CS340 Analysis of Algorithms
Fall 2023

Homework:	**8**	**Professor:**	**Dianna Xu**
Due Date:	**11/21/23 (Tuesday)**	**E-mail:**	`dxu@cs.brynmawr.edu`
Office:	**Park 203**	**URL:**	`http://cs.brynmawr.edu/cs340`

For any network flow problems, you may use Ford-Fulkerson as a black box as usual. No need to provide any pseudo code. Just "run Ford-Fulkerson" would suffice. Of course, this is only if you are proposing to use the Ford-Fulkerson unmodified.

Similiarly, you do not need to prove the correctness of Ford-Fulkerson for max flow. But when you are reducing a problem to max flow, you must have an if and only if equivalence proof that establishes the correctness of your reduction.

1. You live with a black cat and prefer not to have your paths cross on Halloween. Your cat is well-trained and will take himself to the vet, but you need to meet him there (after all, a cat can't be expected to pay the vet too). You are okay with passing the cat on street corners, but don't want to end up walking down the same block with him. Is it possible for both of you to leave your house and get to the vet without ever walking down the same block (even if not at the same time)? Explain how to formulate this problem as a max-flow problem. This is a short-answer question, not a full design question.

2. Full write-up. Your friend has a new drone delivery startup, and he has asked you to help him by designing software to assist with scheduling deliveries.

- There are m drone stations throughout the city. For $1 \leq i \leq m$, let $d_i = (d_{i,x}, d_{i,y})$ denote the (x, y) coordinates of the ith drone station (see Fig. 1(a)). Due to FAA regulations, each drone station can launch no more than 5 drones each day.
- There are n customers expecting to receive a package this day. For $1 \leq j \leq n$, let $c_i = (c_{j,x}, c_{j,y})$ denote the (x, y) coordinates of the jth customer. (You may assume that no two customers occupy the same location, and each customer is expecting exactly one delivery.)

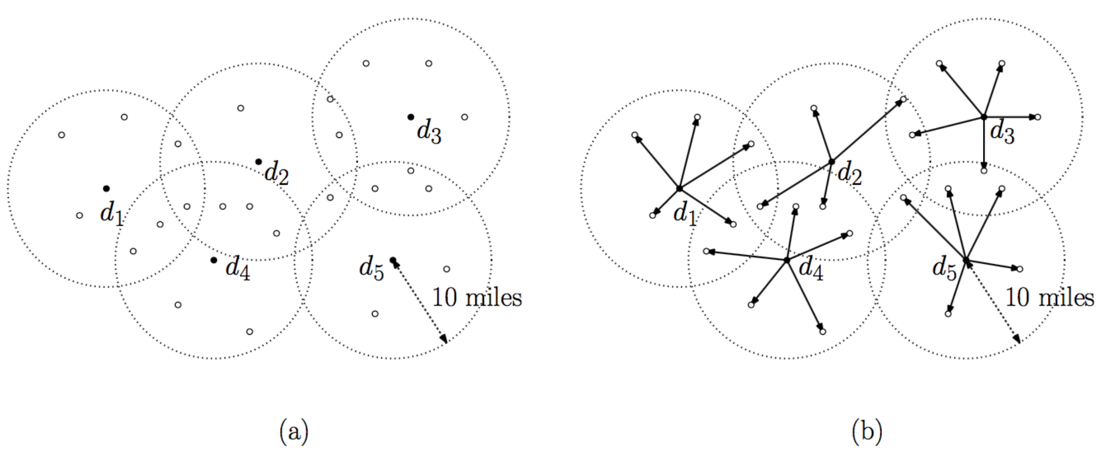


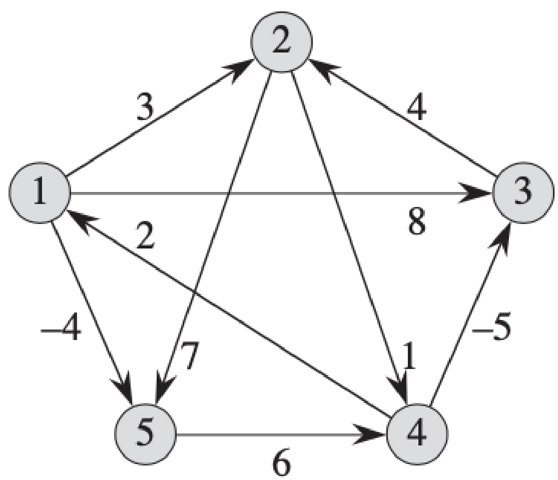
Figure 1: Black points are drone stations and hollow points are customers: (a) Input (b) Possible solution

- Each drone station is attached to a complete warehouse, so in theory a drone from any station can deliver the desired package to any customer. However, because of fuel limitations, each drone can make a delivery only within a 10 mile radius of the station (see Fig. 1(a)). That is, station i can only deliver packages to those customers j such that $dist(d_i, c_j) \leq 10$.

You are given the coordinates of the m drone stations and the coordinates of the n customers. The problem is to determine the maximum number of deliveries that can be made (ideally all n of them), subject to the constraints given above (see Fig. 1(b)).

3. Full write-up. Consider the downtown of a city that is a perfect $n \times m$ grid graph of streets, such that each corner connects via 2-way roads to 4 other corners, except for the intersections at the border of the city. There is a natural disaster and the goal is to evacuate the city (get the people to the border of the city). Because of the heavy traffic expected, it is important that the evacuation routes be vertex-disjoint. Given k starting locations on the grid (the gathering points around the city that are not themselves on the city border), determine if it is possible to evacuate people from those k locations to the city border.

4. Run Floyd-Warshall on the following graph. For each iteration, list the new paths found and show how the matrix is updated, i.e. list the sequence of matrices $d^{(k)}$ you obtain.



Please hand in your assignment electronically on Moodle.