

Computational Geometry

Review

Polygons

- Every polygon admits a triangulation.
- Every triangulation of a polygon with n vertices has $n-2$ triangles and $n-3$ diagonals.
- Every polygon with $n > 3$ vertices has at least two ears.
- $\lfloor n/3 \rfloor$ vertex guards are necessary and sufficient to guard a polygon with n vertices

Area of Polygons

- The cross product and the area of triangle.
- Generalization to arbitrary polygons:

$$2A(P) = \sum_{i=0}^{n-1} (x_i y_{i+1} - y_i x_{i+1})$$

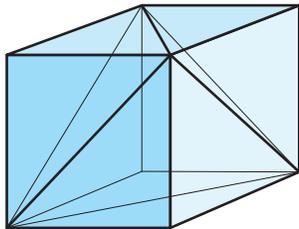
- The number of triangulations of a convex polygon with $n+2$ vertices is the Catalan number

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$$

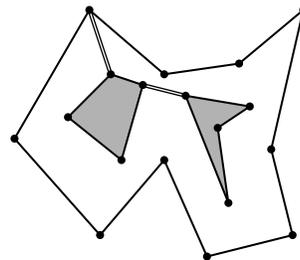
Dissections

- Any two polygons of the same area are scissors congruent.
- Any two polyhedra with the same Dehn invariant are scissors congruent.

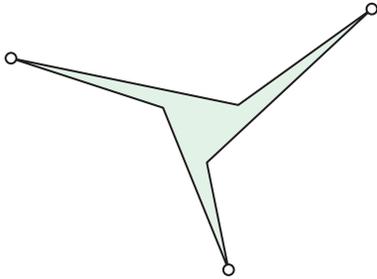
Tetrahedralization of Cube



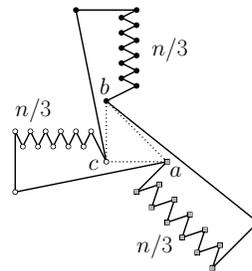
Triangulations of Polygons with Holes



Guarding the Walls

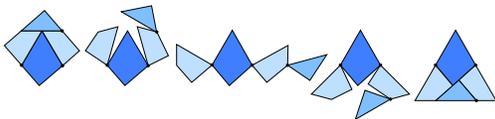


Counter Example

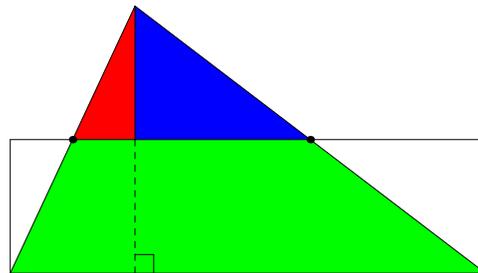


Hinged Dissections

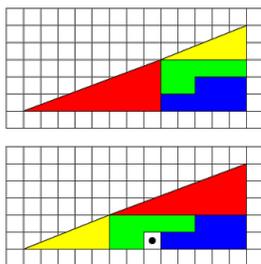
- Dissections with an additional constraint: the polygonal pieces must be hinged together at vertices into a connected assembly.



Hinged Triangle-Rectangle



Dissection Paradox



64 = 65?

