Question 1.

Conduct a minimax search of the tree above. Darken/shade each terminal node examined (evaluated). For each branch show the progressive backed up value. Answer the following questions:

The final backed up value at the root is _____4_____

Which move will be chosen? _____Move2_____

How many static evaluations were performed? _____16_____

The average branching factor of this search tree is _____2_____

Exam 2 Solutions
Deepak Kumar
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Question 2.

Conduct a **minimax search with Alpha-Beta Pruning** of the tree above. Darken/shade each terminal node examined (evaluated). For each branch show alpha, beta, and successive backed up values. Clearly mark the branches where cutoff occurs. Answer the following questions:

The final backed up value at the root is ____4_____

Which move will be chosen? ____Move2_____

How many static evaluations were performed? ____10_____

The average branching factor of this search tree is ____2_____

Question 3 (5+ 5 = 10 points)

1. Describe the static evaluation function used in your Konane program.

\[ e(n) = \begin{cases} 
#Moves_{me} - #Moves_{you} & \text{if I am winning} \\
\infty & \text{if I am winning} \\
-\infty & \text{if I am losing}
\end{cases} \]

2. Is MINIMAX a breadth-first search or a depth-first search algorithm? Why?

Minimax does Depth-First Search since it pursues each successor branch all the way down before exploring its sibling(s). It opens the door for Alpha-Beta pruning so later branches do not have to be fully explored, yet giving the same outcome.
Question 4 (3+3+5+3+3+3=20 points)
Consider the map of states of the North-Eastern United States:

Choose a CSP Formulation where you are required to color the states above using three colors (Red, Blue, and Green) so that no neighboring states can have the same color. Answer the following questions:

• What are the variables in this CSP? That is, define the state, X:
  \[ X = \{\text{PA, NJ, NY, CT, RI, MA, VT, NH, ME}\} \]

• What are the values in this CSP? That is, define the domain set, \( D \):
  \[ D_i = \{\text{red, green, blue}\} \]

• Draw the constraint graph:

Question 4 contd.
• At the start of the BACKTRACK CSP algorithm, all variables’ domains contain all the values. Show this initial state below:

<table>
<thead>
<tr>
<th>PA</th>
<th>NJ</th>
<th>NY</th>
<th>CT</th>
<th>RI</th>
<th>MA</th>
<th>VT</th>
<th>NH</th>
<th>ME</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
<td>RGB</td>
</tr>
<tr>
<td>RGB</td>
<td>RGB</td>
<td>GB</td>
<td>GB</td>
<td>GB</td>
<td>R</td>
<td>GB</td>
<td>GB</td>
<td>RGB</td>
</tr>
</tbody>
</table>

• Suppose, MA (Massachusetts) is picked first, and assigned the color Red. Show, using Forward Checking, how the domains of the other variables will be affected by this assignment.

• Give a final assignment that satisfies the constraints. You may show the assignment below, or on the map itself on the previous page.
**Question 5 (10 Points):** Given the following interpretation:

\[
\begin{align*}
A &= \text{True} \\
B &= \text{True} \\
C &= \text{False}
\end{align*}
\]

• Show which of the following wffs are satisfied by the interpretation above:

\[
\begin{align*}
&\quad \quad \quad \quad \text{Not Satisfied} \\
A \land B \Rightarrow C &\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \q
Question 7 (10 Points) Given the set of wffs:

\[ \Delta = \{ P, P \Rightarrow Q, S, Q \Rightarrow R \} \]

Is it the case that \( \Delta \vdash R \land S \) where

\[ \vdash \{ \text{Modus Ponens, } \land \text{Introduction, } \land \text{Elimination} \} \]

1. \( P \)
2. \( P \Rightarrow Q \)
3. \( S \)
4. \( Q \Rightarrow R \)
5. \( Q \) from 1, 2, and Modus Ponens
6. \( R \) from 5, 4, and Modus Ponens
7. \( R \land S \) from 6, 3, and \( \land \text{Elimination} \)

Therefore, \( \Delta \vdash R \land S \).

Question 8 (10 Points)
Prove the inference in Question 6 using Resolution and proof by refutation (\textit{reductio ad absurdum}).

\[ \{ P, P \Rightarrow Q \} \not\vdash Q \]

1. \( P \)
2. \( \neg P \lor Q \) clause form for \( P \Rightarrow Q \)
3. \( \neg Q \) negation of conclusion
4. \( \neg P \) from 3 and 2, using resolution
5. \( \Phi \) from 4 and 1, using Resolution

Since \( \{ P, P \Rightarrow Q, \neg Q \} \) leads to a contradiction, it must be that \( \{ P, P \Rightarrow Q, Q \} \) is satisfiable. Therefore, \( \{ P, P \Rightarrow Q \} \not\vdash Q \).
Question 9 (10 points)

Move Ordering is a possible heuristic in 2-person-zero-sum games where more promising looking moves are ordered so that they will be examined first. Discuss below:

• How does this heuristic affect a game playing program that uses the Minimax algorithm?

  It doesn’t. Since Minimax always examines all possible moves up to the limit of depth, move ordering will have no impact on the game playing program.

• How does this heuristic affect a game playing program that uses the Minimax algorithm with Alpha-Beta Pruning?

  If move ordering leads to examining more promising moves first (i.e. backed up values), it will result in more cut-offs. This will reduce the size of the search tree and hence the move can be determined much faster.

• How does it affect the quality of the overall game playing program?

  It doesn’t affect the quality of the game at all since both Minimax and Alpha-Beta Pruning ultimately return the same move. It is possible though, with the efficiency gained from pruning, to search further down the tree to determine the best move. This in turn may lead to a potentially better game playing program.