CS380 Computational Geometry
Spring 2013

Homework:	**5**	**Professor:**	**Dianna Xu**
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Assignment 5

1. O'Rourke Exercise 4.8.

2. O'Rourke Exercise 4.16.

3. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n sites. We say that sites p_i and p_j are *Voronoi neighbors* if their Voronoi regions share an edge e in the Voronoi diagram.

1. Give an $O(n \log n)$ algorithm to find: for every site $p \in P$, the site that is closest to it. *Hint:* First prove that for every site $p \in P$, the site that is closest to p must be a Voronoi neighbor of p.
2. Prove or disprove: Suppose p_i and p_j are Voronoi neighbors. Let e be the Voronoi edge between them. Then the midpoint of p_i and p_j must belong to e.

4. In this problem we will investigate the properties of a variant of the Voronoi diagram, in which we replace the notion of "nearest" with "farthest". You will find reading de Berg section 7.4 helpful for this question. Given a set $P = \{p_1, p_2, \ldots, p_n\}$ of point sites and site $p_i \in P$, recall that we defined $\mathcal{V}(p_i)$ to be the set of all points in the plane that are closer to p_i than to any other site of P. Define $\mathcal{F}(p_i)$ to be the set of all points in the plane that are *farther* from p_i than from any other site of P. As we did with Voronoi diagrams, we define the *farthest-point Voronoi diagram* to be the union of the boundaries of $\mathcal{F}(p_i)$ for all points in P. Let $\mathcal{NVD}(P)$ denote the *nearest-point Voronoi diagram* of P and let $\mathcal{FVD}(P)$ denote the *farthest-point Voronoi diagram* of P.

1. Show that $\mathcal{F}(p_i)$ is a convex polygon (possibly unbounded).
2. Unlike the nearest-point Voronoi diagram, a site may not contribute a region to the farthest point diagram. Show that $\mathcal{F}(p_i)$ is nonempty *if and only if* p_i is a vertex of the convex hull of P. This is informally observed in de Berg 7.4. Give a formal proof.
3. Draw a picture of three points in the plane. Show (in different colors) $\mathcal{NVD}(P)$ and $\mathcal{FVD}(P)$. Label each region according to which sites are closest and farthest.
4. Repeat (c), but this time with four points, such that only three of the four points are on the convex hull.
5. Repeat (d), but this time with all four points on the convex hull. (Try to avoid degeneracies, such as four cocircular points or parallel sides.)

5. The Euclidean metric is but one way to measure distances (this is the metric that we used when discussing Voronoi diagrams). In this question, we will examine another distance metric, called the L_∞ metric, which is defined as follows

$$\mathrm{dist}_\infty(p, q) = \max(|p_x - q_x|, |p_y - q_y|).$$

1. Given a point q, describe the set of points that are at L_∞ distance w from q.
2. Given two distinct points p and q, describe the set of points that are equidistant from p and q in the L_∞ distance. *Hint:* The bisector is a polygonal curve. As a function of the coordinates of p and q, indicate how many vertices this curve has, and where these vertices are located. I would like the exact formula, and not simply a high-level description. If you are having trouble visualizing, I recommend writing a small plotting program in Python, Mathematica or some other language that allows easy graphics output. It is not necessary to submit the plotting program or provide its sample runs. It is only meant to guide your insights.