Project 3, Question 2

The `def elapseTime(self, gameState)` function says:

> In order to obtain the distribution over new positions for the ghost, given its previous position (oldPos) as well as Pacman's current position, use this line of code:
>
> ```
> newPosDist =
> self.getPositionDistribution(self.setGhostPosition(gameState, oldPos))
> ```

How are we supposed to find `oldPos`?

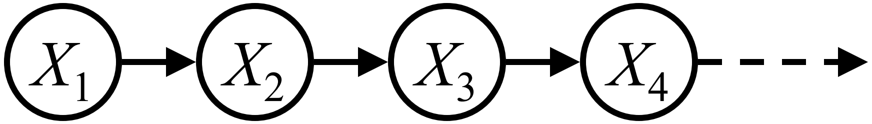
Reasoning over Time

- Often, we want to reason about a sequence of observations
  - Speech recognition
  - Robot localization
  - User attention
  - Medical monitoring

- Need to introduce time into our models
- Basic approach: hidden Markov models (HMMs)
- More general: dynamic Bayes' nets

Markov Models

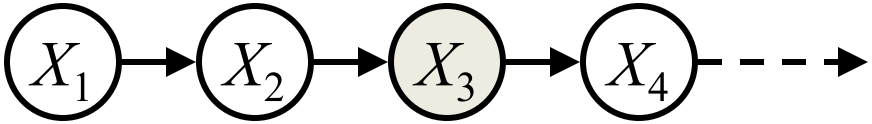
- A Markov model is a chain-structured BN
  - Each node is identically distributed (stationarity)
  - Value of X at a given time is called the state
  - As a BN:



$P(X_1)$ $\qquad$ $P(X|X_{-1})$

  - Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial probs)

Conditional Independence

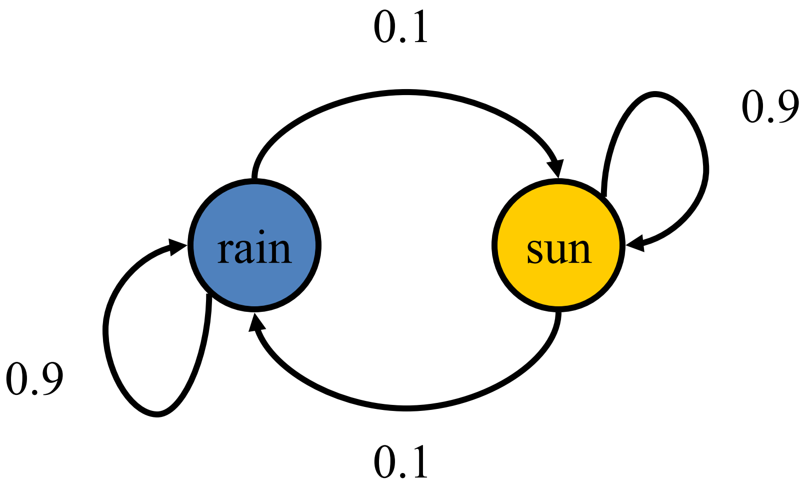


- Basic conditional independence:
  - Past and future independent of the present
  - Each time step only depends on the previous
  - This is called the Markov property

- Note that the chain is just a (growing) BN
  - We can always use generic BN reasoning on it if we truncate the chain at a fixed length

Example: Markov Chain

- Weather:
  - States: X = {rain, sun}
  - Transitions:



  - Initial distribution: 1.0 sun
  - What's the probability distribution after one step?

$$P(X_2 = \text{sun}) = P(X_2 = \text{sun} | X_1 = \text{sun}) P(X_1 = \text{sun}) + P(X_2 = \text{sun} | X_1 = \text{rain}) P(X_1 = \text{rain})$$

$$0.9 \cdot 1.0 + 0.1 \cdot 0.0 = 0.9$$

Mini-Forward Algorithm

- Question: probability of being in state x at time t?
- Slow answer:
  - Enumerate all sequences of length t which end in s
  - Add up their probabilities

$$P(X_t = sun) = \sum_{x_1 \ldots x_{t-1}} P(x_1, \ldots x_{t-1}, sun)$$

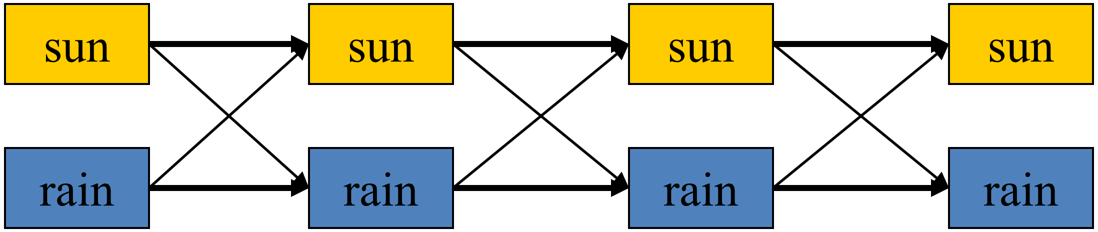
$$P(X_1 = sun)P(X_2 = sun|X_1 = sun)P(X_3 = sun|X_2 = sun)P(X_4 = sun|X_3 = sun)$$

$$P(X_1 = sun)P(X_2 = rain|X_1 = sun)P(X_3 = sun|X_2 = rain)P(X_4 = sun|X_3 = sun)$$

$$\vdots$$

Mini-Forward Algorithm

- Question: What’s P(X) on some day t?
  - An instance of variable elimination!



sun sun sun sun

rain rain rain rain

$$P(x_t) = \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1})$$

$$P(x_1) = \text{known}$$

Forward simulation

Example

- From initial observation of sun

$$\begin{array}{ccccc}
\left\langle \begin{matrix} 1.0 \\ 0.0 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.9 \\ 0.1 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.82 \\ 0.18 \end{matrix} \right\rangle & \Rightarrow & \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle \\
\mathrm{P}(X_1) & \mathrm{P}(X_2) & \mathrm{P}(X_3) & & \mathrm{P}(X_\infty)
\end{array}$$

- From initial observation of rain

$$\begin{array}{ccccc}
\left\langle \begin{matrix} 0.0 \\ 1.0 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.1 \\ 0.9 \end{matrix} \right\rangle & \left\langle \begin{matrix} 0.18 \\ 0.82 \end{matrix} \right\rangle & \Rightarrow & \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle \\
\mathrm{P}(X_1) & \mathrm{P}(X_2) & \mathrm{P}(X_3) & & \mathrm{P}(X_\infty)
\end{array}$$

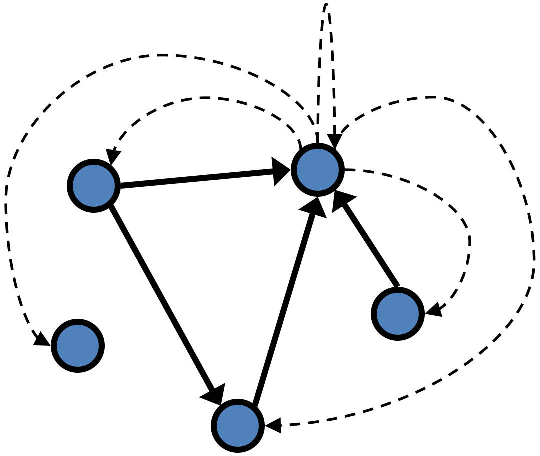
Stationary Distributions

- If we simulate the chain long enough:
  - What happens?
  - Uncertainty accumulates
  - Eventually, we have no idea what the state is!

- Stationary distributions:
  - For most chains, the distribution we end up in is independent of the initial distribution
  - Called the stationary distribution of the chain
  - Usually, can only predict a short time out

Web Link Analysis

- PageRank over a web graph
  - Each web page is a state
  - Initial distribution: uniform over pages
  - Transitions:
    - With prob. c, uniform jump to a random page (dotted lines, not all shown)
    - With prob. 1-c, follow a random outlink (solid lines)

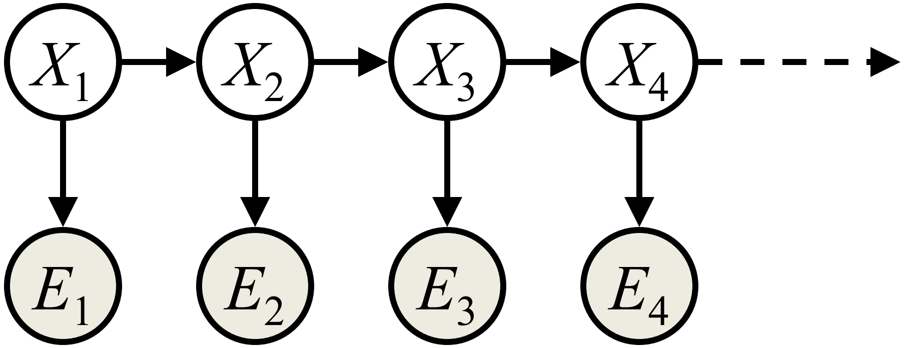


- Stationary distribution
  - Will spend more time on highly reachable pages
  - E.g., many ways to get to the Acrobat Reader download page
  - Somewhat robust to link spam
  - Google 1.0 returned the set of pages containing all your keywords in decreasing rank, now all search engines use link analysis along with many other factors (rank actually getting less important over time)

Hidden Markov Models

- Markov chains not so useful for most agents
  - Eventually you don't know anything anymore
  - Need observations to update your beliefs

- Hidden Markov models (HMMs)
  - Underlying Markov chain over states S
  - You observe outputs (effects) at each time step
  - As a Bayes' net:

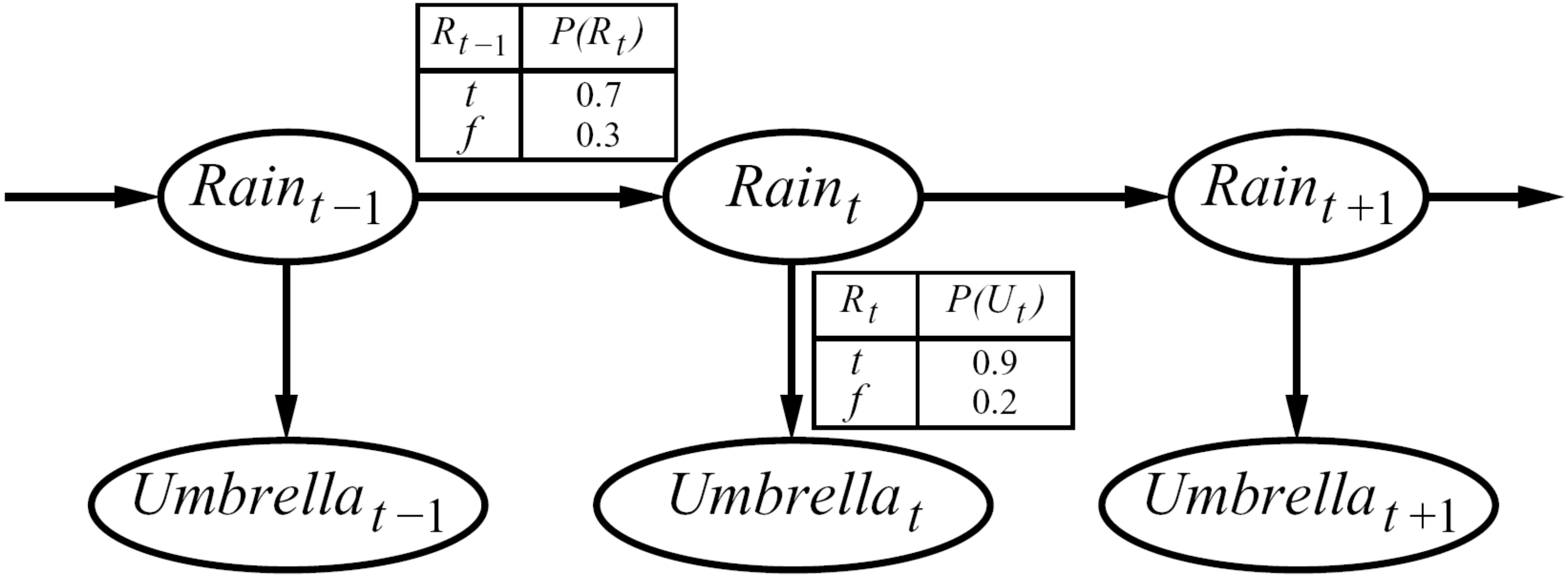


Example

- Security guard works and lives underground
- Friend comes in each day and may have umbrella

- Is the sun shining, or is it raining?

Robyn Fenty Example



R_{t-1}	$P(R_t)$
t	0.7
f	0.3

R_t	$P(U_t)$
t	0.9
f	0.2

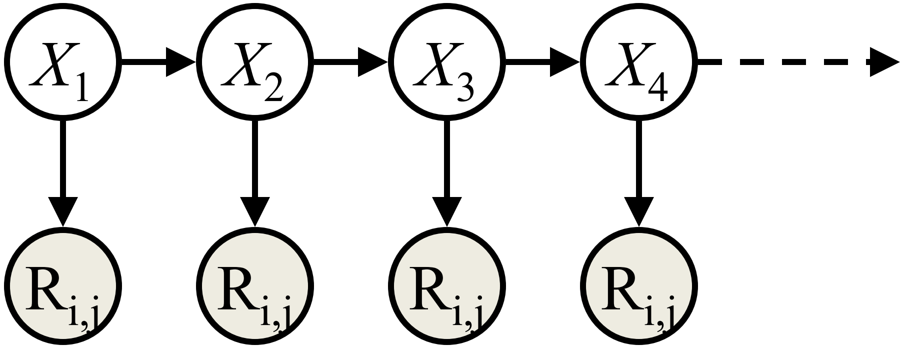
- An HMM is defined by:
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X|X_{-1})$
  - Emissions: $P(E|X)$

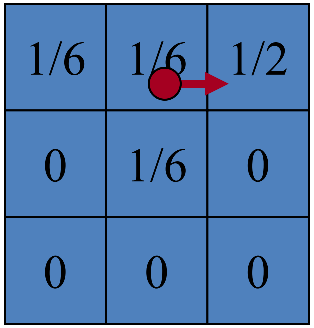
Ghostbusters HMM

- $P(X_1)$ = uniform
- $P(X|X')$ = usually move clockwise, but sometimes move in a random direction or stay in place
- $P(R_{ij}|X)$ = same sensor model as before: red means close, green means far away.

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

$P(X_1)$





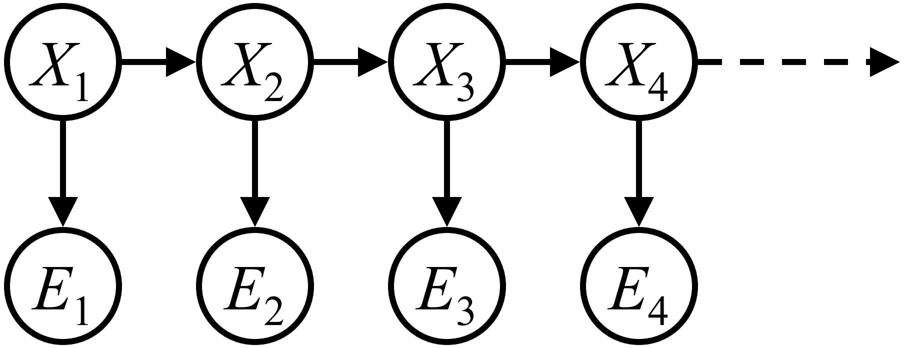
1/6	1/6	1/2
0	1/6	0
0	0	0

$P(X|X'=<1,2>)$

[Demo]

Conditional Independence

- HMMs have two important independence properties:
  - Markov hidden process, future depends on past via the present
  - Current observation independent of all else given current state



- Quiz: does this mean that observations are independent given no evidence?
  - [No, correlated by the hidden state]

Real HMM Examples

- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)

- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options (dozens per word)

- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

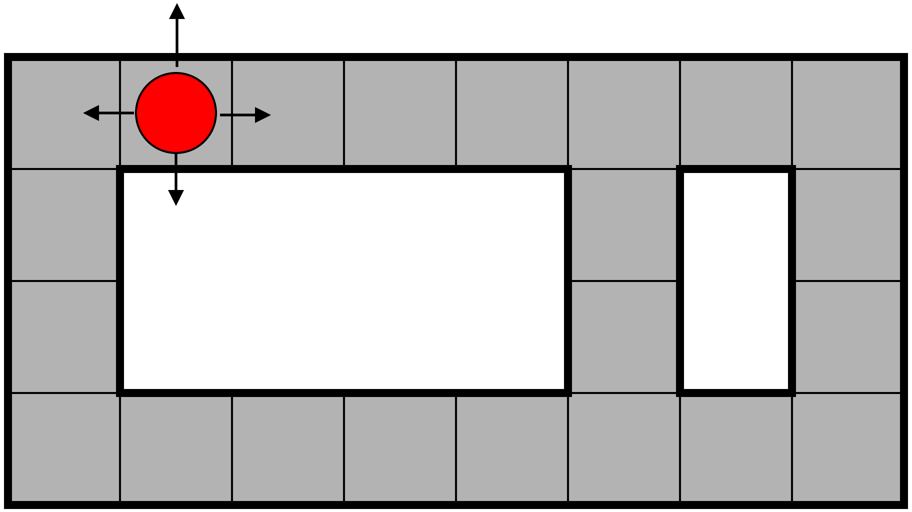
Filtering / Monitoring

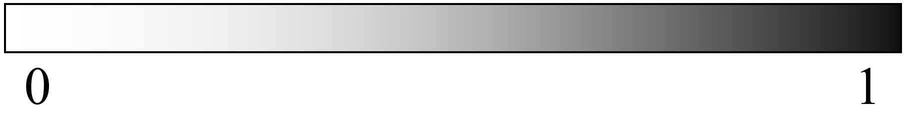
- Filtering, or monitoring, is the task of tracking the distribution B(X) (the belief state) over time
- We start with B(X) in an initial setting, usually uniform
- As time passes, or we get observations, we update B(X)
- The Kalman filter was invented in the 60's and first implemented as a method of trajectory estimation for the Apollo program

[Demo]

Example: Robot Localization

Example from Michael Pfeiffer



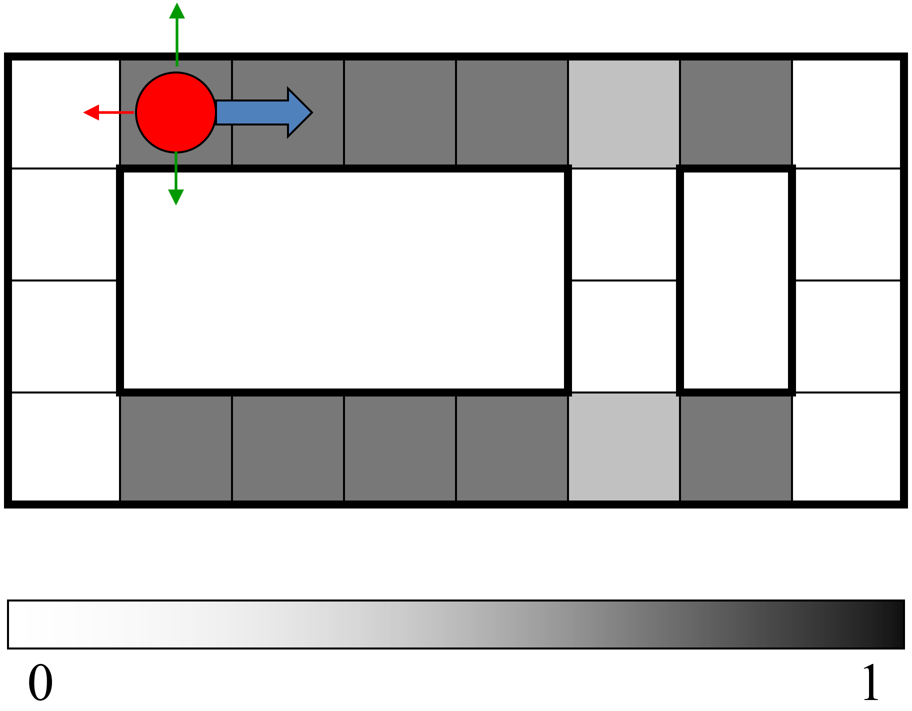


t=0

Sensor model: never more than 1 mistake

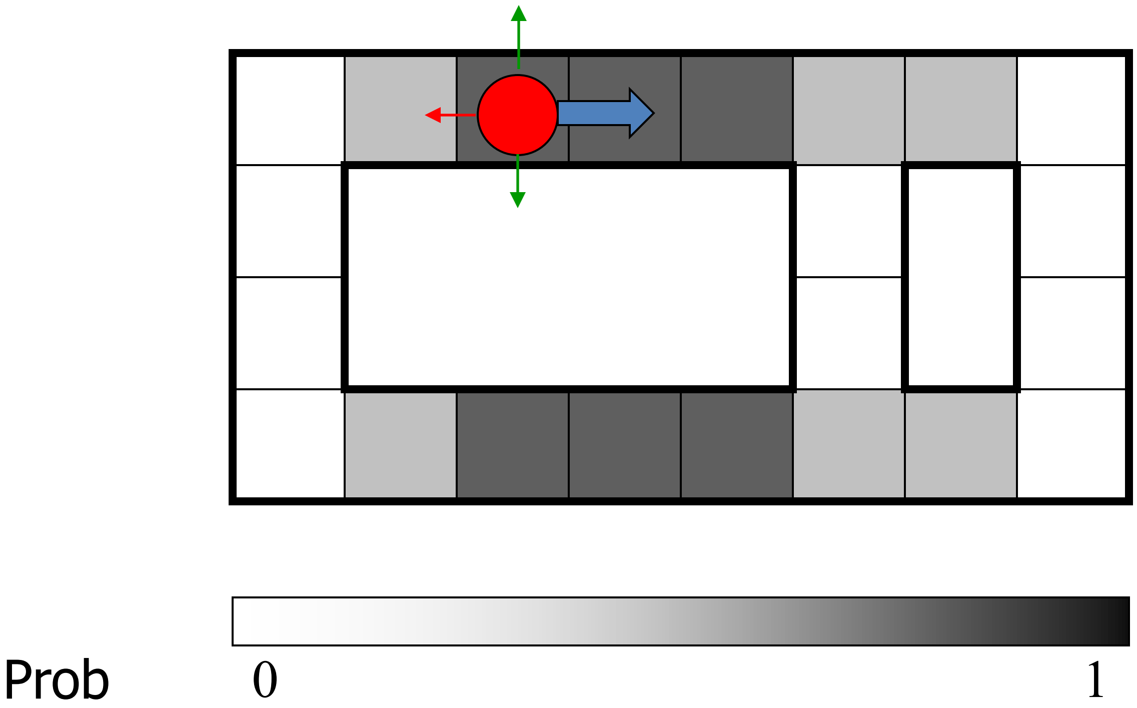
Motion model: may not execute action with small prob.

Example: Robot Localization



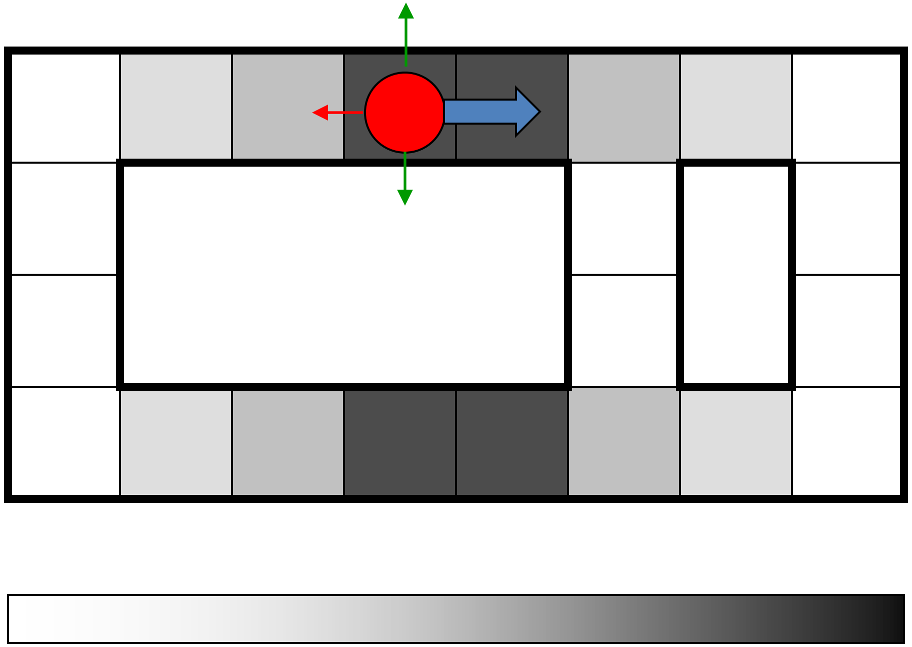
t=1

Example: Robot Localization



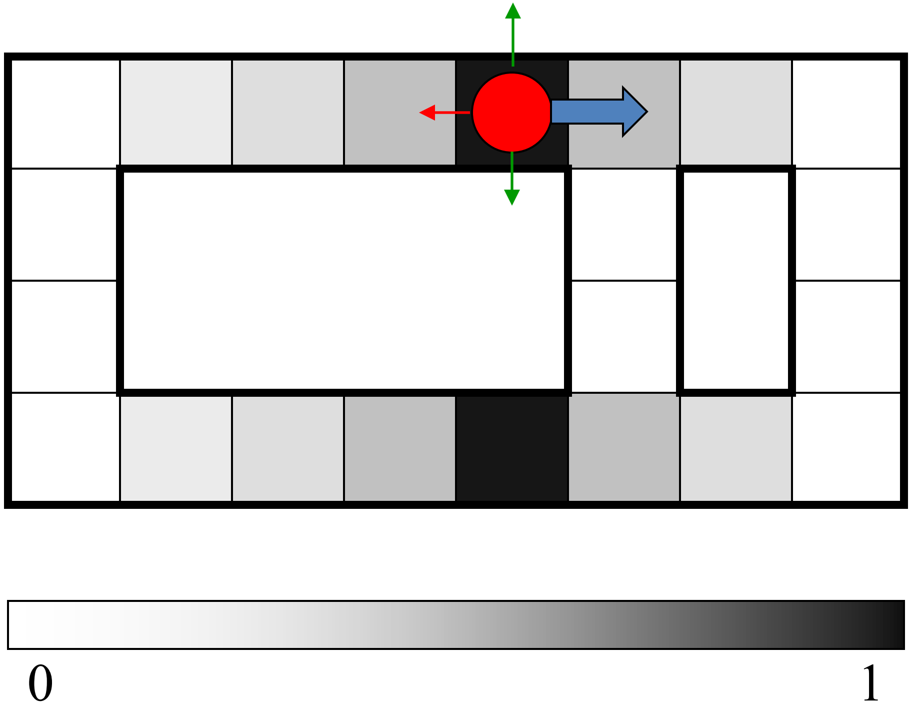
t=2

Example: Robot Localization



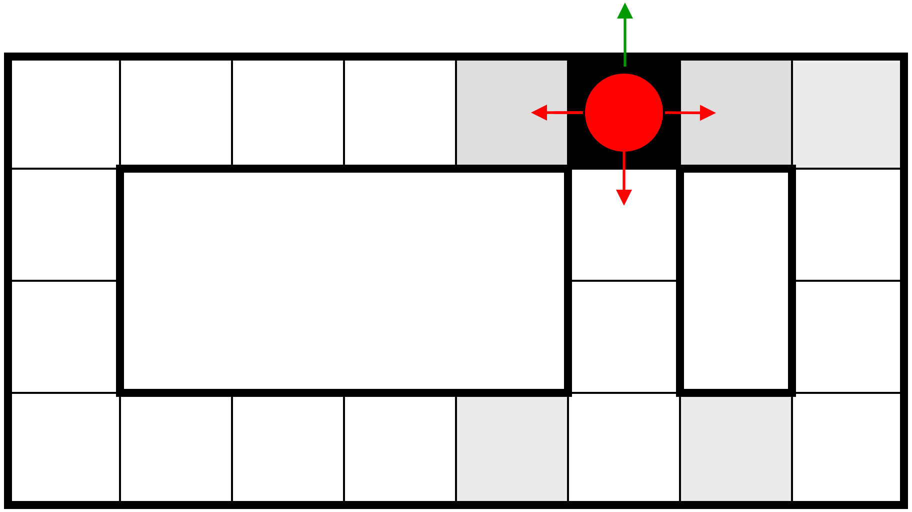
t=3

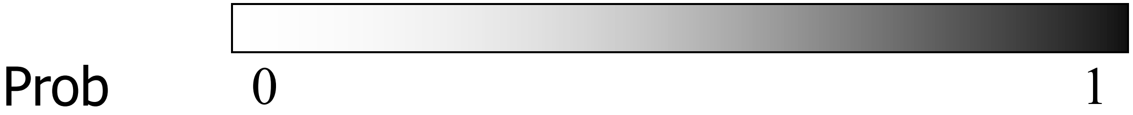
Example: Robot Localization



t=4

Example: Robot Localization





t=5