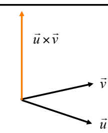
## Computational Geometry

### Area and Triangulation

## The Cross Product

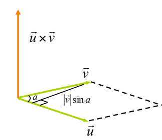



- A vector operation

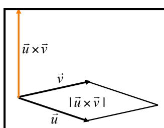
$$\vec{u} \times \vec{v} = \begin{bmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{bmatrix} = \begin{bmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_z v_x \\ u_x v_y - u_y v_x \end{bmatrix}$$

- Geometric significance

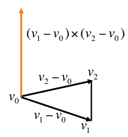
$$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin a$$




## Area of Triangle



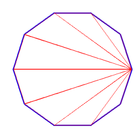

- A(△) = ½|$(v_1$-$v_0)$X$(v_2$-$v_0)$|




$$(v_1 - v_0) \times (v_2 - v_0) = \begin{bmatrix} i & j & k \\ x_1 - x_0 & y_1 - y_0 & 0 \\ x_2 - x_0 & y_2 - y_0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ (x_1 - x_0)(y_2 - y_0) - (y_1 - y_0)(x_2 - x_0) \end{bmatrix}$$

## Area of Polygon

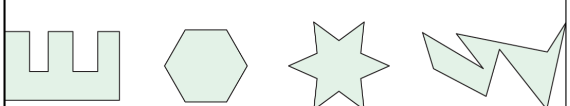
- Convex polygons:



- General?

$$2A(P) = \sum_{i=0}^{n-1} (x_i y_{i+1} - y_i x_{i+1})$$

## How Many Distinct Triangulations?



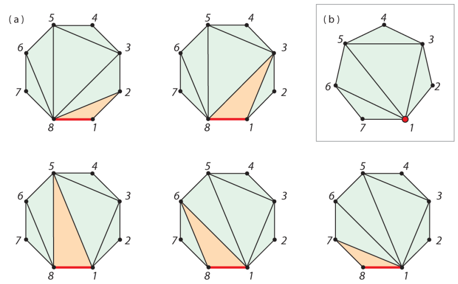
(a) (b) (c) (d)

- A diagonal exists between any two non-adjacent vertices of a polygon *P* iff *P* is a convex polygon

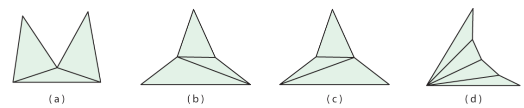
## Theorem

- The number of triangulations of a convex polygon with *n*+2 vertices is the Catalan number: $C_n = \frac{1}{n+1}\begin{pmatrix} 2n \\ n \end{pmatrix} = \frac{(2n)!}{(n+1)!n!}$

## Edge Contraction



## Reflex Vertices?

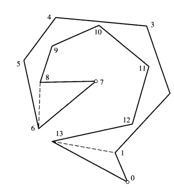



## Big-O Notation

- Upper bounds on the growth rate of functions and running time of algorithms.
- *n* represents input size, i.e. a list of *n* elements.
- Constants are ignored and only the dominant term matters
- A for-each sequential loop is O(*n*)
- A nested for where inner loop runs *y* times and outer loop runs *x* times is O(xy), or O($n^2$)

## Triangulation

- By ear removal
  - Search for an ear-diagonal
  - Cut off ear and repeat on the rest of the polygon, which now has *n*-1 vertices
- Complexity?



## Monotone Polygon