CMSC 373 Artificial Intelligence
Fall 2025
07-Logic

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1

AI Approaches to *Knowledge*

- Expert Systems ✓
- Logic
- Frames, Scripts, and Semantic Networks
- Knowledge Graphs

2

2

Representing Knowledge using Logic

- Aristotle: What are correct arguments/thought processes?
- Formal Logics:

  Socrates is human.
  All humans are mortal.
  Therefore, Socrates is mortal.

- Laws of thought govern the operation of the mind.

- Logic uses **declarative symbolic representations**.

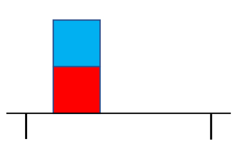
3

3

Example

- BLOCKSWORLD with declarative knowledge

  if a block is on top of another block
  then
  the latter block is not clear



Facts

blue block is on top of red block
red block is on the table
blue block is clear

∴ red block is not clear

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4

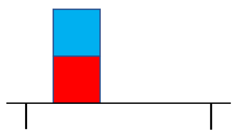
Example

Represents the "what" and not "how".

- BLOCKSWORLD with declarative knowledge

if a block is on top of another block
then
the latter block is not clear

These are called **propositions**.



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5

5

Propositions

- Propositions are something that can be expressed by a declarative sentence of English (or another language)

  B is a block.
  The sky is blue.
  The snark was a boojum.
  Dinosaurs were warm-blooded.
  The stock market index will double its current value within two years.

- These sentences are true if the proposition they express holds. It is false if the proposition does not hold.

- If a sentence is assumed to be **true**, one can answer questions about it without knowing what the words in it mean.

6

6

Propositions

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- These sentences are true if the proposition they express holds. It is false if the proposition does not hold.

- If a sentence is assumed to be **true**, one can answer questions about it without knowing what the words in it mean:

  What kind of a thing is a snark?
  A boojum.

  Is it true that a snark is either a beejum or a boojum?
  Yes, because a snark is a boojum.

  If no boojum is ever a beejum, was the snark ever a beejum?
  No.

  What is an example of something that was a boojum?
  The snark.

Entailment

- Entailment enables us to answer questions from sentences without knowing what the words in a sentence stand for.

  Deepak is in the classroom or in his office.
  Nobody is in the classroom.
  So: Deepak is in the office.

  Evan is married to Chris or Pat.
  Nobody is married to Chris.
  So: Evan is married to Pat.

  The frumble is frimble or framble.
  Nothing is frimble.
  So: The frumble is a framble.
- Without knowing what the words mean, we can arrive at a correct conclusion.

Logical Entailment

- A collection of sentences $S_1, S_2, \ldots, S_n$ **logically entails** another sentence S if the truth of S is implicit in the truth of the S_i sentences.
- That is, no matter what certain words (like *boojum*, *framble*, etc.) in sentences S_i mean, if S_i are all true, then the sentence S is also true.

  The snark was a boojum.

  Logically entails

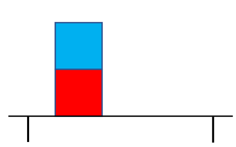
  Something was a boojum.

9

9

Knowledge Representation in Logic

- Syntax: How sentences are formed/written
- Semantics: Meaning/Interpretation of sentences
- Computation: How sentences are manipulated



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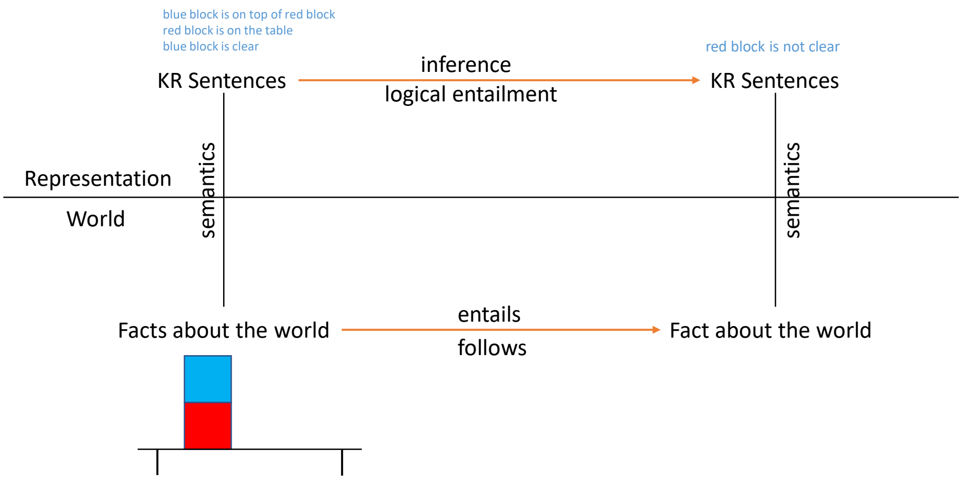
10

10

Knowledge Representation



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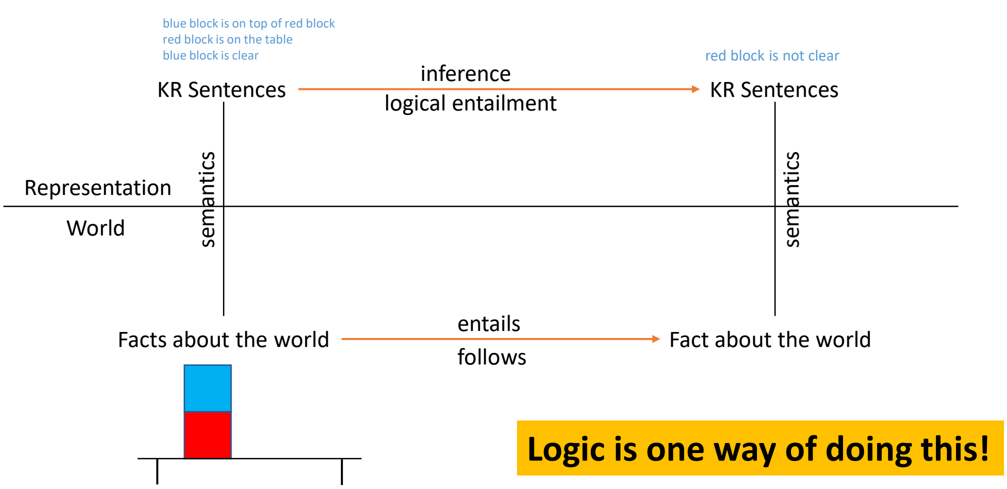
11

Knowledge Representation



12

12

What is a logic?

- Study of correct inferences

Premises

…
… } True
…

Conclusion

∴ … } is also True

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∴ … } is also True

If it is cold then my car will not start.
My car will not start.

∴ It is cold.

If it is cold then my car will not start.
It is cold.

∴ My car will not start.

What is a logic?

- Study of correct inferences – Truth preserving consequences

Premises

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15

15

What is a logic?

- Study of correct inferences – Truth preserving consequences

Premises

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… } True
…

Conclusion

∴ … } is also True

If it is cold then my car will not start.
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← This is NOT Truth preserving.

If it is cold then my car will not start.
It is cold.

∴ My car will not start.

← This is Truth preserving.

16

16

What is a logic?

- Study of correct inferences – Truth preserving consequences

Premises

…
… } True
…

Conclusion

∴ … } is also True

If it is cold then my car will not start.
My car will not start.

∴ It is cold.

This is NOT Truth preserving. This is sometimes used in diagnosis! (aka abduction)

If it is cold then my car will not start.
It is cold.

∴ My car will not start.

This is Truth preserving. (aka **deduction**)

17

17

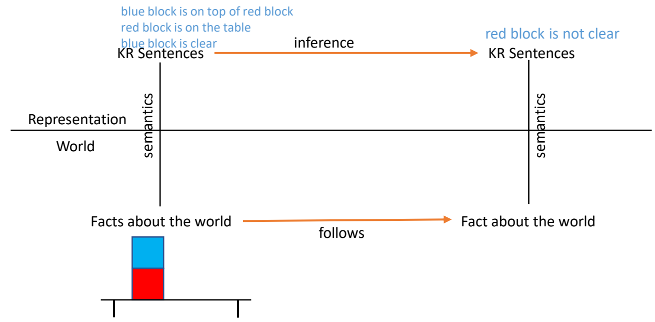
What is a logic?

- Study of correct inferences
- Formalize the notion of correct inference
  - Step1: Define a formal language to write sentences – syntax
    well-formed sentences (wffs)
  - Step 2: What do the wffs mean? – Semantics/Model Theory
    Need an interpretation for wffs
  - Step 3: Rules of Inference – Proof Theory

18

18

Rules of Inference



Properties

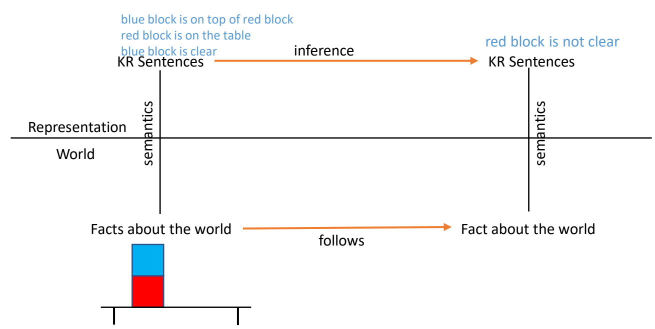
Soundness: Every new sentence that can be derived from KR is a valid consequence.

Completeness: Every valid consequence of KR can be formally derived.

19

19

Rules of Inference



Properties

Soundness: Every new sentence that can be derived from KR is a valid consequence. ← This is easy to show

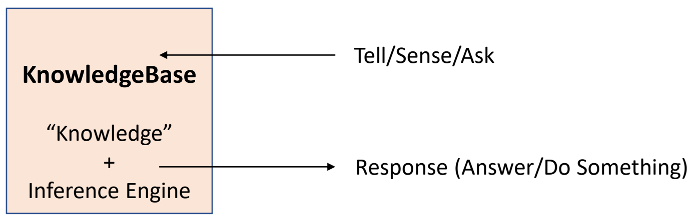
Completeness: Every valid consequence of KR can be formally derived. ← This is true for some logics.

20

20

Knowledge Representation & Reasoning

- Knowledge Representation & Reasoning (KRR) Systems



- Knowledge – set of sentences that describe facts about the world (or domain)
- Inferences – procedures/rules that operate on facts to infer new facts

There are many types of logic

- Propositional Logic
- First-Order Logic
- Second-Order Logic
- Temporal Logic
- Modal Logic
- Constraint Logic
- Etc.

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- Propositional Logic
- First-Order Logic (semi-formally!)
- Second-Oder Logic
- Temporal Logic
- Modal Logic
- Constraint Logic
- Etc.

23

First-Order Predicate Calculus (FOPC)

Components

- Object constants: A, B, Deepak, etc.
- Function Constants: *fatherOf*, *colorOf*, etc.
- Relation Constants: Parent, On, Clear, Sibling, etc.
- Variables: x, y, z, u, v, w, etc.
- Connectives: $\vee$, $\wedge$, $\neg$, $\Rightarrow$
- Quantifiers: $\forall$, $\exists$
- Delimiters: (,), [,]

24

Objects, Functions, Relations

Object constants: these are objects or individuals in the domain…

A, B, Deepak, Red, Car54, etc.

Relation Constants: Denote properties of/between objects

$Parent^2$, On^2, $Clear^1$, $Sibling^2$, etc.

25

FOPC - Syntax

Terms

- An object is a term

Wffs – represent propositions

- Atomic Sentences (wffs) relation-constant^n(term$_1$, term$_2$,…term$_n$)

  e.g. Parent(Ginny, Lily)

- Propositional wffs
  If ω_1 and ω_2 are wffs, then so are
  ω_1
  $\omega_1 \vee \omega_2$ disjunction
  $\omega_1 \wedge \omega_2$ conjunction
  $\omega_1 \Rightarrow \omega_2$ implication
  $\neg \omega_1$ negation

26

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ω_1	
$\omega_1 \vee \omega_2$	disjunction
$\omega_1 \wedge \omega_2$	conjunction
$\omega_1 \Rightarrow \omega_2$	implication
$\neg\, \omega_1$	negation

Save for later…

Variables: x, y, z, u, v, w, etc.
Quantifiers: ∀

27

27

FOPC - Semantics

Objects The world can have infinite objects/individuals

- concrete
- abstract
- fictional

If it is a name and we need to say something about it.

Relations Denote properties

Heavy(x)	On(A, B)
Big(y)	Clear(A)
Human(Deepak)	Block(A)

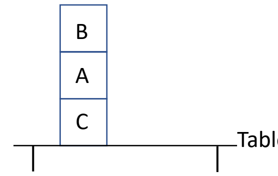
Interpretation

- An atomic wff is True/False just in case the relation denoted by it holds for its arguments.

28

28

FOPC Example - Blocksworld



- **Objects** A, B, C, Table
- **Relations** On^2, $Clear^1$

On(C, Table) On(A, C) On(B, A)
Clear(B) Clear(A) Clear(C) Clear(Table)

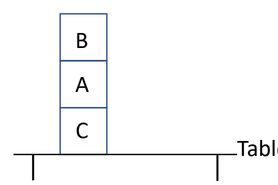
- Example Knowledge Base

Δ = {On(C, Table), On(A, C), On(B, A), Clear(B)}

29

29

FOPC Example - Blocksworld



- **Objects** A, B, C, Table
- **Relations** On^2, $Clear^1$

On(C, Table) On(A, C) On(B, A)
Clear(B) Clear(A) Clear(C) Clear(Table)

- Example Knowledge Base

But, how to make general Statements about the world?

Δ = {On(C, Table), On(A, C), On(B, A), Clear(B)}

30

30

Variables & Quantifiers

- **Variable**s – A variable is a term. i.e. it denotes/can denote an object.
- **Universal Quantifier (∀ - "for-all")**

if ω is a wff and x is a variable then

$(\forall x)\ \omega$ is a wff

$\forall x\ (\omega)$ is a wff

$\forall x\ [\omega]$ is a wff

ω is the scope of the variable.

E.g.

$\forall x\ [\ P(x) \Rightarrow R(x)\ ]$

Read as: All P's are R's.
Or: If something is a P, then it is an R

31

31

Variables & Quantifiers

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ω is the scope of the variable.

E.g.

$\forall x\ [\ P(x) \Rightarrow R(x)\ ]$

These are also called conditional sentences.

Read as: All P's are R's.
Or: If something is a P, then it is an R

32

32

Semantics of ∀

In a given domain

$\forall x$ ω(x) has value True

Just in case ω(x) has value True for all assignments of x to objects in the domain.

e.g. On^2, $Clear^1$

$\forall x$ [On(x, C) ⇒ ¬Clear(C)] *If anything is on C then C is not clear.*

Just in case x=A/B/C/Table are True.

33

33

Example

Tell King(John)
Tell Person(Richard)
Tell $\forall x$ [King(x) ⇒ Person(x)]

Queries

1. King(John)?
   True

2. Who is a person?
   Person(x)?
   x = Richard

3. What about John??? Requires inference!

Is Person(x) a King(x)?
Since King(John)
Person(John)

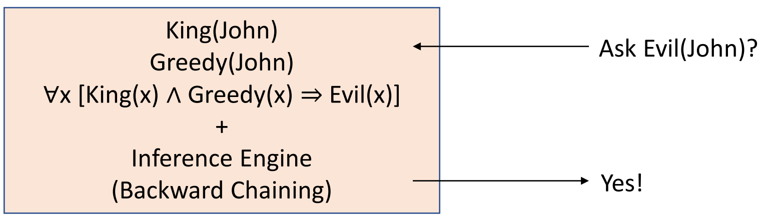
x = John

34

34

KRR Systems

- Tell-Ask Systems



35

35

Example

Tell King(John)
Tell Person(Richard)
Tell $\forall x$ [King(x) ⇒ Person(x)]

Queries

1. King(John)?

   True

2. Person(x)?

   x = Richard
   x = John

Tell ∀x [King(x) ∧ Greedy(x) ⇒ Evil(x)]
Tell King(John)
Tell Greedy(John)

Query

Evil(John)?
True

36

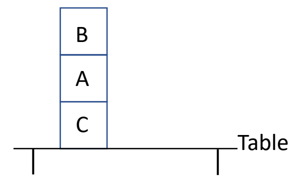
36

Blocksworld

On(x, y) : x is on top of y

Clear(x) : x is clear

Block(x) : x is a block

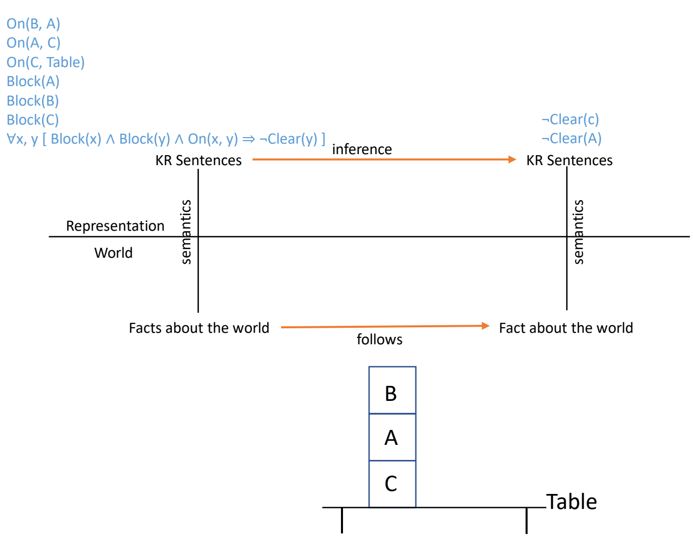


On(B, A)
On(A, C)
On(C, Table)
Block(A)
Block(B)
Block(C)
∀x, y [Block(x) ∧ Block(y) ∧ On(x, y) ⇒ ¬Clear(y)]

37

37

Reasoning with Logic



38

38

Blocksworld - Alternatives

On(x, y) : x is on top of y

Clear(x) : x is clear

Block(x) : x is a block

Isa(x, y) : x is a y

Isa(A, Block)
Isa(B, Block)
Isa(C, Block)

On(B, A)
On(A, C)
On(C, Table)
Block(A)
Block(B)
Block(C)
∀x, y [Block(x) ∧ Block(y) ∧ On(x, y) ⇒ ¬Clear(y)]

39

39

Blocksworld - Alternatives

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Isa(x, y) : x is a y

Isa(A, Block)
Isa(B, Block)
Isa(C, Block)

On(B, A)
On(A, C)
On(C, Table)
Block(A)
Block(B)
Block(C)
∀x, y [Isa(x, Block) ∧ Isa(y, Block) ∧ On(x, y) ⇒ ¬Clear(y)]

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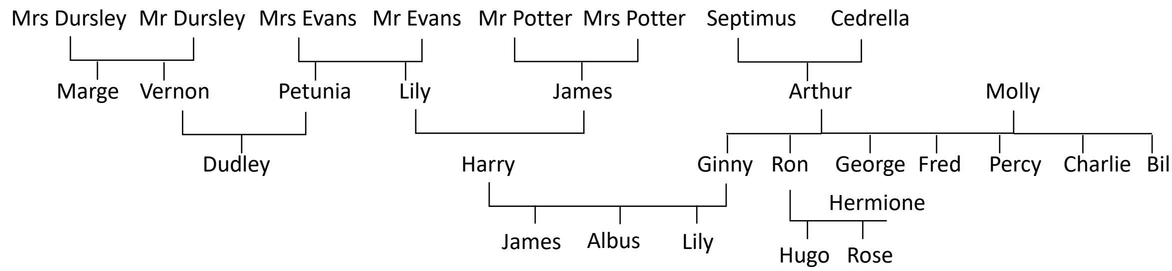

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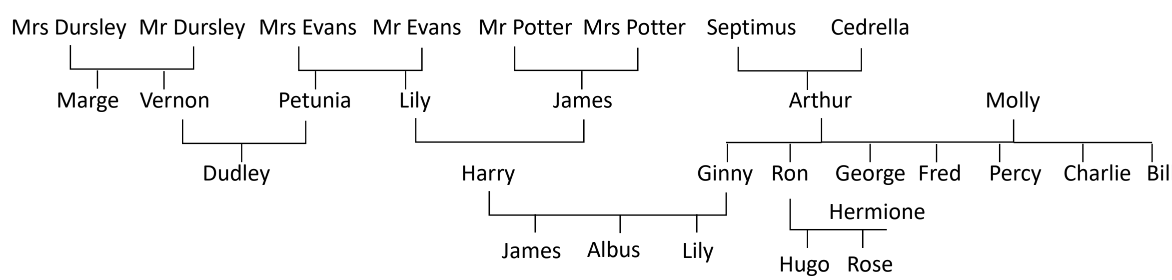
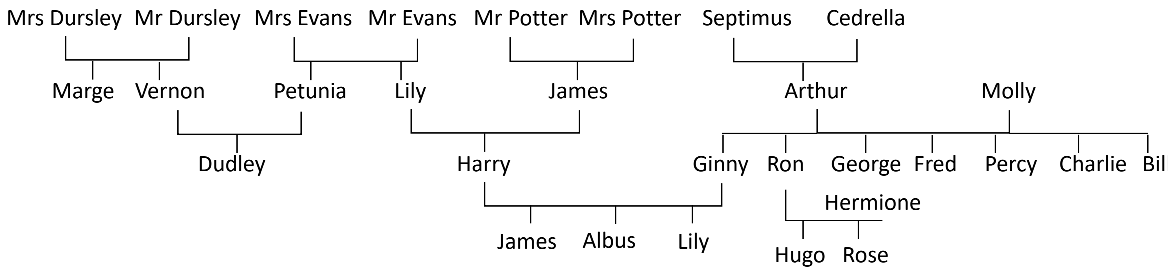


<u>Predicates</u>

1. Female(x) : x is female
2. Male(x) : x is male
3. Parent(x, y) : x is a parent of y
4. Married(x, y) : x is married to y

42

42



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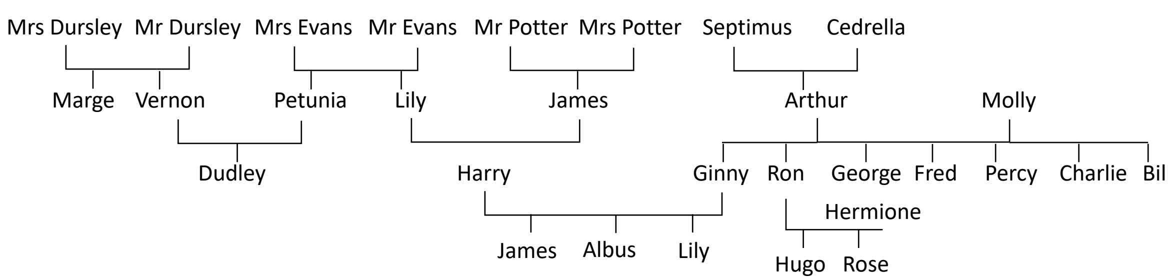
Symbols

Marge, Vernon, James1, James2, Albus, Lily, Harry, etc.

Female(Marge)	**Married(Vernon, Petunia)**	**Parent(Harry, James2)**
Female(Petunia)	**Married(Lily, James1)**	**Parent(Ginny, James2)**
Female(Lily)	**Married(Arthur, Molly)**	**Parent(Petunia, Dudley)**
Male(James1)	**Married(Harry, Ginny)**	**Parent(Vernon, Dudley)**
Male(Vernon)	**Married(Ron, Hermione)**	**Parent(Molly, Bill)**
Etc.	**Etc.**	**Etc.**

43

43



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Female(Petunia)	**Married(Lily, James1)**	**Parent(Ginny, James2)**
Female(Lily)	**Married(Arthur, Molly)**	**Parent(Petunia, Dudley)**
Male(James1)	**Married(Harry, Ginny)**	**Parent(Vernon, Dudley)**
Male(Vernon)	**Married(Ron, Hermione)**	**Parent(Molly, Bill)**
Etc.	**Etc.**	**Etc.**

Facts

44

44

Modeling Relationships

- Spouse(x, y) : x is a spouse of y
- Wife(x, y) : x is a wife of y
- Husband(x, y)
- Father(x, y), Mother(x, y)
- Sibling(x, y), Brother(x, y), Sister(x, y)
- GrandParent/GrandMother/GrandFather
- GrandChild/GrandDaughter/GrandSon
- Aunt/Uncle/AuntOrUncle
- Cousin
- Niece/Nephew/NieceOrNephew
- Child/Daughter/Son
- Etc.

45

45

Modeling Relationships

- Spouse(x, y) : x is a spouse of y
- Husband/Wife
- Mother/Father

46

46

Modeling Relationships

- Spouse(x, y) : x is a spouse of y

  $\forall x, y\ [Married(x, y) \Rightarrow Spouse(x, y)]$

- Husband/Wife*

  $\forall x, y\ [Female(x) \land Married(x, y) \Rightarrow Wife(x, y)]$

- Father/Mother *

  $\forall x, y\ [Female(x) \land Parent(x, y) \Rightarrow Mother(x, y)]$

47

47

Modeling Relationships

- Spouse(x, y) : x is a spouse of y
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- Father(x, y), Mother(x, y)
- Sibling(x, y), Brother(x, y), Sister(x, y)
- GrandParent/GrandMother/GrandFather
- GrandChild/GrandDaughter/GrandSon
- Aunt/Uncle/AuntOrUncle
- Cousin
- Niece/Nephew/NieceOrNephew
- Child/Daughter/Son
- Etc.

48

48

Vocabulary

Declarative Knowledge
Propositions
Entailment
Knowledge Representation
What is a logic?
Rules of Inference
Soundness
Completeness
First-Order Logic (FOPC)
Syntax
Semantics
Knowledge Representation & Reasoning Systems

References

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- Hector Levesque, *Thinking as Computation*, MIT Press, 2017.