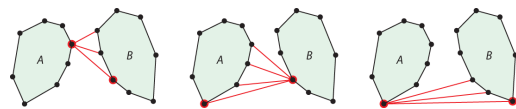
### Divide-and-Conquer



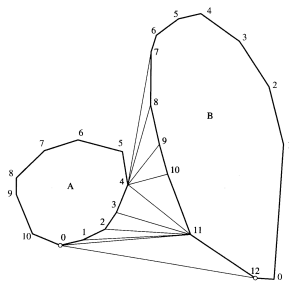
### Tangent Lines of Two Hulls



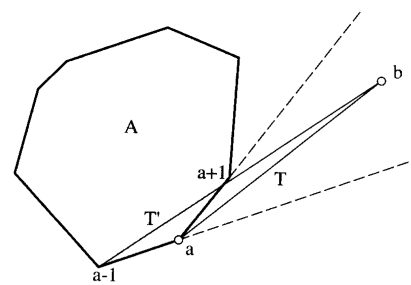
### Tangent Line in $O(n)$



### Lower Tangent



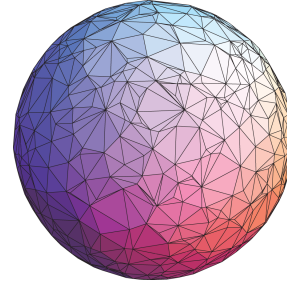
### ab Does Not Cross A



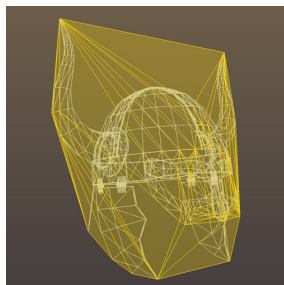
### Complexity

- $O(n)$  to find the tangent lines
- $T(n) = 2T(n/2) + O(n) = 2T(n/2) + cn$
- $= 2(2T(n/4) + cn/2) + cn$
- $= 2(2(2T(n/8) + cn/4) + cn/2) + cn$
- $= 2(2(2(2T(n/16) + cn/8) + cn/4) + cn/2) + cn$
- ...
- $= 2^{\lg(n)} + (2^{\lg(n)-1}/2^{\lg(n)-1} + 2^{\lg(n)-2}/2^{\lg(n)-2} + \dots + 2^0) cn$

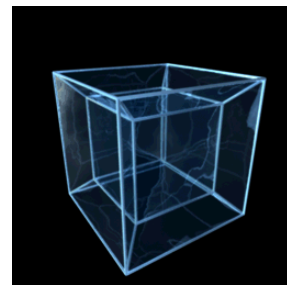
### Hull of 758 Random 3D Points



### Helmet Hull



### Hyper Cube

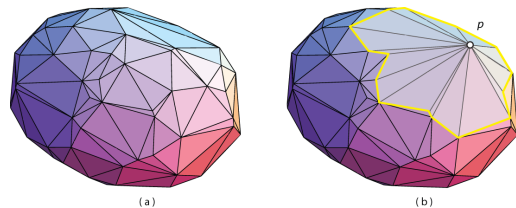


### Bounds

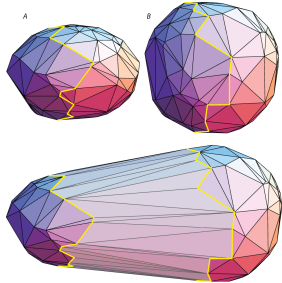
- Convex hull of  $n$  points in  $d$  dimension has a lower bound of  $\Omega(n^{\lfloor d/2 \rfloor})$

| Algorithm          | 2D            | 3D            |
|--------------------|---------------|---------------|
| Incremental        | $O(n^2)$      | $O(n^2)$      |
| Gift wrapping      | $O(nh)$       | $O(nf)$       |
| Divide-and-Conquer | $O(n \log n)$ | $O(n \log n)$ |
| Graham scan        | $O(n \log n)$ |               |

### 3D Incremental Hull



### 3D Divide-and-Conquer



### Possible Shadow Boundary Edges

