CMSC 373 Artificial Intelligence Fall 2023
08-Logic in PROLOG

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Knowledge Engineering in FOPC
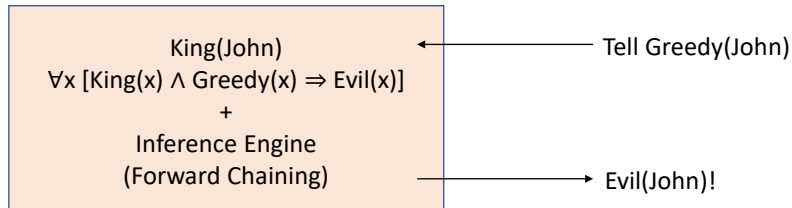
• Identify the task
• Assemble relevant knowledge
• Decide on a vocabulary of predicates, functions, and constants
• Encode general knowledge about the domain
• Encode a description of the specific problem instance
• Pose queries to the inference procedure and get answers
• Debug the knowledge base
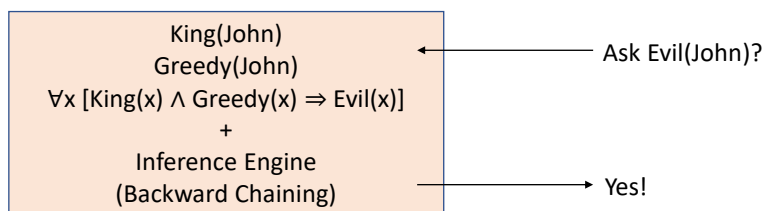
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Forward Chaining Inference

• Tell-Ask Systems

King(John)
$\forall x$ [King(x) $\land$ Greedy(x) $\Rightarrow$ Evil(x)]
+
Inference Engine
(Forward Chaining)

Tell Greedy(John)

Evil(John)!

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Backward Chaining Inference

• Tell-Ask Systems

King(John)
Greedy(John)
$\forall x$ [King(x) $\land$ Greedy(x) $\Rightarrow$ Evil(x)]
+
Inference Engine
(Backward Chaining)

Ask Evil(John)?

Yes!

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Backward Chaining Inference

- Tell-Ask Systems

King(John)
Greedy(John)
$\forall x$ [King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x)]
+
Inference Engine
(Backward Chaining)

Ask Evil(John)?

Yes!
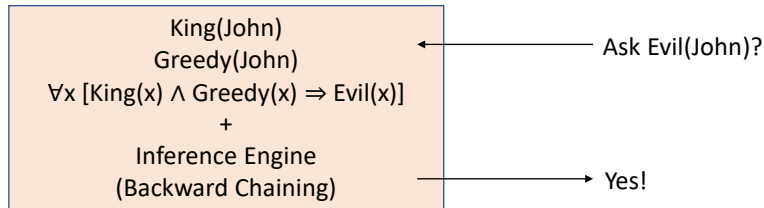
- **Requires wffs to be in Definite Clause form!**

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Definite Clauses in FOPC

- Disjunction of literals of which exactly one is positive

  i.e. $\neg\omega_1 \vee \neg\omega_2 \vee \dots \vee \neg\omega_{n-1} \vee \omega_n \quad \equiv \quad \omega_1 \wedge \omega_2 \wedge \dots \wedge \omega_{n-1} \Rightarrow \omega_n$

- A definite clause is either a fact: American(JoeBiden)

- Or, an implication whose antecedent is a conjunction of positive literals

- Can have variables, but all must be universally quantified ($\forall$)

  $\forall x$ [King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x)]

- We can then rewrite the wff without the quantifier: King(x) $\wedge$ Greedy(x) $\Rightarrow$ Evil(x)

  $\neg$ King(x) $\vee \neg$ Greedy(x) $\vee$ Evil(x)

- Most Knowledge bases can be converted to this form.

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Logic Programming (Prolog)

- A program is a set of definite clauses (facts, $\omega_1 \wedge \omega_2 \wedge ... \wedge \omega_{n-1} \Rightarrow \omega_n$)

- The Syntax is different from FOPC

    - Variables   : written in uppercase
    - Constants   : written in lowercase
    - Relations   : written beginning with lowercase letter
    - Conjunction ($\wedge$)   : comma (,)
    - Implications   : written as <u>Prolog rules</u>

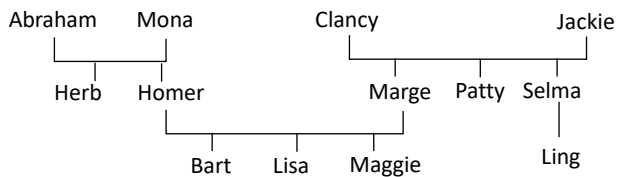 $\omega_1 \wedge \omega_2 \Rightarrow \omega_n$: ω_n :- ω_1, ω_2.

    $\forall x [Missile(x) \Rightarrow Weapon(x)]$:   weapon(X) :- missile(X)

    $\forall x [King(x) \wedge Greedy(x) \Rightarrow Evil(x)]$:   evil(X) :- king(X), greedy(X).
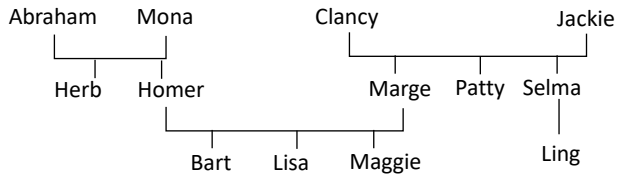
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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

Herb   Homer   Marge Patty Selma

Bart   Lisa   Maggie   Ling

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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

      Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(homer, marge).
true
```
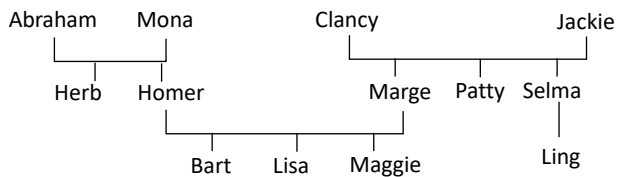
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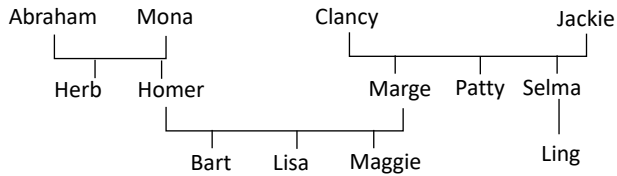# A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

      Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(homer, marge).
true

?- married(homer, jackie).
false
```

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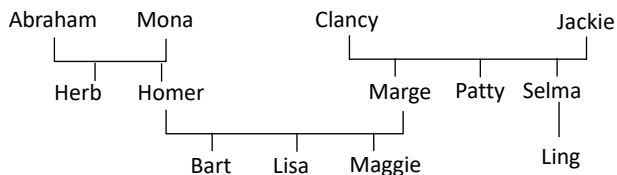# A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(homer, marge).
true

?- married(homer, jackie).
false

?- married(deepak, jackie).
false
```
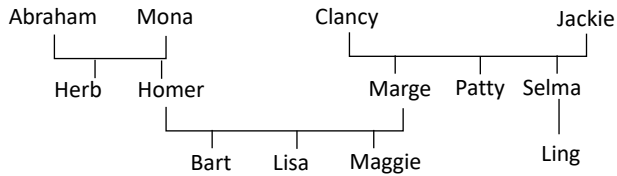
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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, homer).
X = marge
```
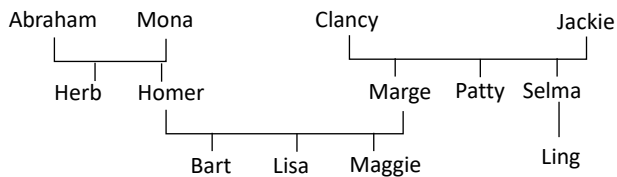
12

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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, homer).
X = marge

?- married(homer, X).
X = marge
```
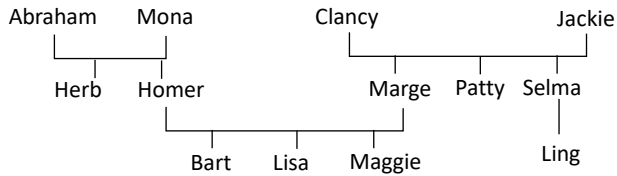
13

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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, Y).
X = Abraham
Y = mona
```
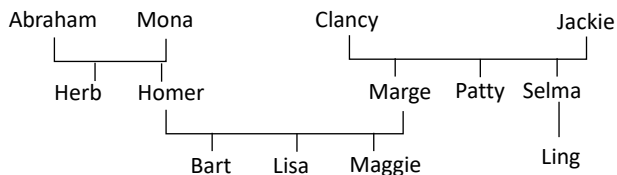
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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

       Bart   Lisa   Maggie   Ling

% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, Y).
X = abraham
Y = mona;
X = mona
Y = abraham

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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie

   Herb   Homer   Marge   Patty   Selma

       Bart   Lisa   Maggie   Ling

% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, Y).
X = abraham
Y = mona;
X = mona
Y = Abraham
X = clancy,
Y = jackie ;
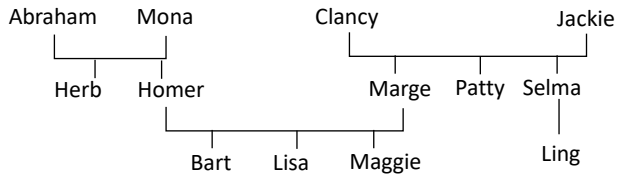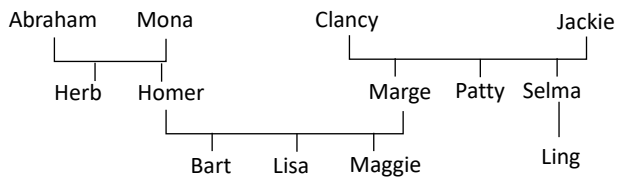X = jackie,
Y = clancy ;
X = homer,
Y = marge ;
X = marge,
Y = homer

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A Simple Prolog Program

Abraham — Mona   Clancy   Jackie

Herb   Homer   Marge   Patty   Selma

Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

?- married(X, Y).
X = abraham
Y = mona;
X = mona
Y = Abraham
X = clancy,
Y = jackie .
```

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A Simple Prolog Program

Abraham — Mona   Clancy   Jackie

Herb   Homer   Marge   Patty   Selma

Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

% males and females
% female(x) : x is female
% male(x)   : x is male
male(abraham).
male(clancy).
male(herb).
male(homer).
male(bart).
female(mona).
female(jackie).
female(marge).
female(patty).
female(selma).
female(ling).
```

```
% parents
% parent(x, y) : x is a parent of y
parent(abraham, herb).
parent(mona, herb).
parent(abraham, homer).
parent(mona, homer).
parent(clancy, marge).
parent(jackie, marge).
parent(clancy, patty).
parent(jackie, patty).
parent(clancy, selma).
parent(jackie, selma).
parent(homer, bart).
parent(marge, bart).
parent(homer, lisa).
parent(marge, lisa).
parent(homer, maggie).
parent(marge, maggie).
parent(selma, ling).
```
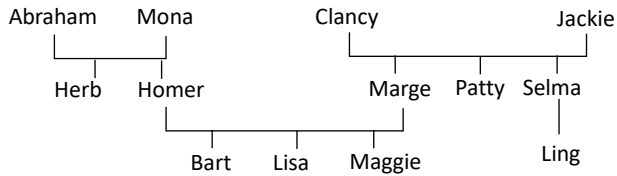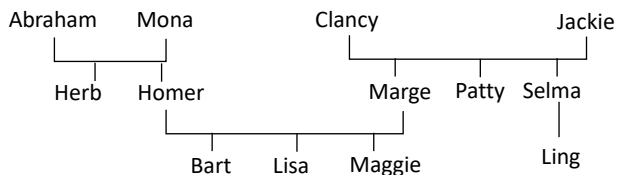
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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie
   |_____| |_____|
Herb   Homer   Marge Patty Selma
        |_____| |
     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

% males and females
% female(x) : x is female
% male(x)   : x is male
male(abraham).
male(clancy).
male(herb).
male(homer).
male(bart).
female(mona).
female(jackie).
female(marge).
female(patty).
female(selma).
female(ling).
```

```
% parents
% parent(x, y) : x is a parent of y
parent(abraham, herb).
parent(mona, herb).
parent(abraham, homer).
parent(mona, homer).
parent(clancy, marge).
parent(jackie, marge).
parent(clancy, patty).
parent(jackie, patty).
parent(clancy, selma).
parent(jackie, selma).
parent(homer, bart).
parent(marge, bart).
parent(homer, lisa).
parent(marge, lisa).
parent(homer, maggie).
parent(marge, maggie).
parent(selma, ling).

%who is Marge the mother of?
?- parent(marge, X), female(marge).
X = bart ;
X = lisa ;
X = maggie.
```

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A Simple Prolog Program

Abraham   Mona   Clancy   Jackie
   |_____| |_____|
Herb   Homer   Marge Patty Selma
        |_____| |
     Bart   Lisa   Maggie   Ling

```
% married(x, y) : x is married to y
married(abraham, mona).
married(mona, abraham).
married(clancy, jackie).
married(jackie, clancy).
married(homer, marge).
married(marge, homer).

% males and females
% female(x) : x is female
% male(x)   : x is male
male(abraham).
male(clancy).
male(herb).
male(homer).
male(bart).
female(mona).
female(jackie).
female(marge).
female(patty).
female(selma).
female(ling).
```

```
% parents
% parent(x, y) : x is a parent of y
parent(abraham, herb).
parent(mona, herb).
parent(abraham, homer).
parent(mona, homer).
parent(clancy, marge).
parent(jackie, marge).
parent(clancy, patty).
parent(jackie, patty).
parent(clancy, selma).
parent(jackie, selma).
parent(homer, bart).
parent(marge, bart).
parent(homer, lisa).
parent(marge, lisa).
parent(homer, maggie).
parent(marge, maggie).
parent(selma, ling).

% Define mother(x, y) rule
?- mother(X, Y) :-  parent(X, Y),
                    female(X).
```

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z ∀w [father(z, x) ∧ father(z, y) ∧ mother(w, x) ∧ mother(w, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancestor
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

grandparent(X, Z) :- parent(X, Y), parent(Y, Z).

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancestor
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z ∀w [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancester
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

```
grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
sibling(X, Y) :- parent(Z, X), parent(Z, Y), not(X=Y).
```

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancester
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

```
grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
sibling(X, Y) :- parent(Z, X), parent(Z, Y), not(X=Y).
auntoruncle(X, W) :- sibling(X, Y), parent(Y, W).
auntoruncle(X, Z) :- married(X, Y), sibling(Y, W), parent(W, Z).
aunt(X, W) :- female(X), auntoruncle(X, W).
uncle(X, W) :- male(X), auntoruncle(X, W).
```

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancestor
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

```
grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
sibling(X, Y) :- parent(Z, X), parent(Z, Y), not(X=Y).
auntoruncle(X, W) :- sibling(X, Y), parent(Y, W).
auntoruncle(X, Z) :- married(X, Y), sibling(Y, W), parent(W, Z).
```

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancestor
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

```
grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
sibling(X, Y) :- parent(Z, X), parent(Z, Y), not(X=Y).
auntoruncle(X, W) :- sibling(X, Y), parent(Y, W).
auntoruncle(X, Z) :- married(X, Y), sibling(Y, W), parent(W, Z).
aunt(X, W) :- female(X), auntoruncle(X, W).
uncle(X, W) :- male(X), auntoruncle(X, W).
```

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From FOPC To Prolog

- **Knowledge Base – General statements about family relationships**

GrandParent(x, y) : x is a grand parent of y
∀x ∀y ∀z [Parent(x, y) ∧ Parent(y, z) ⇒ GrandParent(x, z)]

Sibling(x, y): x is a sibling of y
∀x ∀y ∀z [parent(z, x) ∧ parent(z, y) ∧ (x≠y) ⇒ Sibling(x, y)]

AuntOrUncle(x, y): x is an aunt or uncle of y
∀x ∀y ∀w [Sibling(x, y) ∧ Parent(y, w) ⇒ AuntOrUncle(x, w)]

∀w ∀x ∀y ∀w [Married(x, y) ∧ Sibling(y, w) ∧ Parent(w, z) ⇒ AuntOrUncle(x, z)]

Aunt(x, y): x is an aunt of y
∀x ∀y [Female(x) ∧ AuntOrUncle(x, y) ⇒ Aunt(x, y)]

Ancester(x, y): x is an ancestor
∀x ∀y [Parent(x, y) ⇒ Ancester(x, y)]

∀x ∀y ∀z [Parent(x, z) ∧ Ancestor(z, y) ⇒ Ancester(x, y)]

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grandparent(X, Z) :- parent(X, Y), parent(Y, Z).
sibling(X, Y) :- parent(Z, X), parent(Z, Y), not(X=Y).
auntoruncle(X, W) :- sibling(X, Y), parent(Y, W).
auntoruncle(X, Z) :- married(X, Y), sibling(Y, W), parent(W, Z).
aunt(X, W) :- female(X), auntoruncle(X, W).
uncle(X, W) :- male(X), auntoruncle(X, W).
ancester(X, Y) :- parent(X, Y).
ancester(X, Y) :- parent(X, Z), ancester(Z, Y).
```

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