CS380 Computational Geometry
Spring 2013

Homework:	**1**	**Professor:**	**Dianna Xu**
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Assignment 1

Give written (and legible) answers to the following questions. Your answers should be precise and mathematically accurate. In general, numerical answers should be followed by justification. You should give formal proofs if at all possible, or at the minimum, give strong arguments. Rigorous proofs can be difficult to write and digest in geometry, so you are encouraged to use intuition and give illustrations whenever appropriate. Beware however, that a poorly drawn figure can make certain erroneous hypotheses appear to be "obviously correct".

Throughout the semester, unless otherwise stated, you may assume that input objects are in *general position.* For example, you may assume that no two points have the same coordinates, no three points are collinear and no four points are cocircular, etc.

You are encouraged to discuss the problems with other students in class, but the write-up should be entirely your own.

1. *Meister's two ears.* Prove that every triangulation has at least two ears by taking the dual graph and prove that every tree has at least two leaves.

2. Find a tetrahedralization of the cube into five tetrahedra.

3. Three consecutive vertices a, b, c form a mouth of a polygon if ac is an external diagonal of the polygon, a segment wholly outside. Formulate and prove a theorem about the existence of mouths.

4. Prove that every polygonal region with polygonal holes admits a triangulation of its interior.

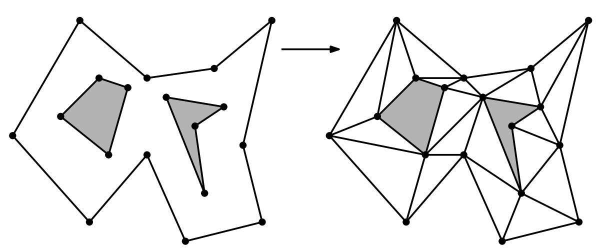

Figure 1: Triangulation of polygonal region with holes.

5. *Guarding the walls.* Construct a polygon P and a placement of guards such that the guards see every point of δP, but there is at least one point interior to P not seen by any guard.

6. *Clear visibility, point guards.* Recall that visibility is defined to allow grazing contact with δP, in other words, vertices and edges parallel to line of sight do not block vision. *Clear visibility* is defined to not allow grazing contact with δP. What is the answer to Klee's question for clear visibility? More specifically, what is the smallest number of *point guards* that suffice to clearly see every point in any polygon of n vertices? Recall that point guards are guards who may stand at any point of P; these are distinguished from *vertex guards* who may be stationed only at vertices. Are clearly seeing guards stronger or weaker than regular guards? Does Fisk's proof establish $\lfloor n/3 \rfloor$ sufficiency for clear visibility? Try to determine the number exactly. Note that a point guard does clearly see the point on which it stands.

7. *clear visibility, vertex guards.* Answer the last question, but for *vertex guards*, stationary guards placed on vertices. Note that a vertex guard with clear visibility can not see the other end point, because the edge is part of the boundary.

8. Given a polygon with n vertices, recall that the addition of any diagonal splits P into two polygons with n_1 and n_2 vertices, where $n_1 + n_2 = n + 2$.

1. Show that for any $n > 3$ there exists a diagonal that splits P such that $min(n_1, n_2) \geq \lfloor n/3 \rfloor$.
2. Show that the constant $1/3$ is the best possible, in that for any $c > 1/3$, there exists a polygon such that *any* diagonal chosen results in a split such that $min(n_1, n_2) < cn$. You can provide a drawing, but you should give justification on how your drawing can be generalized to all sufficiently large values of n.

Please hand in your assignment in class.