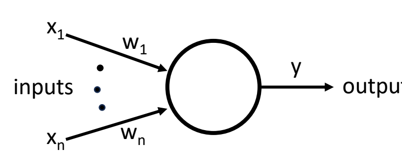
# CMSC 373 Artificial Intelligence
# Fall 2023
# 13-Backpropagation

Deepak Kumar
Bryn Mawr College

1

## The Perceptron (1958)

- A single "neuron" (**unit**)
  *aka* Threshold Logic Unit (TLU)




- **Transfer Function**
  T is the Threshold value (assume $T = 0$)

$$I = \sum_{i=1}^{i=n} w_i x_i$$

$$y = \begin{cases} +1, \text{if } I \geq T \\ -1, \text{if } I < T \end{cases}$$

2

2

# Learning in Neural Networks (so far)

Epochs
Forward Pass
Hyperparameters
Labelled Dataset
Learning Rule
Model
Parameters

- A **labelled dataset**
  e.g., Iris Dataset, MNIST Numbers Dataset
- A **model** for the network
  e.g., the Perceptron (single TLU)
  **Parameters** refer to the number of weights
- A **forward pass/prediction algorithm**
  Compute **net input**. Compute **activation**.
- A **Learning Rule/Algorithm**
  Learning occurs by changing weights.
  Specifies change in the weights using the **Error/Loss**
  In Perceptrons, weights are changed after every prediction.
- Several **epochs** of training are needed (how many???)
  This is a **hyperparameter**.

$$I = \sum_{i=1}^{i=n} w_i x_i$$

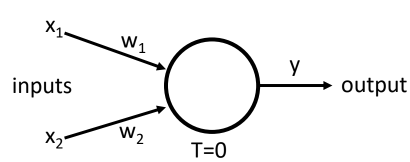
$$y = \begin{cases} +1, \text{if } I \geq T \\ -1, \text{if } I < T \end{cases}$$

Perceptron Learning Rule: $\overline{\boldsymbol{w_{new}}} = \overline{\boldsymbol{w_{old}}} + \beta y^* \,\overline{\boldsymbol{x}}$

3

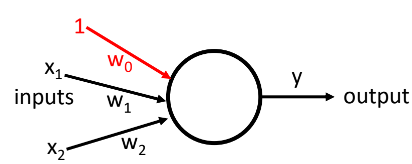
3

# Introducing Bias




- Instead of using an arbitrary Threshold value, we can turn it into an input (=1)
- The weight on the bias, $w_0$ can then be learned using the same algorithm.

$$\overline{\boldsymbol{w}} = [w_0, w_1, w_2]$$
$$\overline{\boldsymbol{x}} = [x_0, x_1, x_2]$$




- More often, in other networks, the net input is determined using the following (and no bias is used for output layer):

$$I = \sum_{i=1}^{i=n} w_i x_i + \overline{\boldsymbol{b}}$$

4

4

# Multi-Layer Perceptron Network

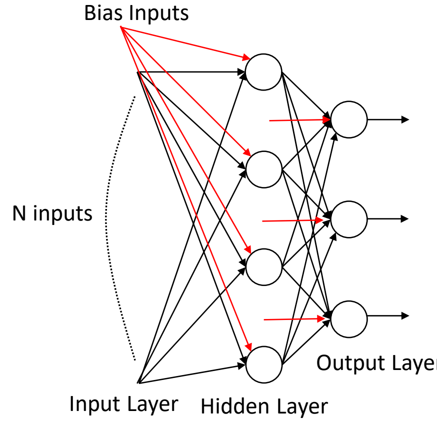
- **Example:** This could be a network that can recognize all three categories of irises from the Iris dataset.

  4 inputs, 3 outputs (Hyperparameters)
  4x4 (input to hidden) + 4x3 (hidden to output) weights + 7 bias inputs

  #Parameters = 16+12+7 = 35

- Since all units are linear TLUs this network can **only learn linear functions**.

- **We need to make each unit non-linear**.








# Backpropagation Network (Classic Version)

- **Net Input**

$$I = \sum_{i=1}^{i=n} w_i x_i + \overline{\boldsymbol{b}}$$

- **Activation Function** (Sigmoid)

$$f(I) = \frac{1}{1 + e^{I}}$$

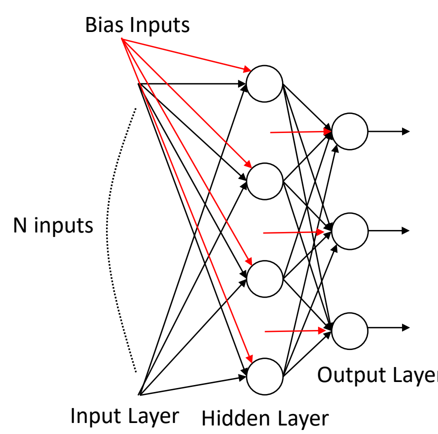
- **Learning Rule**

$$\Delta w_{ij} = \beta * E * f(I_j)$$

- **Error/Loss**

$$E_j^{output} = y_j^{desired} - y_j^{actual}$$

$$E_i^{hidden} = \frac{df(I_i^{hidden})}{dI} \sum_{j=1}^{n} (w_{ij} E_j^{output})$$



# Backpropagation Network (Classic Version)

- **Net Input**

$$I = \sum_{i=1}^{i=n} w_i x_i + \bar{b}$$

- **Activation Function** (Sigmoid)

$$f(I) = \frac{1}{1+e^{I}}$$

- **Learning Rule**

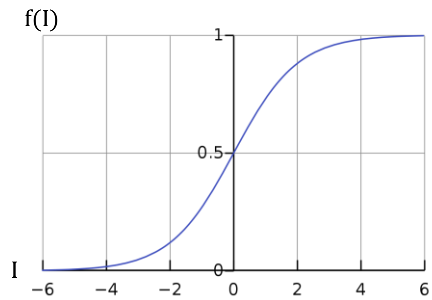
$$w_{ij}^{new} = w_{ij}^{old} + \beta * E * f(I_j)$$

where $\beta$ is the **Learning Constant** ($0 < \beta < 1$), E is the error/loss

- **Error/Loss (E)**

$$E_j^{output} = y_j^{desired} - y_j^{actual}$$

$$E_i^{hidden} = \frac{df(I_i^{hidden})}{dI} \sum_{j=1}^{n} (w_{ij} E_j^{output})$$




From: https://en.wikipedia.org/wiki/Sigmoid_function

**Sigmoid Function**
- Output is always between 0 and 1
- Its derivative (slope) is always positive
- $\frac{df(I)}{dI} = f(I)(1 - f(I))$

7

7

# Backpropagation Network (Classic Version)

- **Net Input**

$$I = \sum_{i=1}^{i=n} w_i x_i + \bar{b}$$

- **Activation Function** (Sigmoid)

$$f(I) = \frac{1}{1+e^{I}}$$

- **Learning Rule**

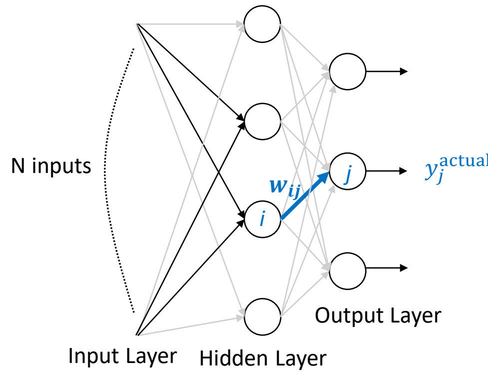
$$w_{ij}^{new} = w_{ij}^{old} + \beta * E * f(I_j)$$

where $\beta$ is the **Learning Constant** ($0 < \beta < 1$), E is the error/loss

- **Error/Loss (E)**

$$E_j^{output} = y_j^{desired} - y_j^{actual}$$

$$E_i^{hidden} = \frac{df(I_i^{hidden})}{dI} \sum_{j=1}^{n} (w_{ij} E_j^{output})$$




8

8

# Backpropagation Network (Classic Version)

- **Net Input**

$$I = \sum_{i=1}^{i=n} w_i x_i + \bar{\boldsymbol{b}}$$

- **Activation Function** (Sigmoid)

$$f(I) = \frac{1}{1 + e^{I}}$$

- **Learning Rule**

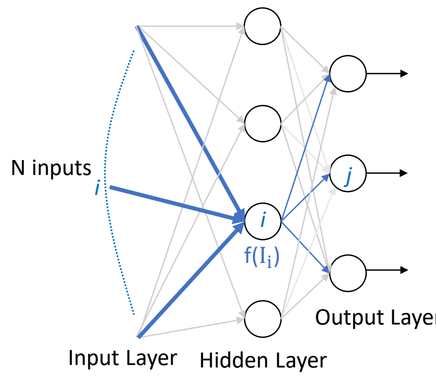
$$w_{ij}^{new} = w_{ij}^{old} + \beta * E * f(I_j)$$

where $\beta$ is the **Learning Constant** ($0 < \beta < 1$), E is the error/loss

- **Error/Loss (E)**

$$E_j^{output} = y_j^{desired} - y_j^{actual}$$

$$E_i^{hidden} = \frac{df(I_i^{hidden})}{dI} \sum_{j=1}^{n} (w_{ij} E_j^{output})$$




9

9

# Backpropagation Network (Classic Version)

- **Net Input**

$$I = \sum_{i=1}^{i=n} w_i x_i + \bar{\boldsymbol{b}}$$

- **Activation Function** (Sigmoid)

$$f(I) = \frac{1}{1 + e^{I}}$$

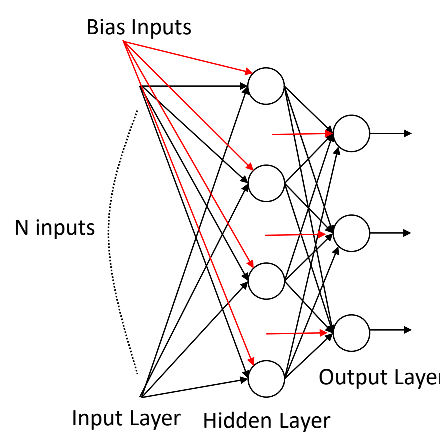
- **Learning Rule**

$$\Delta w_{ij} = \beta * E * f(I_j)$$

- **Error/Loss**

$$E_j^{output} = y_j^{desired} - y_j^{actual}$$

$$E_i^{hidden} = \frac{df(I_i^{hidden})}{dI} \sum_{j=1}^{n} (w_{ij} E_j^{output})$$




10

10

# Backpropagation (Classic) Training Algorithm

set minimum acceptable error and #epochs to train
set #epochs = 0

**repeat**

total error = o

**for** each pattern in training set **do**
**do a forward pass**
for each unit in the hidden layer do
compute net input I, and activation, f(I)
save f(I) for backpropagation
for each unit in the output layer do
compute net input I, and activation f(I)
output y = f(I)

**do backward pass**
for each unit in the **output layer** do
compute error = desired – actual output
total error = total error + error

for each unit in the **middle layer do**
compute incoming error = weighted sum of output later errors
compute final error = incoming error * $\mathrm{f(I)}$ * $(1 - \mathrm{I})$ [derivative]

for each unit in the **output layer** do
for each weight from a hidden layer to unit do
compute weight change $\beta * \mathrm{error} * f(\mathrm{I})$ and update weight

for each unit in the **middle** layer do
for each weight from an input layer unit do
compute weight change $\beta * \mathrm{final\ error} * \mathrm{f(I)}$ and update weight

#epochs = #epochs + 1

**until** total error < maximum acceptable error or #epochs reaches limit

11

11

# Example: Recognizing Handwritten Digits

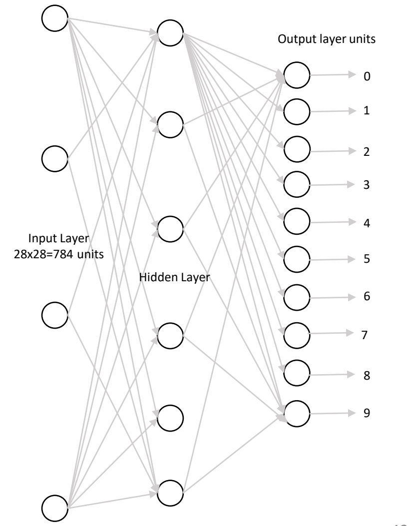
- MNIST Dataset
  70,000 images (28x28 pixels), grayscale values (in range 0 (white) to 255 (black).
- Training set: 60,000 images
- Testing set: 10,000 images
- Task: Given an image, classify it as [0,1,…,9]

12

12

# MNIST Digit Recognition: The Design

- Assume we will use a 3-leayer network: input, hidden, and output
- Input will be 28x28=784 units
- 10 outputs, one for each digit. Say the network is shown a 6, we would then expect the output of 6 to be high (closer to 1.0) compared to others.
- How many hidden units???

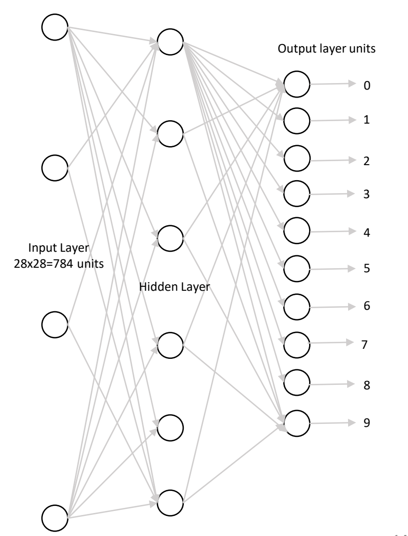



13

13

# MNIST Digit Recognition: The Design

- Assume we will use a 3-leayer network: input, hidden, and output
- Input will be 28x28=784 units
- 10 outputs, one for each digit. Say the network is shown a 6, we would then expect the output of 6 to be high (closer to 1.0) compared to others.
- How many hidden units???
  No known science to this.
  Lets say we have 512 units in hidden layer. (Why??)

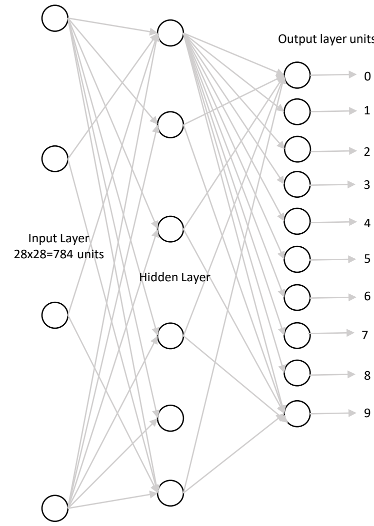



14

14

# MNIST Digit Recognition: The Design

- A 3-leayer network: input, hidden, and output layers
- Input will be 28x28=784 units
- 10 outputs, one for each digit. 512 units in hidden layer. (Why??)
- All units will have a Sigmoid activation function.
- How to determine error/loss at output? (use our formulation?)
- What should be the value of $\beta$?



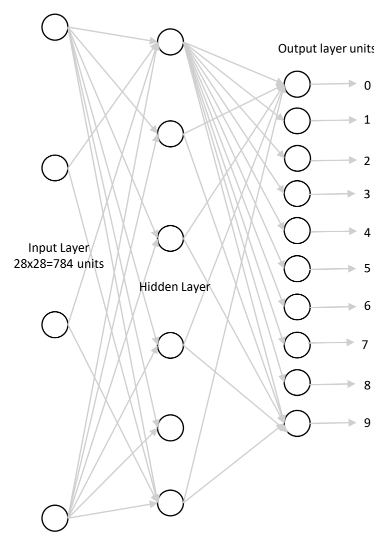

15

15


# MNIST Digit Recognition: The Design

- A 3-leayer network: input, hidden, and output layers
- Input will be 28x28=784 units
- 10 outputs, one for each digit. 512 units in hidden layer. (Why??)
- All units will have a Sigmoid activation function.
- How to determine error/loss at output? (use our formulation?)
- What should be the value of $\beta$?

- How many parameters are there?

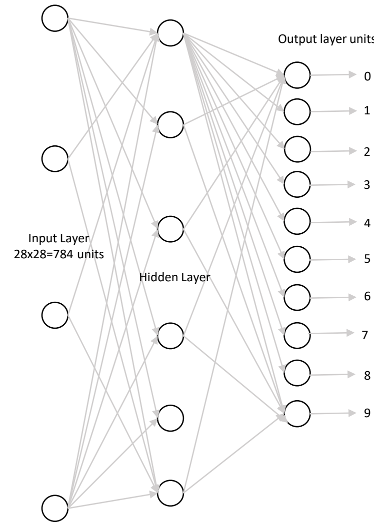



16

16

# MNIST Digit Recognition: The Design

- A 3-leayer network: input, hidden, and output layers
- Input will be 28x28=784 units
- 10 outputs, one for each digit. 512 units in hidden layer. (Why??)
- All units will have a Sigmoid activation function.
- How to determine error/loss at output? (use our formulation?)
- What should be the value of $\beta$?
- How many parameters are there?
  784x512 + 512 (bias) + 512x10 + 10 (Bias) = 407,050

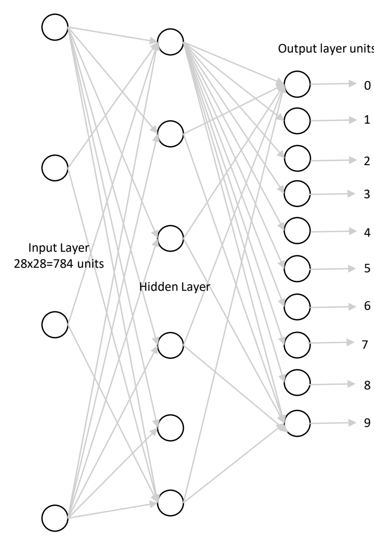






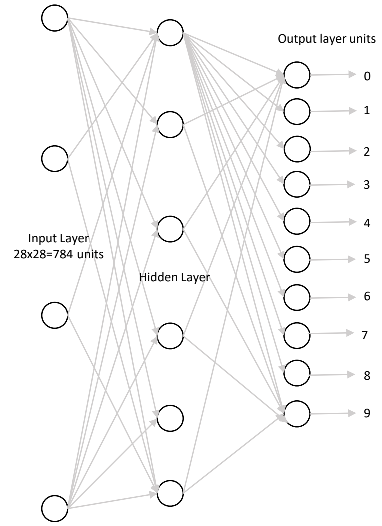

# MNIST Digit Recognition: The Design

- A 3-leayer network: input, hidden, and output layers
- Input will be 28x28=784 units
- 10 outputs, one for each digit. 512 units in hidden layer. (Why??)
- All units will have a Sigmoid activation function.
- How to determine error/loss at output? (use our formulation?)
- What should be the value of $\beta$?
- How many parameters are there?
  784x512 + 512 (bias) + 512x10 + 10 (Bias) = 407,050
- What are the hyperparameters?



# MNIST Digit Recognition: The Design

- A 3-leayer network: input, hidden, and output layers
- Input will be 28x28=784 units
- 10 outputs, one for each digit. 512 units in hidden layer. (Why??)
- All units will have a Sigmoid activation function.
- How to determine error/loss at output? (use our formulation?)
- What should be the value of $\beta$?
- How many parameters are there?
  784x512 + 512 (bias) + 512x10 + 10 (Bias) = 407,050
- What are the hyperparameters?
  # Layers, # Units in each layer, Activation Function, $\beta$, # epochs, Minimum acceptable error, etc.

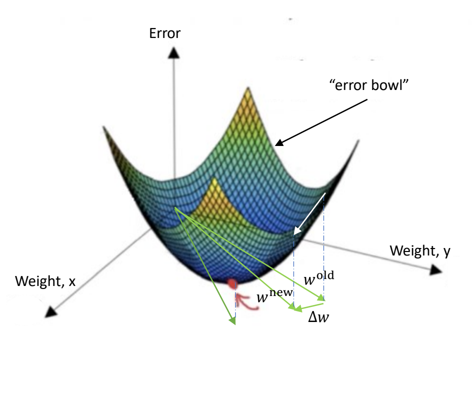



19

19

# Backpropagation: Gradient Descent

- Learning in a neural network using Backpropagation is essentially a Gradient Descent process.
- Each change in the weights is an attempt to reduce error and descend into the lowest possible position in the "error bowl" (as shown in a 2-D weight vector case)
- In higher dimensional weight vectors (typical ML situations), the error surface can be quite complex.



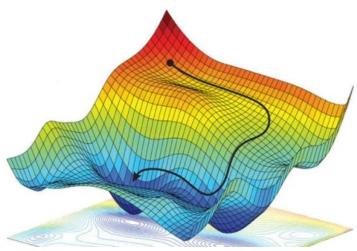

From: https://builtin.com/data-science/gradient-descent

20

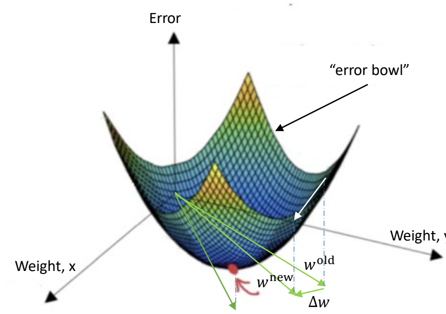
20

# Backpropagation: Gradient Descent

- In higher dimensional weight vectors (typical ML situations), the error surface can be quite complex.



From: https://poissonisfish.com/2023/04/11/gradient-descent/



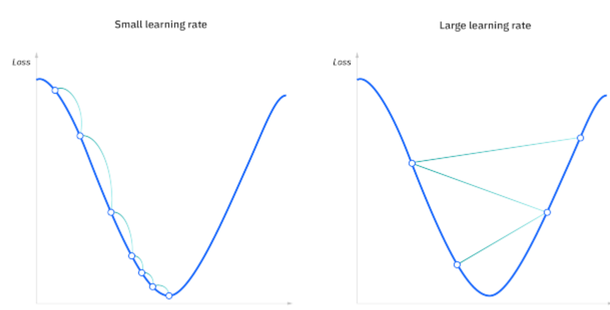

From: https://builtin.com/data-science/gradient-descent





# Backpropagation: Learning rate

- Learning Rate, $\beta$ (0 < 1)
- The value of $\beta$ determines how fast or slow the gradient descent takes place.
- Typically, one starts with a higher value (say 0.5 or 0.6) and then decrease it as the learning/epochs progresses. This is called a **Learning Rate Schedule**.




From: https://www.ibm.com/topics/gradient-descent

# Vocabulary

- In what order do we present the patterns? As they are in the training set? Or, randomly?
  If patterns are chosen at random (without replacement), we call it **Stochastic Gradient Descent (SGD)**

  Choices:

  Do a backpropagation pass after every input. **True SGD**

  Do the backward pass after all the inputs have been seen, and errors recorded. **Full batch SGD**

  Do a small batch of data and the do a backward pass. **Mini Batch SGD** (each batch is a power of 2)
- Is there a better way to assess error/loss? **Loss Functions**
- Are there any other weight update mechanisms? **Optimizers** (also manage Learning rate schedules)

  Most of the period from 1986 until now has been spent on studying these questions.

Backpropagation
Bias
Epochs
Forward Pass
Full Batch SGD
Gradient Descent
Hyperparameters
Labelled Dataset
Learning Rule
Loss Function
Mini Batch SGD
Model
Optimizer
Parameters
SGD
True SGD

23

23

# MNIST Digit Recognition: More Decisions

- In what order do we present the patterns? As they are in the training set? Or, randomly?
- Do we do a backpropagation pass after every input?

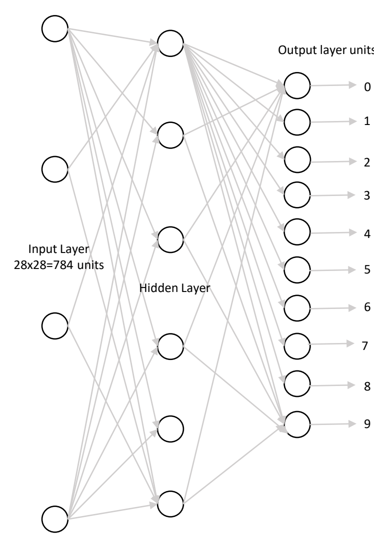
  Choices:

  Do a backpropagation pass after every input.

  Do the backward pass after all the inputs have been seen, and errors recorded.

  Do a small batch of data and the do a backward pass.
- Is there a better way to assess error/loss?
- Are there any other weight update mechanisms?

  Most of the period from 1986 until now has been spent on studying these questions.




24

24

# Advances in NN

- **Better hardware**
  Laptops became 5000 times faster between 1990 and 2010
  Use of GPUs for faster processing (NVIDIA, AMD). Took off in 2011
  Google's Tensor Processing Units (TPUs), 2016
- **Wide availability of datasets and benchmarks**
  MNIST, ImageNet, etc.
- **Better Algorithms**
  Better activation functions
  Better weight initialization schemes
  Better optimization schemes (RMSprop, Adam)
- **Widely available toolsets for creating and training NNs**
  Theano, TensorFlow, Scikit Learn, **Keras**

25

25

# Backpropagation Network (Updated)

- **Net Input**

$$\mathrm{I} = \sum_{i=1}^{i=n} w_i x_i + \overline{\boldsymbol{b}}$$

- **Activation Functions**

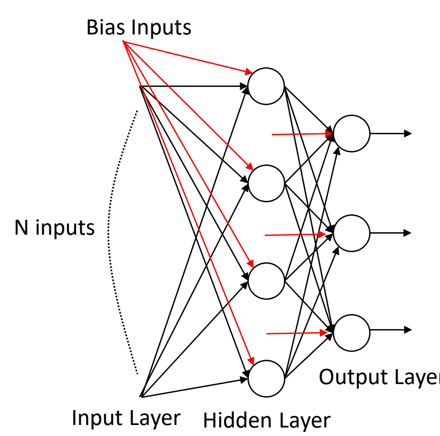
  Sigmoid, Relu, Softmax, tanh, exponential, etc.

- **Loss Functions**

  Binary Cross Entropy, Categorical Cross Entropy, Poisson, KL Divergence, Mean Squared Error, Mean Absolute Error, Cosine Similarity, etc.

- **Optimizer (Learning Rule)**

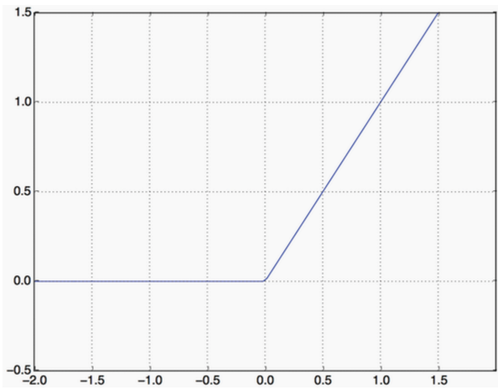
  SGD, RMSprop, Adam, Adadelta, Adamax, Namad, etc.

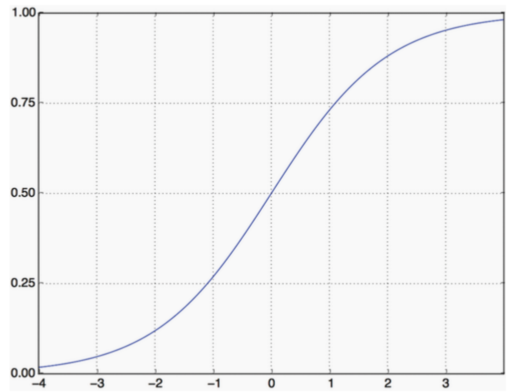



26

26

# Popular Activation Functions




**Relu** – Rectified Linear Unit: $f(I) = \max(0, I)$

**Sigmoid**: $\text{f(I)} = \frac{1}{1+e^{I}}$



27

# Vocabulary

- In what order do we present the patterns? As they are in the training set? Or, randomly?
  If patterns are chosen at random (without replacement), we call it **Stochastic Gradient Descent (SGD)**

  **Choices:**

  Do a backpropagation pass after every input. **True SGD**

  Do the backward pass after all the inputs have been seen, and errors recorded. **Full batch SGD**

  Do a small batch of data and the do a backward pass. **Mini Batch SGD** (each batch is a power of 2)
- Is there a better way to assess error/loss? **Loss Functions**
- Are there any other weight update mechanisms? **Optimizers** (also manage Learning rate schedules)

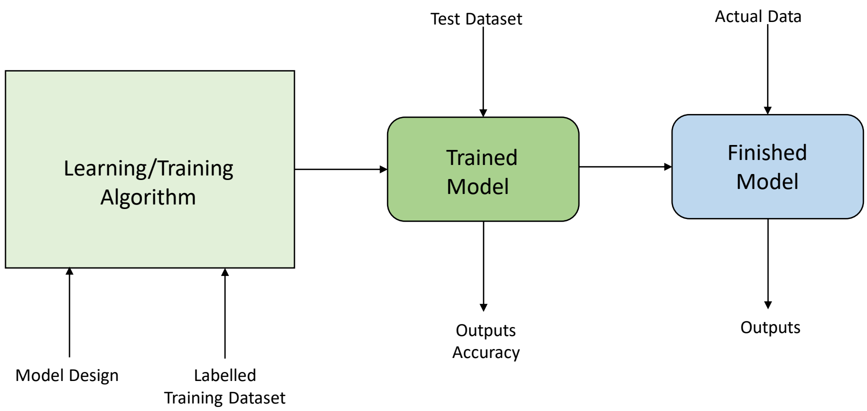
  Most of the period from 1986 until now has been spent on studying these questions.

Adam
Backpropagation
Bias
Binary Cross Entropy
Categorical Cross Entropy
Epochs
Exponential
Forward Pass
Full Batch SGD
Gradient Descent
Hyperparameters
Labelled Dataset
Learning Rule
Loss Function
Mean Absolute Error
Mean Squared Error
Mini Batch SGD
Model
Optimizer
Parameters
Relu
RMSProp
SGD
Sigmoid
Softmax
Tanh
True SGD



28

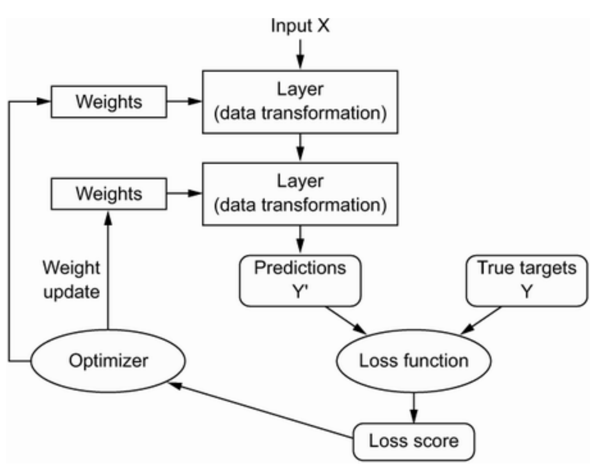
# The Learning Paradigm




29

29

# How NN Learning Works

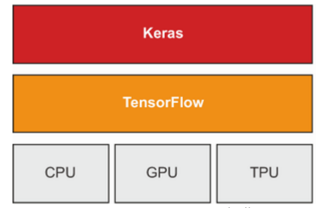



From: Chollet, 2021.

30

30

# Introducing Keras




From: Chollet, 2021.

- Deep Learning API for Python (2016-17)
- Built on top of TensorFlow (2015)
- Can run on a typical CPU, or can be accelerated with specialized hardware, if available. GPUs (Graphics Processing Units), TPUs (Tensor Processing Units)
- Makes Neural Network design, implementation, and exploration akin to building with LEGOs!



31

# Typical Keras Workflow

- **Acquire, prepare, and load the dataset**
  Keras has several predefined datasets available: MNIST Digits, CIFAR10, CIFAR100, IMDB Reviews for sentiment classification, Reuters Newswire classification, Fashion MNIST, Boston Housing price regression (see https://keras.io/api/datasets/)
- **Design and Build the Model**
  How many layers to use? How many units in each later? What activation function to use? (see https://keras.io/api/layers/activations/)
- **Compile the Model**
  Decide which **optimizer** to use, **loss function**, **accuracy metric**
  https://keras.io/api/optimizers/, https://keras.io/api/losses/, https://keras.io/api/metrics/
- **Train/Fit the Model**
  Provide the training data and its labels, number of epochs to train, batch size
- **Test/Validate the Model**
  Use the test data to test how well the trained model performs



32

# Over to Colab…

- See Lab for Recognizing Handwritten Digits

33

33

# References


- M. Caudill and C. Butler: Understanding Neural Networks, Volume 1, MIT Press, 1993.
- F. Chollet: *Deep Learning with Python, Second Edition*, Manning2021.
- A Geron: *Hands-on Machine Learning with SciKit-Learn, Keras and TensorFlow*, Oreilly, 2019.
- M. Mitchell: *Artificial Intelligence: A Guide For Thinking Humans*, Farrar, Strouss, Giroux, 2019.
- Rumelhart, McClelland, and the PDP Research Group: *Parallel Distributed Processing*, Volumes 1 & 2. MIT Press, 1986.


34

34