CS340 Analysis of Algorithms
Fall 2023

Homework:	**7**	**Professor:**	**Dianna Xu**
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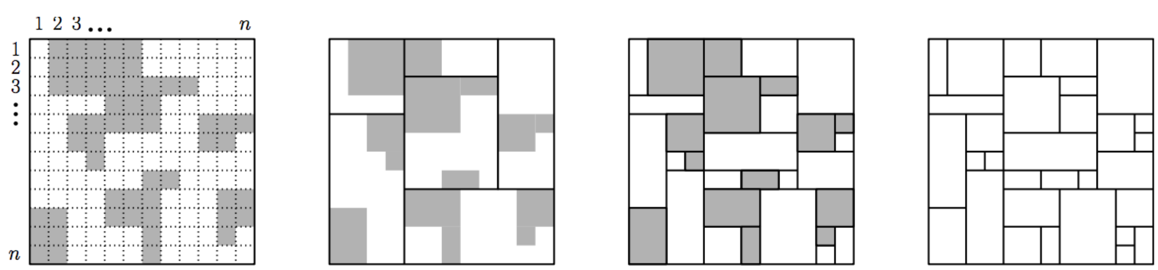
1. Full write-up. You are looking at the performance of a given stock for n consecutive days, at some point in the past. The days are numbered $1, 2, \ldots, n$. You are given $P[1..n]$ where $P[i]$ denotes the price per share for the stock on the ith day. For a given integer k you want to know what is the maximum profit of a so-called k-buy-sell strategy. A k-buy-sell strategy is a collection of m pairs of days $(b_1, s_1), (b_2, s_2), \ldots, (b_m, s_m)$ for some $1 \leq m \leq k$ and $b_1 < s_1 < b_2 < s_2 < \ldots < b_m < s_m$. This can be viewed as a set of at most k non-overlapping intervals, during each of which you buy certain (say $1,000$) shares of the stock (on day b_i) and then sell the same (on day s_i). The profit of such a strategy is simply the profit of the m buy-sell transactions,

$$1000\Sigma_{i=1}^{m}(P[s_i] - P[b_i])$$

Given P and k, design an efficient DP solution to find the maximum profit obtained by a k-buy-sell strategy throughout days 1 to n.

2. Full write-up. The following problem arises in image processing and compression. You are given a black-and-white digitized picture in the form of a two dimensional $n \times n$ matrix P. For $1 \leq i$, $j \leq n$, $P[i, j]$ is 0 if the pixel on row i and column j is black and 1 if it is white. We want to decompose the picture into a minimum number of monochromatic rectangles, which means that each rectangle is either all white or all black. The decomposition must be performed in the following hierarchical manner. Starting with the full image as the starting rectangle, we can split it into two rectangles either by a vertical line or a horizontal line that cuts through the entire rectangle. After this, we can split each of these rectangles again either by a vertical or horizontal line that cuts through the entire rectangles, and so on. The process stops when a rectangle is either all white or all black. An example of such a decomposition is shown in Fig. 1(c).



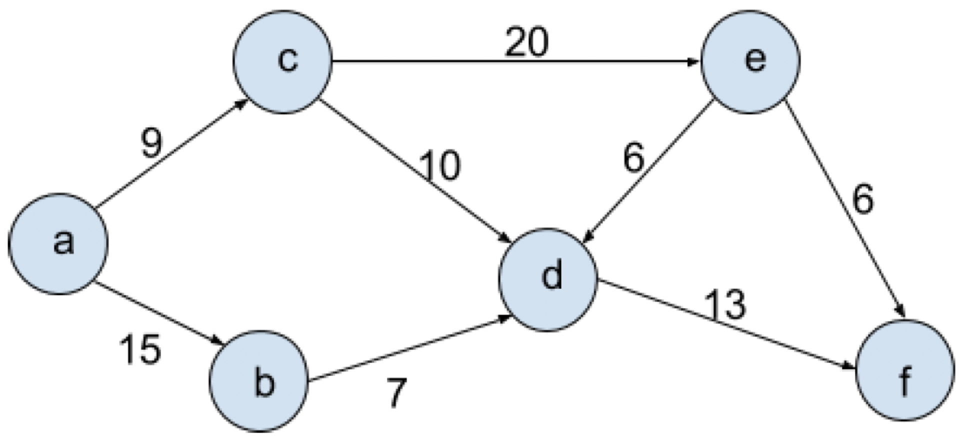
(a) (b) (c) (d)

Figure 1: (a) the image rectangle (b) a partial decomposition after 5 cuts (c) the final decomposition into 31 monochromatic rectangles (d) the cuts of the final decomposition

Design a dynamic programming solution, which given an image P, determines the minimum number of rectanges in a hierarchial monochromatic decomposition. You may assume that you have access to a function `monochrome(`i_0`, `i_1`, `j_0`, `j_1`)` that returns true if the image rectangle $P[i, j]$, for $i_0 \leq i \leq i_1$ and $j_0 \leq j \leq j_1$ is monochromatic in $O(1)$ time. (This is possible, but not trivial. See extra credit.)

Extra Credit: Show that, given the $n \times n$ input image, it is possible to build a data structure (after $O(n^2)$ preprocessing) from which `monochrome(`i_0`, `i_1`, `j_0`, `j_1`)` can be computed in $O(1)$ time. Provide a time analysis of your preprocessing step and argue why $O(1)$ is possible.

3. Run the Ford-Fulkerson algorithm on the graph below with source a and sink f. Draw the residual graph, and the updated graph after each augmenting path is chosen. For each iteration, pick the augmenting path that is lexicographically the smallest (using the node letters). State resulting max flow.



Please hand in your assignment electronically on Moodle.