Computer Graphics

From Vertices to Fragments:
Clipping, HSR, Rasterization and Anti-aliasing

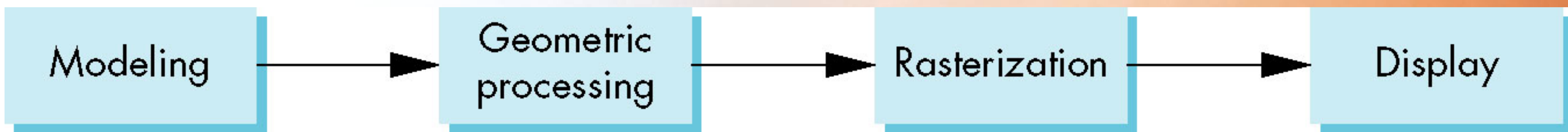
Based on slides by Dianna Xu, Bryn Mawr College

Rendering Algorithms

- Rendering a scene with opaque objects
  - –For every pixel, determine which object that projects on the pixel is closest to the viewer and compute the shade of this pixel
  - –Ray tracing paradigm
  - –For every object, determine which pixels it covers and shade these pixels
    - Pipeline approach
    - Must keep track of depths

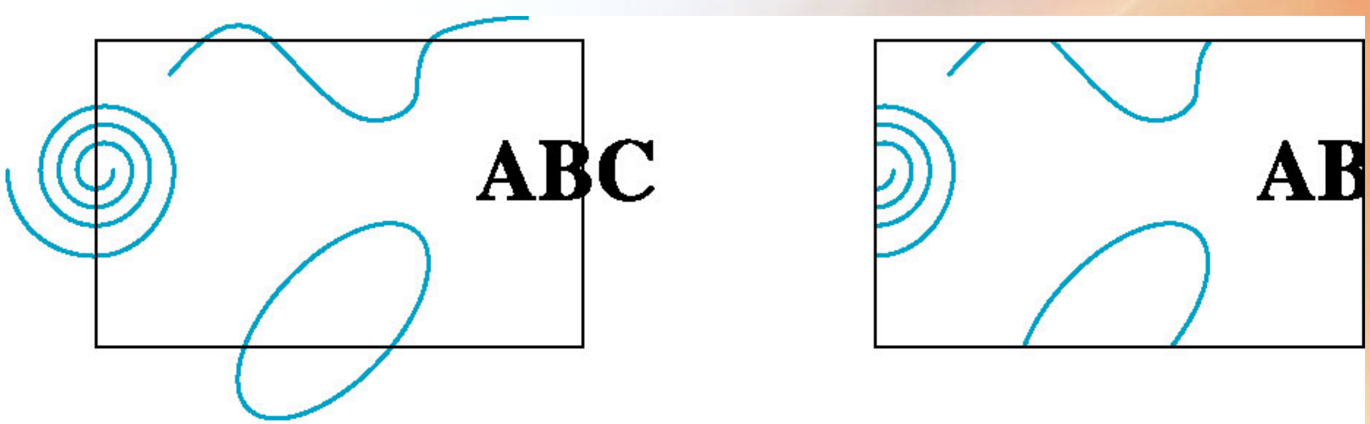
Common Tasks

- Clipping
- Hidden surface removal
- Rasterization or scan conversion
- Antialiasing



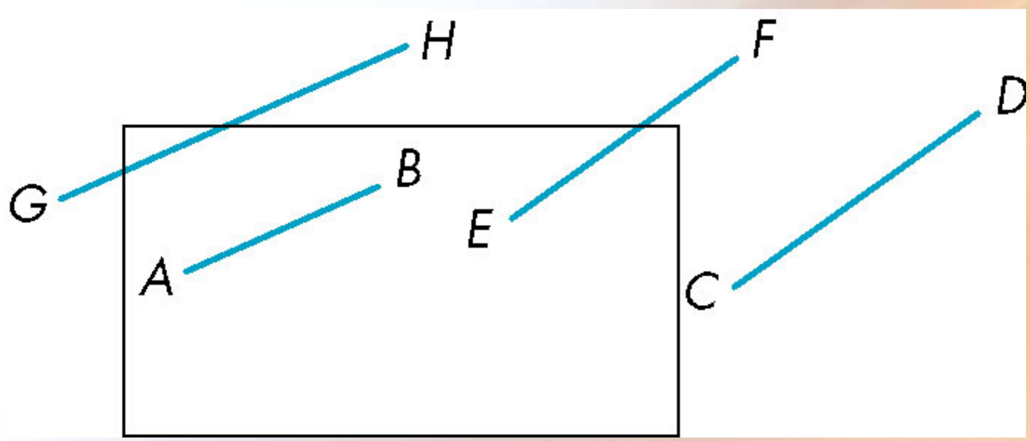
Clipping

- 2D against clipping window
- 3D against clipping volume
- Easy for line segments polygons
- Hard for curves and text
  - Convert to lines and polygons first



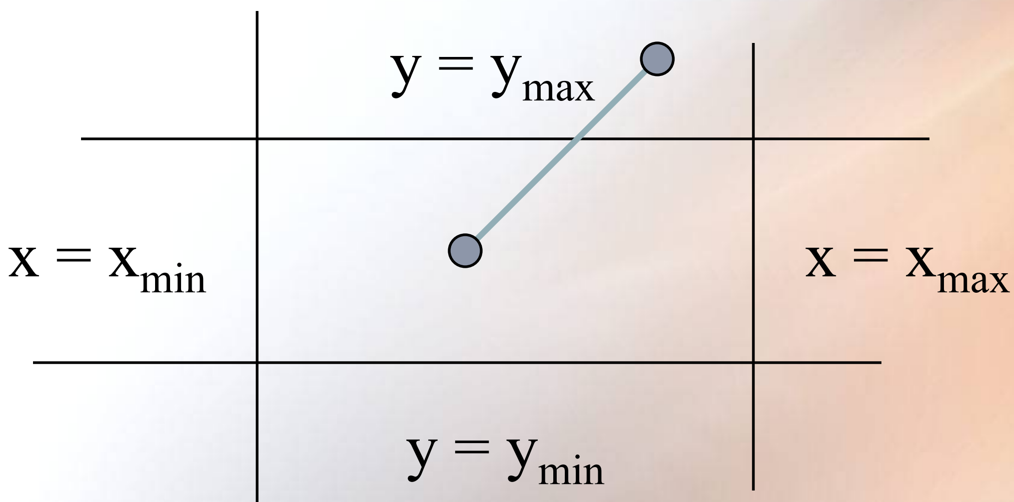
Clipping 2D Line Segments

- Brute force approach: compute intersections with all sides of clipping window
  - Inefficient: one division per intersection



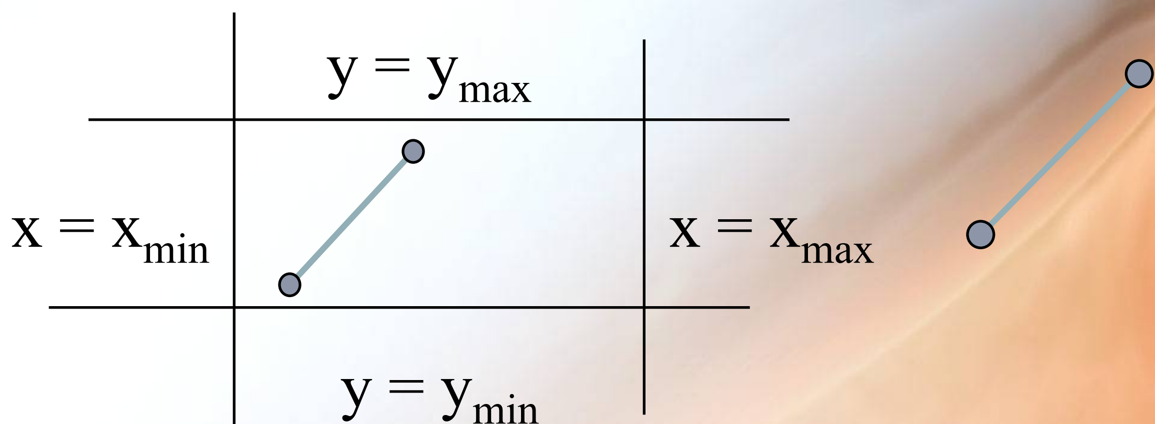
Cohen-Sutherland Algorithm

- Idea: eliminate as many cases as possible without computing intersections
- Start with four lines that determine the sides of the clipping window



The Cases

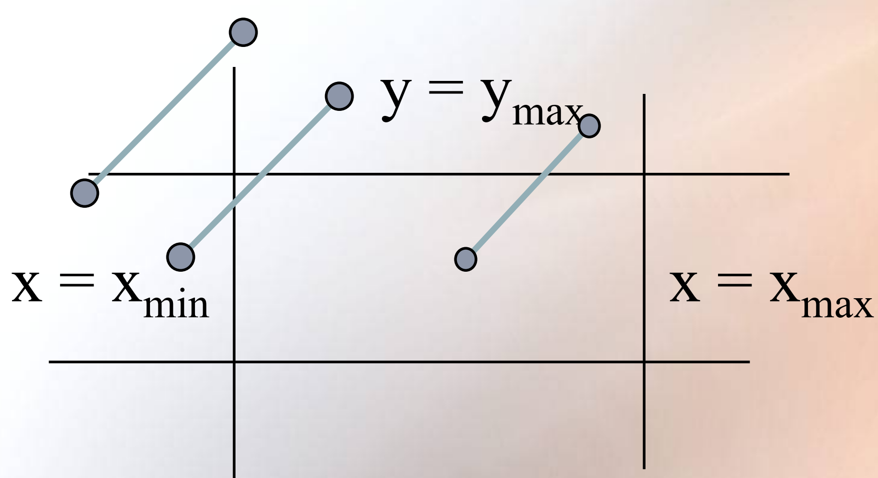
- Case 1: both endpoints of line segment inside all four lines
  - Draw (accept) line segment as is



- Case 2: both endpoints outside all lines and on same side of a line
  - Discard (reject) the line segment

The Cases

- Case 3: One endpoint inside, one outside
  - Must do at least one intersection
- Case 4: Both outside
  - May have part inside
  - Must do at least one intersection

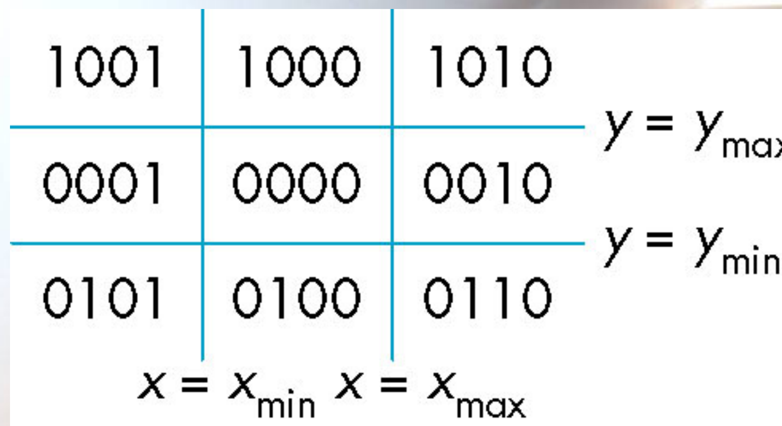


Defining Outcodes

- For each endpoint, define an outcode

$$b_0b_1b_2b_3$$

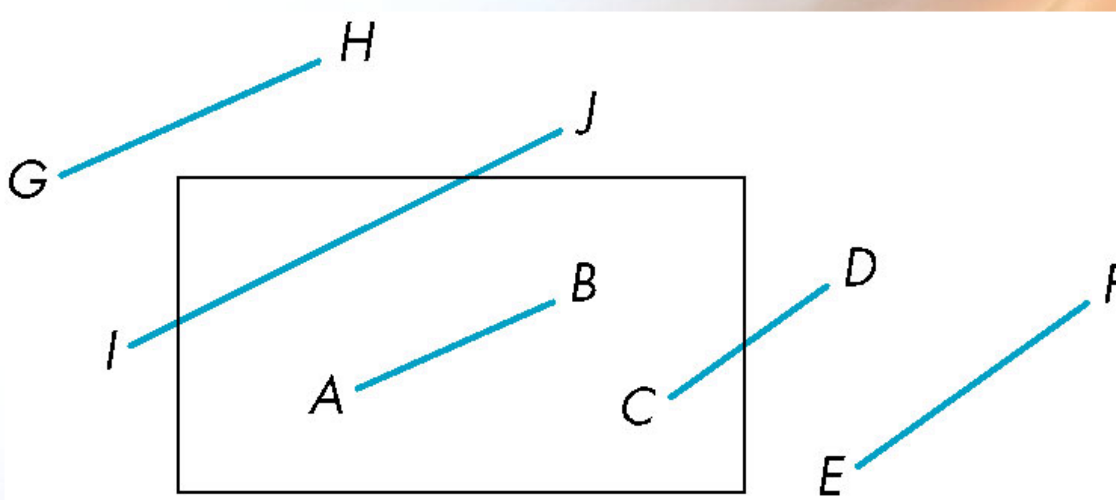
$b_0 = 1$ if $y > y_{max}$, 0 otherwise
$b_1 = 1$ if $y < y_{min}$, 0 otherwise
$b_2 = 1$ if $x > x_{max}$, 0 otherwise
$b_3 = 1$ if $x < x_{min}$, 0 otherwise



- Outcodes divide space into 9 regions
- Computation of outcode requires at most 4 subtractions

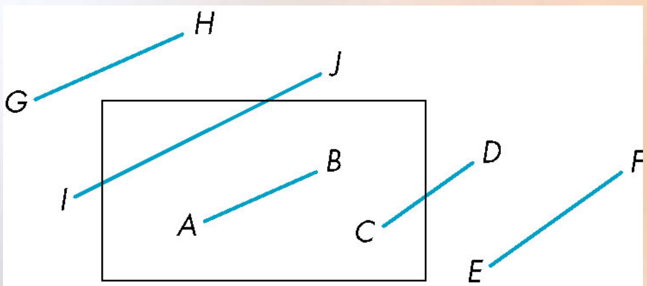
Using Outcodes

- Consider the 5 cases below
- AB: outcode(A) = outcode(B) = 0
  – Accept line segment



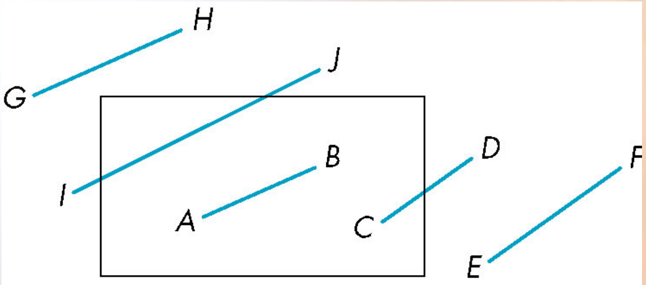
Using Outcodes

- CD: outcode (C) = 0, outcode(D) $\neq$ 0
  - Compute intersection
  - Location of 1 in outcode(D) determines which edge to intersect with
  - Note if there were a segment from C to a point in a region with 2 ones in outcode, we might have to do two intersections



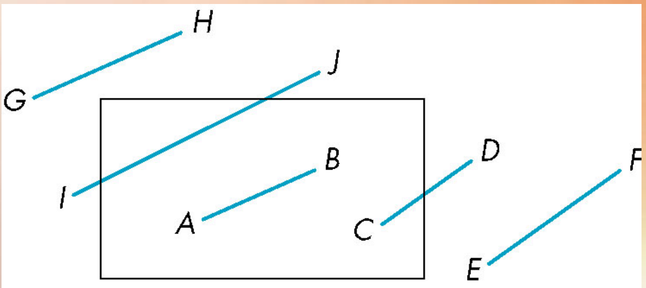
Using Outcodes

- EF: outcode(E) & outcode(F) (bitwise) != 0
  - Both outcodes have a 1 bit in the same place
  - Line segment is outside of corresponding side of clipping window
  - reject



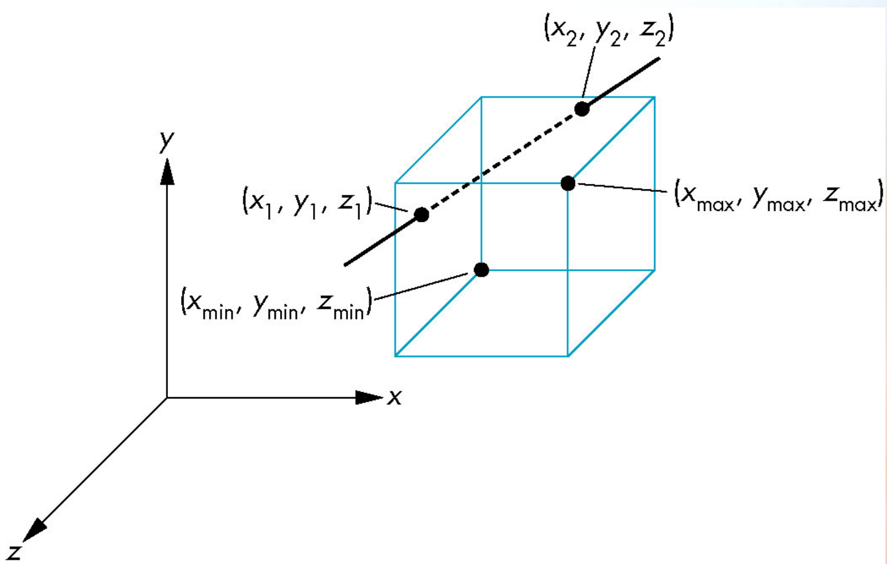
Using Outcodes

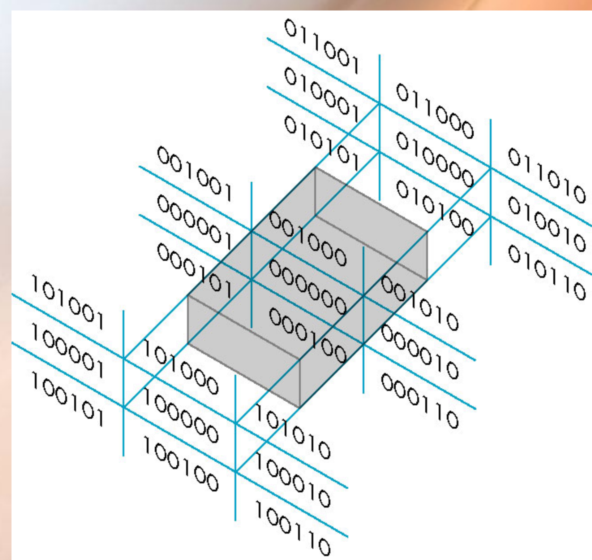
- GH and IJ: same outcodes, neither zero but logical AND yields zero
- Shorten line segment by intersecting with one of sides of window
- Compute outcode of intersection (new endpoint of shortened line segment)
- Re-execute algorithm



Cohen Sutherland in 3D

- Use 6-bit outcodes
- When needed, clip line segment against planes

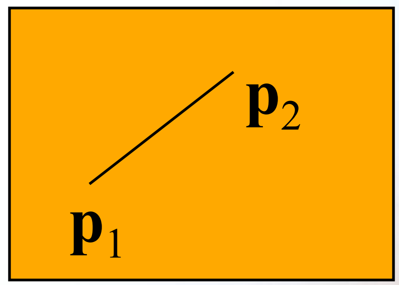




Liang-Barsky Clipping

- Consider the parametric form of a line segment

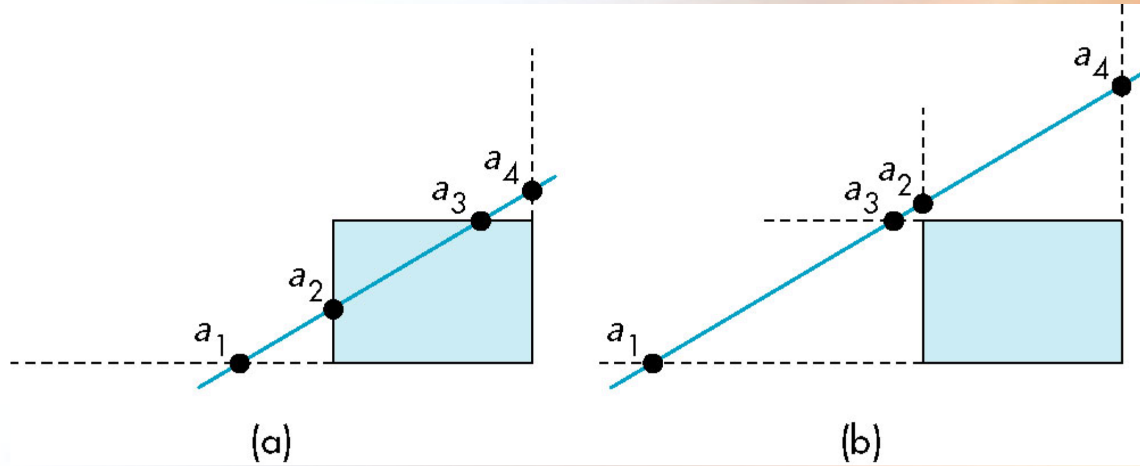
$$\mathbf{p}(\alpha) = (1-\alpha)\mathbf{p}_1 + \alpha\mathbf{p}_2 \quad 1 \geq \alpha \geq 0$$



- We can distinguish between the cases by looking at the ordering of the values of α where the line determined by the line segment crosses the lines that determine the window

Liang-Barsky Clipping

- In (a): $\alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$
  - Intersect right, top, left, bottom: shorten
- In (b): $\alpha_4 > \alpha_2 > \alpha_3 > \alpha_1$
  - Intersect right, left, top, bottom: reject

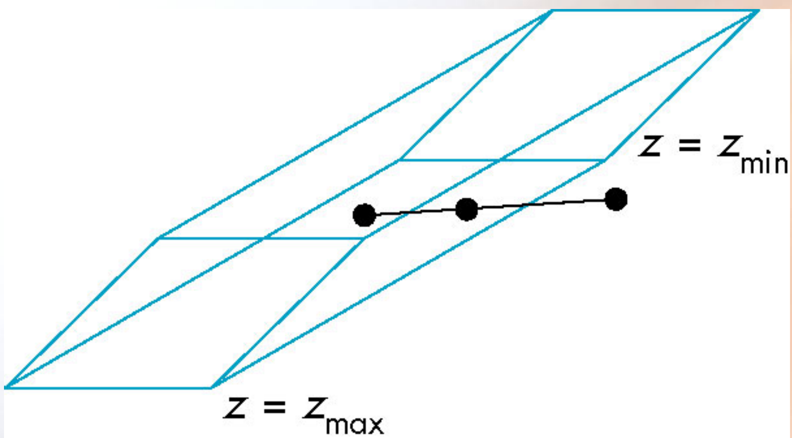


Advantages

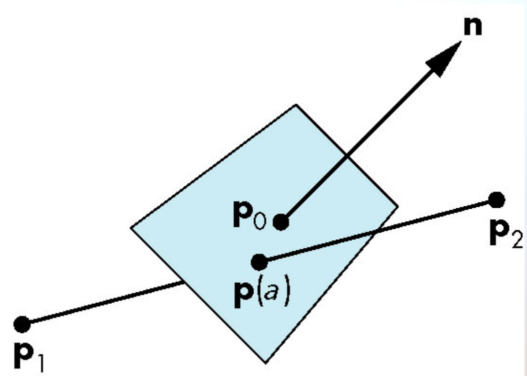
- Can accept/reject as easily as with Cohen-Sutherland
- Using values of α, we do not have to use algorithm recursively as with C-S
- Extends to 3D

Clipping and Normalization

- General clipping in 3D requires intersection of line segments against arbitrary plane
- Example: oblique view



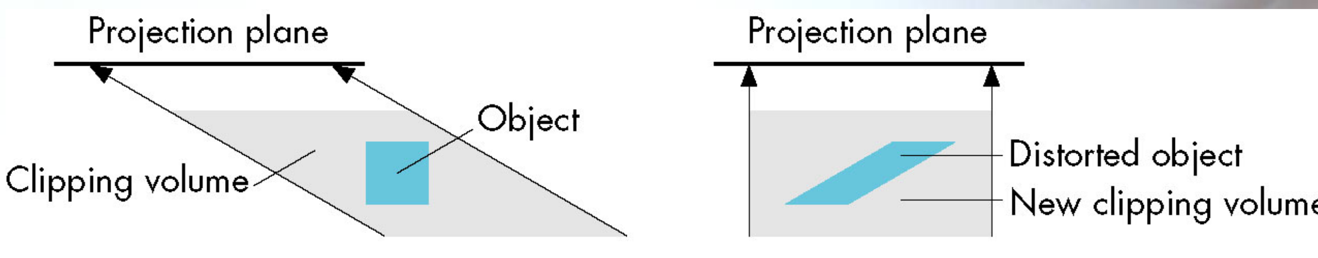
Plane-Line Intersections



$$a = \frac{n \bullet (p_o - p_1)}{n \bullet (p_2 - p_1)}$$

Normalized Form

top view



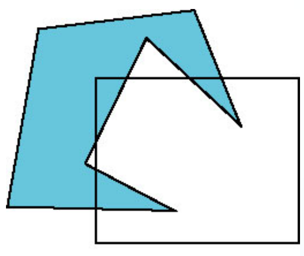
before normalization after normalization

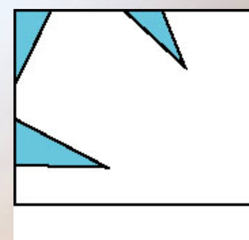
Normalization is part of viewing (pre clipping) but after normalization, we clip against sides of right parallelepiped

Typical intersection calculation now requires only a floating point subtraction, e.g. is $x > x_{max}$?

Polygon Clipping

- Not as simple as line segment clipping
  - Clipping a line segment yields at most one line segment
  - Clipping a polygon can yield multiple polygons

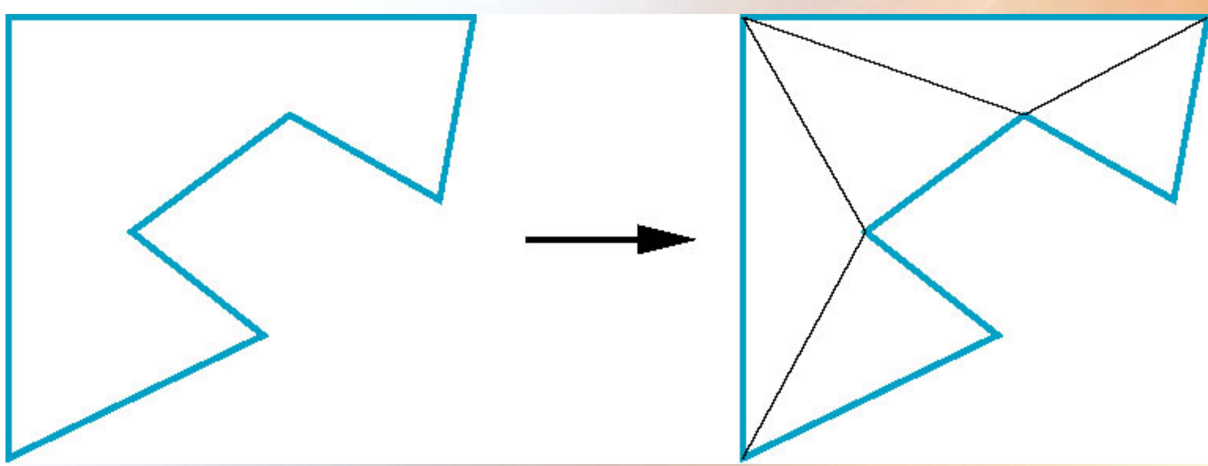




- However, clipping a convex polygon can yield at most one other polygon

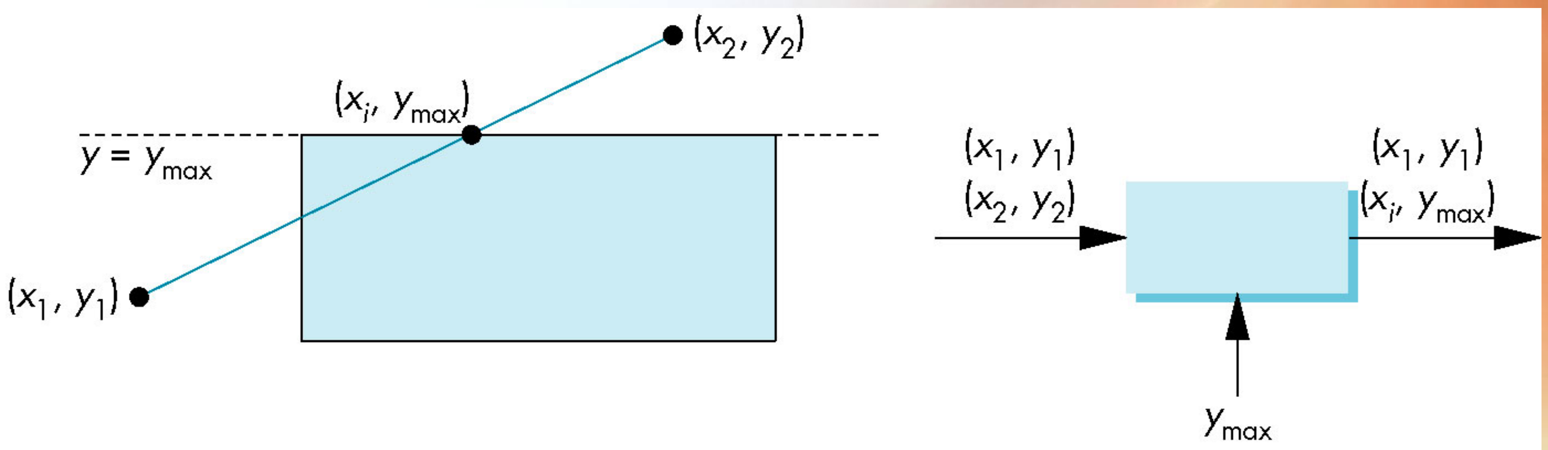
Tessellation and Convexity

- **One strategy is to replace nonconvex (*concave*) polygons with a set of triangular polygons (a *tessellation*)**
- **Also makes fill easier**
- **Tessellation code in GLU library**



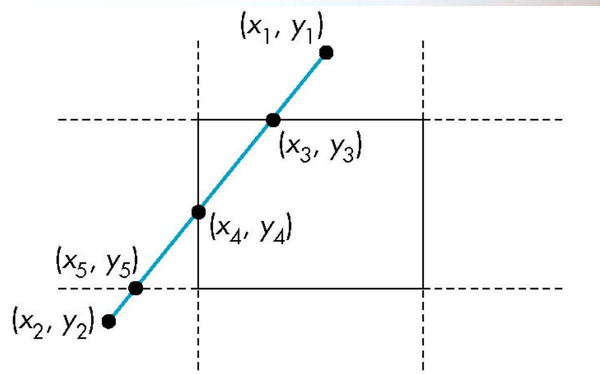
Clipping as a Black Box

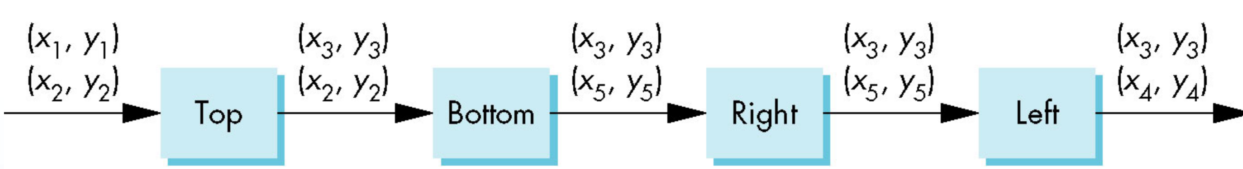
- Can consider line segment clipping as a process that takes in two vertices and produces either no vertices or the vertices of a clipped line segment



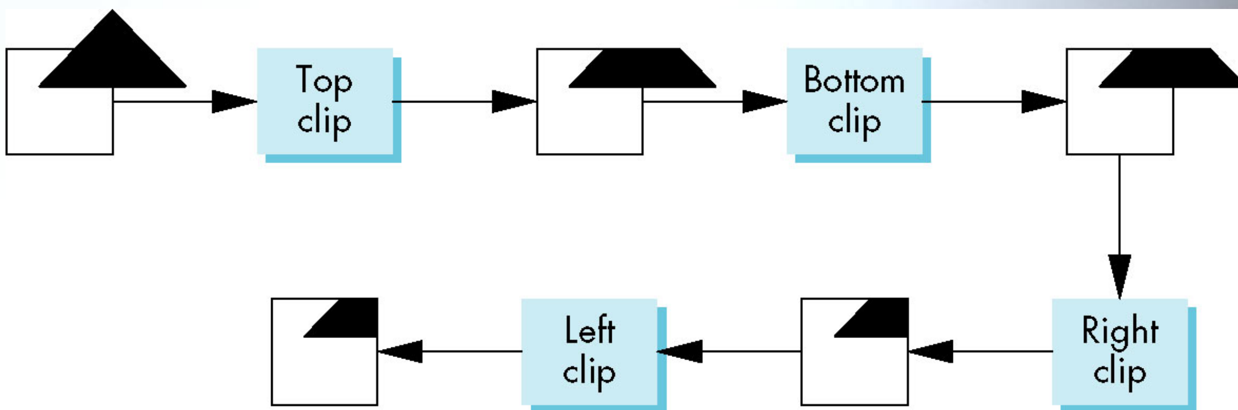
Pipeline Clipping of Line Segments

- Clipping against each side of window is independent of other sides
  - Can use four independent clippers in a pipeline





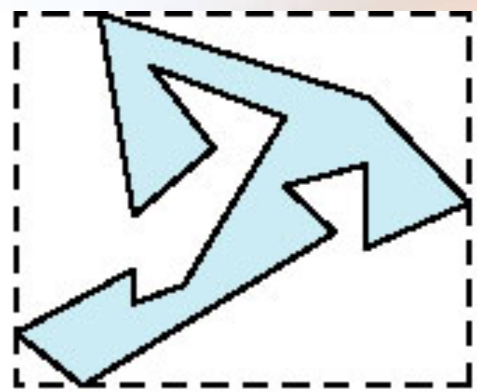
Pipeline Clipping of Polygons



- Three dimensions: add front and back clippers
- Strategy used in SGI Geometry Engine
- Small increase in latency

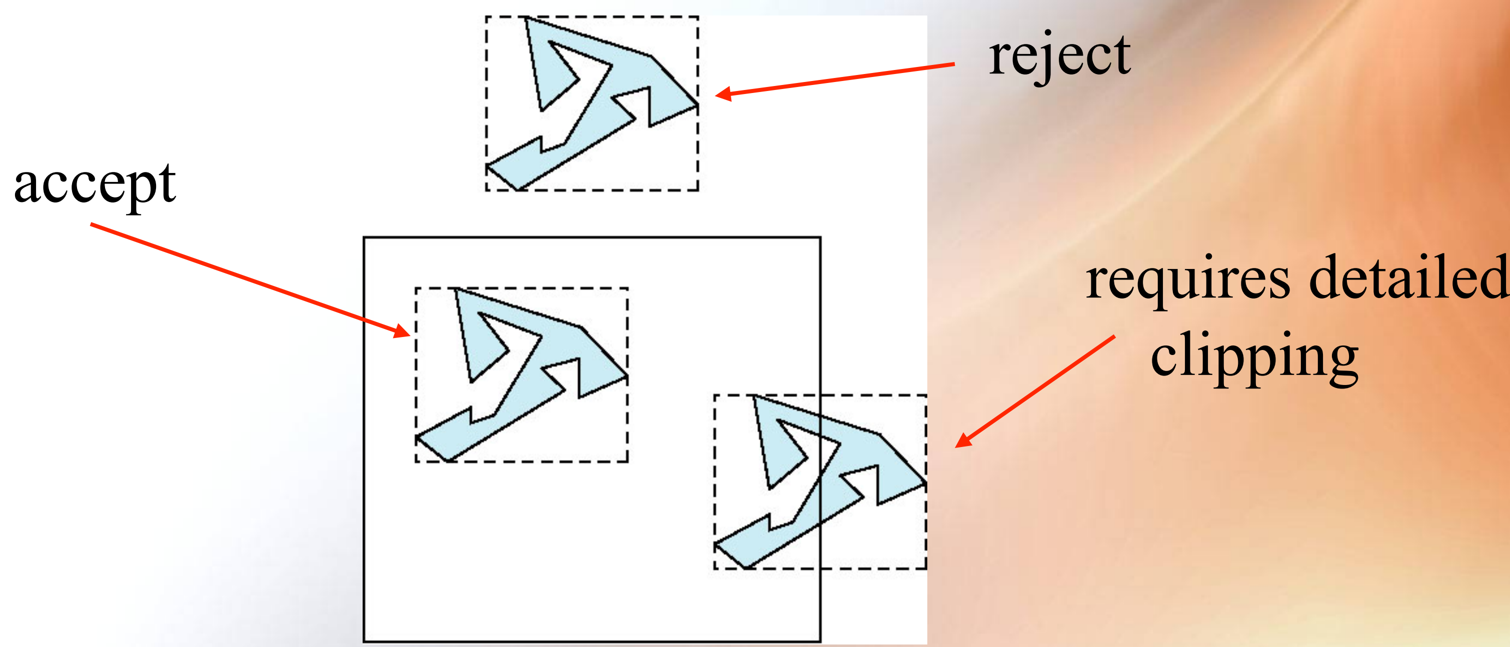
Bounding Boxes

- Rather than doing clipping on a complex polygon, we can use an *axis-aligned bounding box* or *extent*
  - Smallest rectangle aligned with axes that encloses the polygon
  - Simple to compute: max and min of x and y



Bounding boxes

Can usually determine accept/reject based only on bounding box

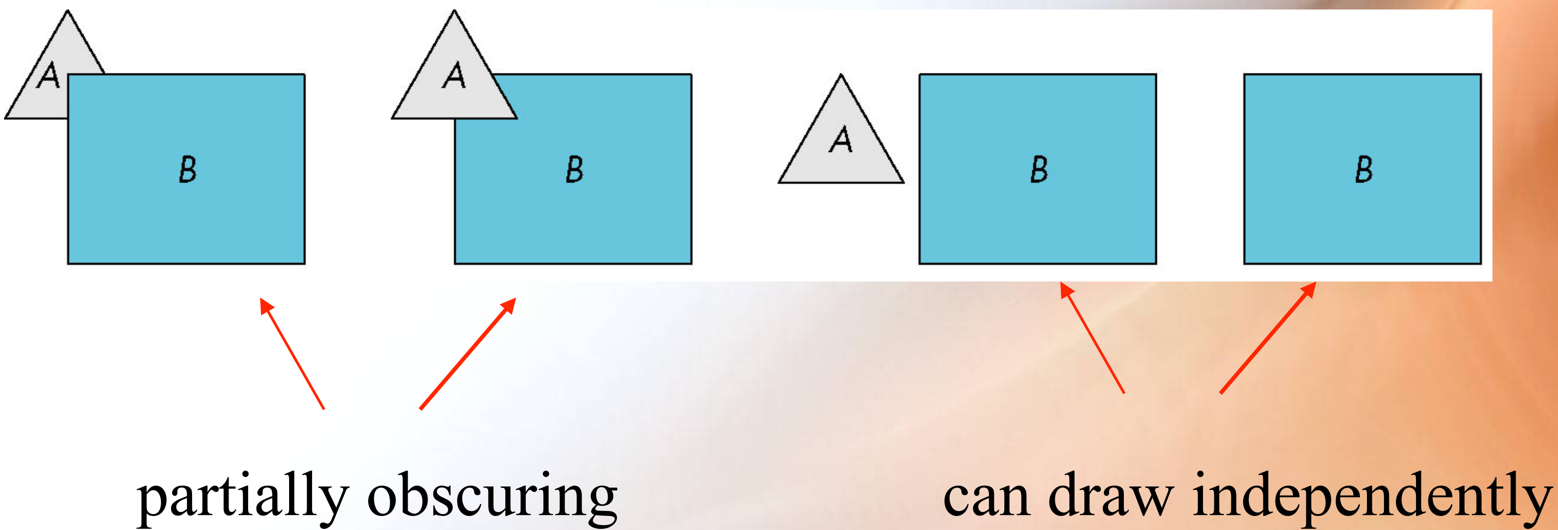


Clipping and Visibility

- Clipping has much in common with hidden-surface removal
- In both cases, we are trying to remove objects that are not visible to the camera
- Often we can use visibility or occlusion testing early in the process to eliminate as many polygons as possible before going through the entire pipeline

Hidden Surface Removal

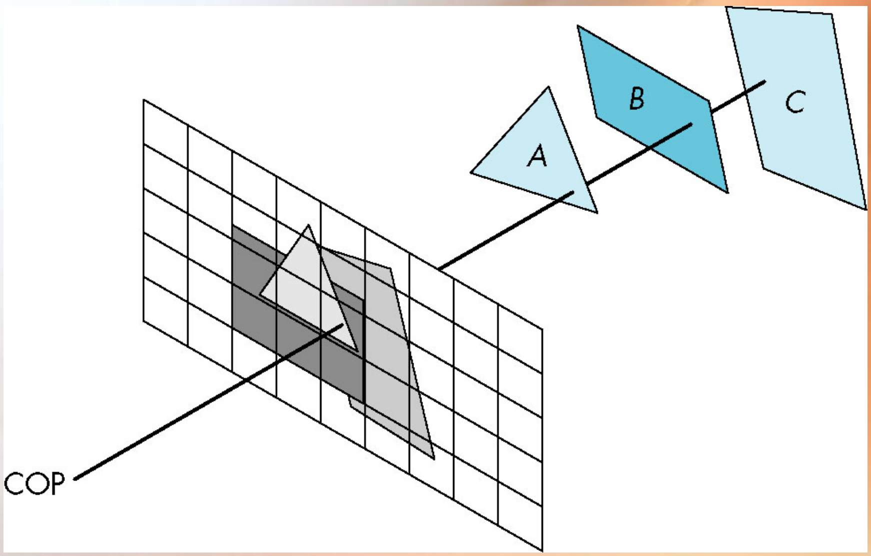
- Object-space approach: use pairwise testing between polygons (objects)



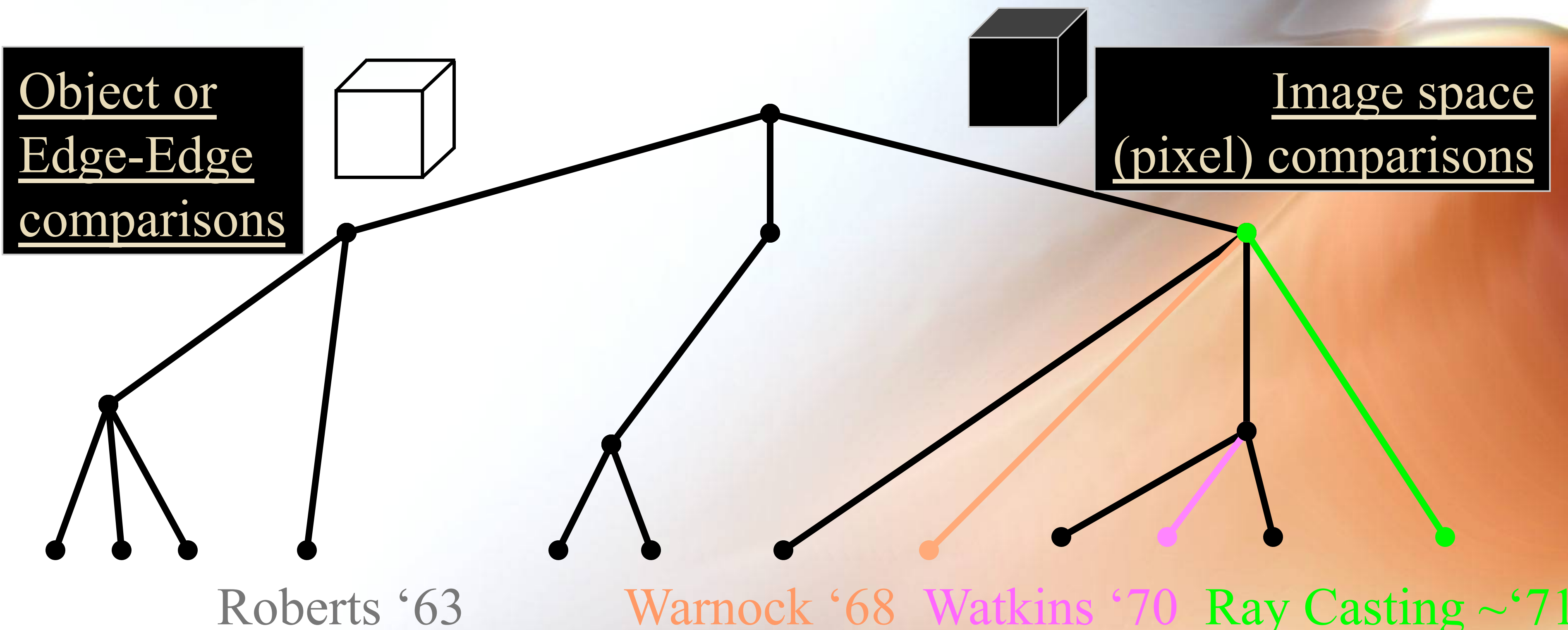
- Worst case complexity $O(n^2)$ for n polygons

Image Space Approach

- Look at each projector (nm for an $n \times m$ frame buffer) and find closest of k polygons
- Complexity $O(nmk)$
- Ray tracing
- z-buffer



Visible Surface Algorithms



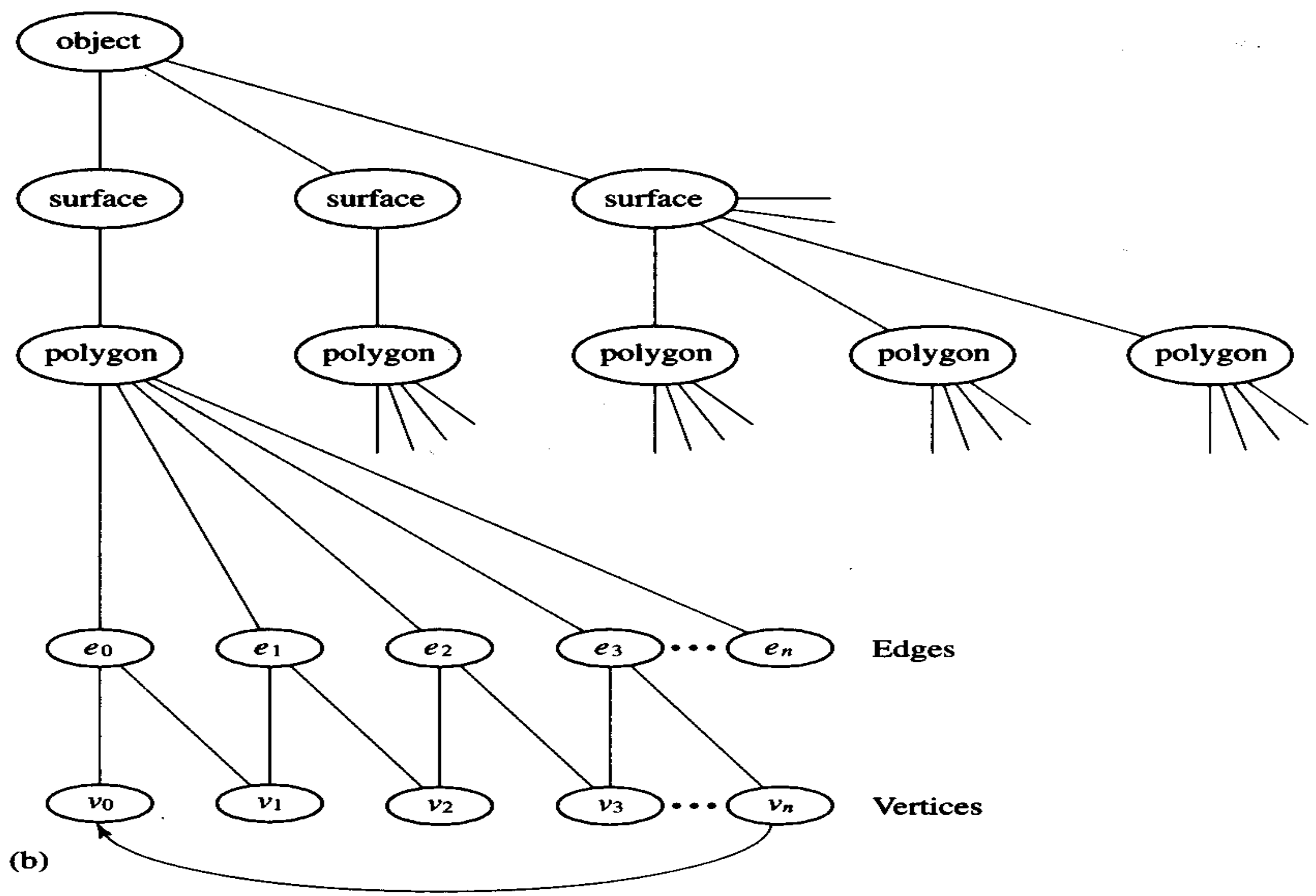
Complexity grows $O(n^2)$
(n=number of objects)

Complexity ~ visual complexity
Bounded by sorting cost $O(n \log n)$

Polyhedral Object Model Assumptions

- Clip geometry to view volume.
- Planar polygon faces (convex or concave).
- Consistent edge traversal order -- to establish uniform notion of inside and outside.
  - Surface normal points outward in a right-handed world modeling coordinate system.
  - In GL, make sure you list the vertices in a consistent order (all clockwise or counterclockwise (default) when viewed from outside).

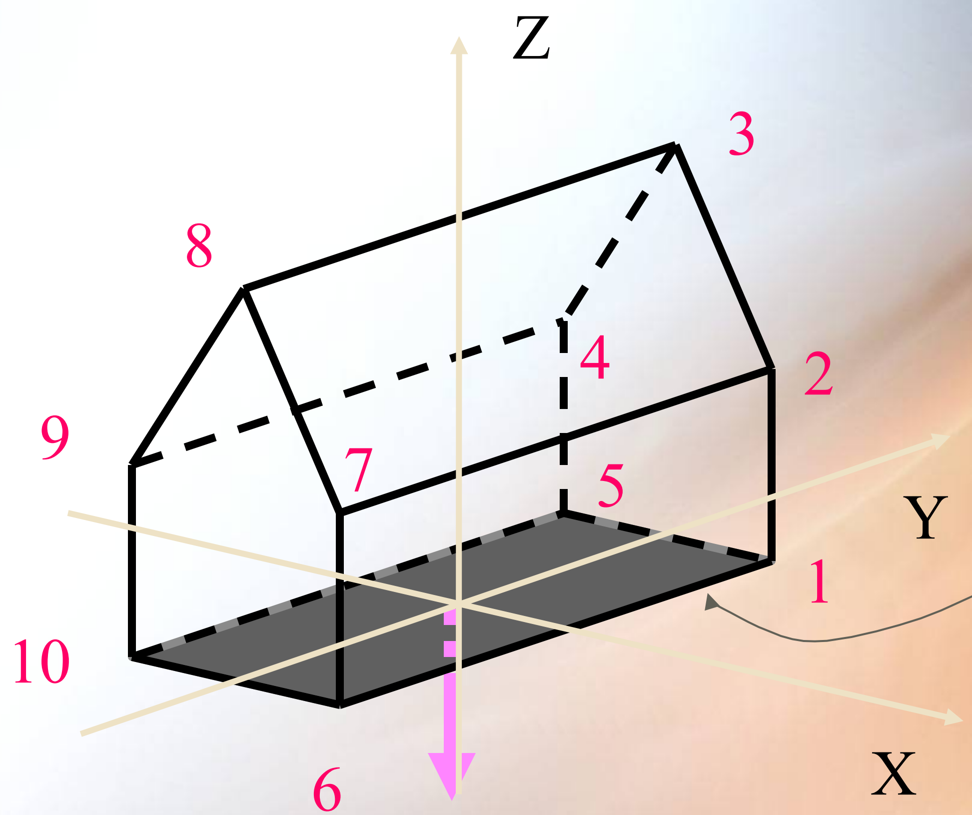
Polygon Model Conceptualization



A Polygonal Model

POINTS

P1 = (1, 2, 0)
P2 = (1, 2, 3)
P3 = (0, 2, 5)
P4 = (-1, 2, 3)
P5 = (-1, 2, 0)
P6 = (1, -2, 0)
P7 = (1, -2, 3)
P8 = (0, -2, 5)
P9 = (-1, -2, 3)
P10 = (-1, -2, 0)



POLYGONS

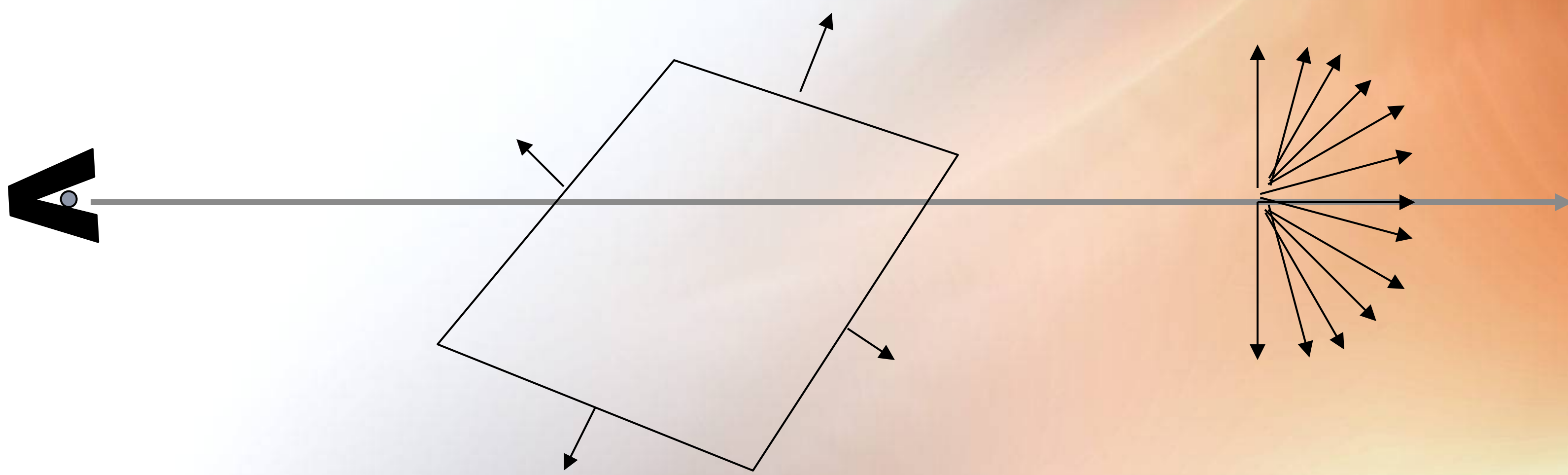
P1 P5 P4 P3 P2
P6 P7 P8 P9 P10
P1 P2 P7 P6
P2 P3 P8 P7
P3 P4 P9 P8
P4 P5 P10 P9
P1 P6 P10 P5

ATTRIBUTES:

name = ‘floor’, normal = (0, 0, -1), color = (R=0.1, G=0.1, B=0.1), fill = yes, edge-color = (R=1, G=1, B=1), ...

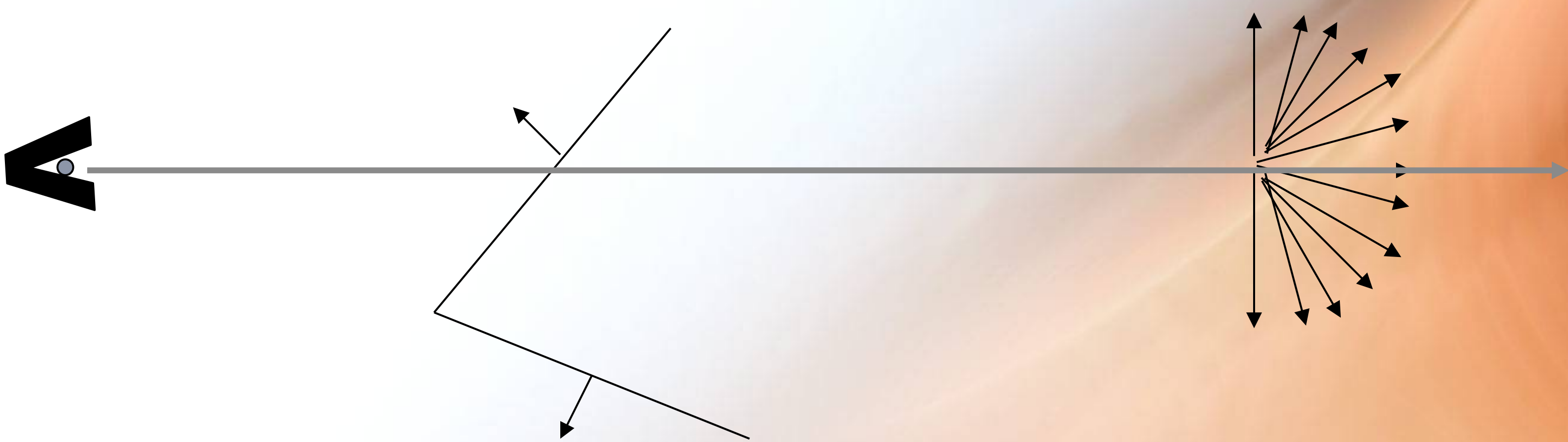
Back Face Cull

- `glEnable(GL_CULL_FACE);`
- **Throw out polygons facing away from eye -- that is, any polygon with a** BACK-facing normal:



Back Face Cull

- Only FRONT-facing ones left to process further.



THIS IS ONLY SUFFICIENT AS A VISIBLE SURFACE DISPLAY FOR A SCENE CONSISTING OF A SINGLE CONVEX POLYHEDRON

Depth or Z-Buffer

- **Each pixel stores COLOR and DEPTH.**
- **Algorithm:**

  Initialize all elements of buffer (COLOR(row, col), DEPTH(row, col)) to (background-color, maximum-depth);

  FOR EACH polygon:

  Rasterize polygon to frame;

  FOR EACH pixel center (x, y) that is covered:

  IF polygon depth at (x, y) < DEPTH(x, y)

  THEN COLOR(x, y) := polygon color at (x, y)

  AND DEPTH(x, y) := polygon depth at (x, y)

Depth Buffer Operation

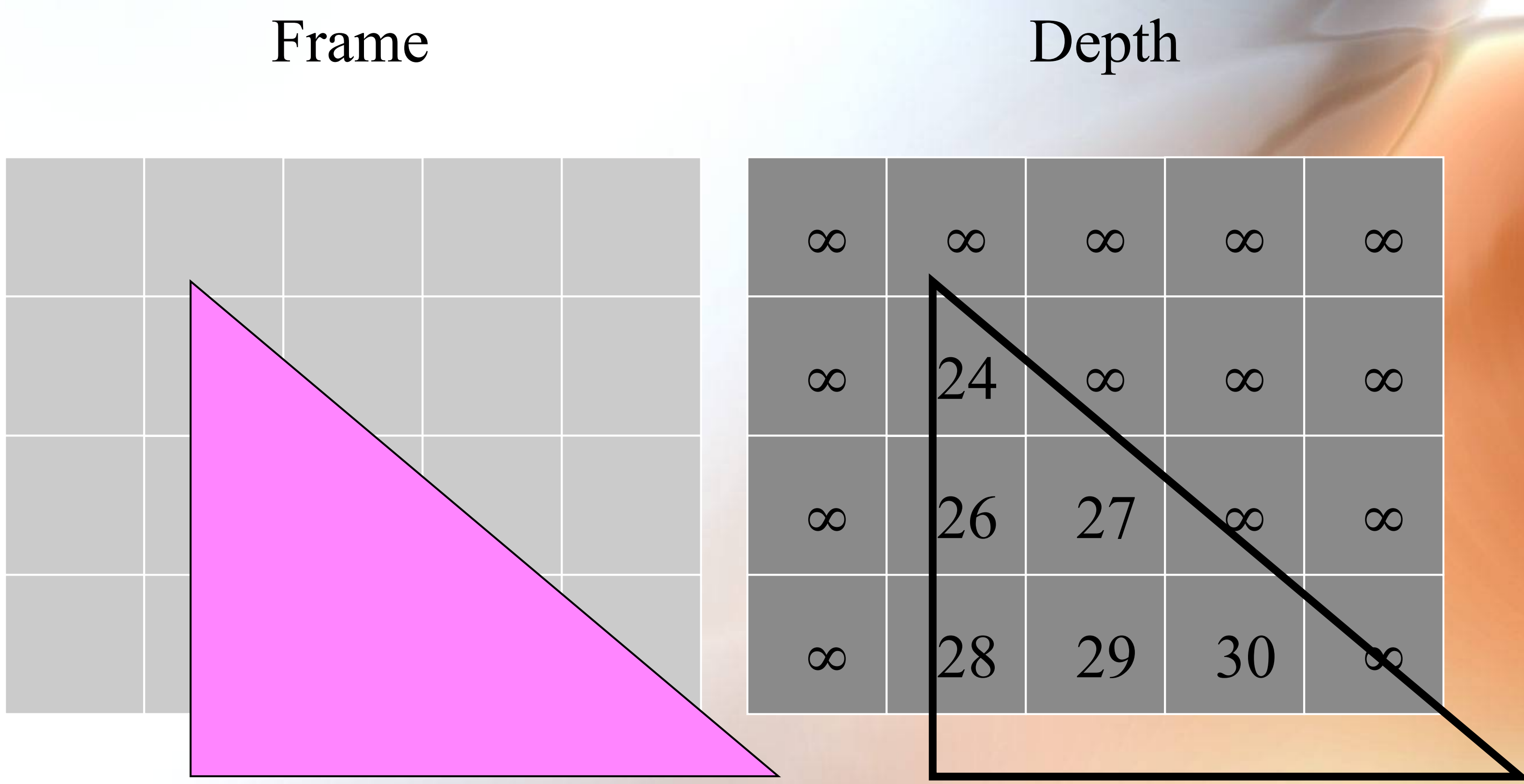
Frame

Depth

∞	∞	∞	∞	∞
∞	∞	∞	∞	∞
∞	∞	∞	∞	∞
∞	∞	∞	∞	∞

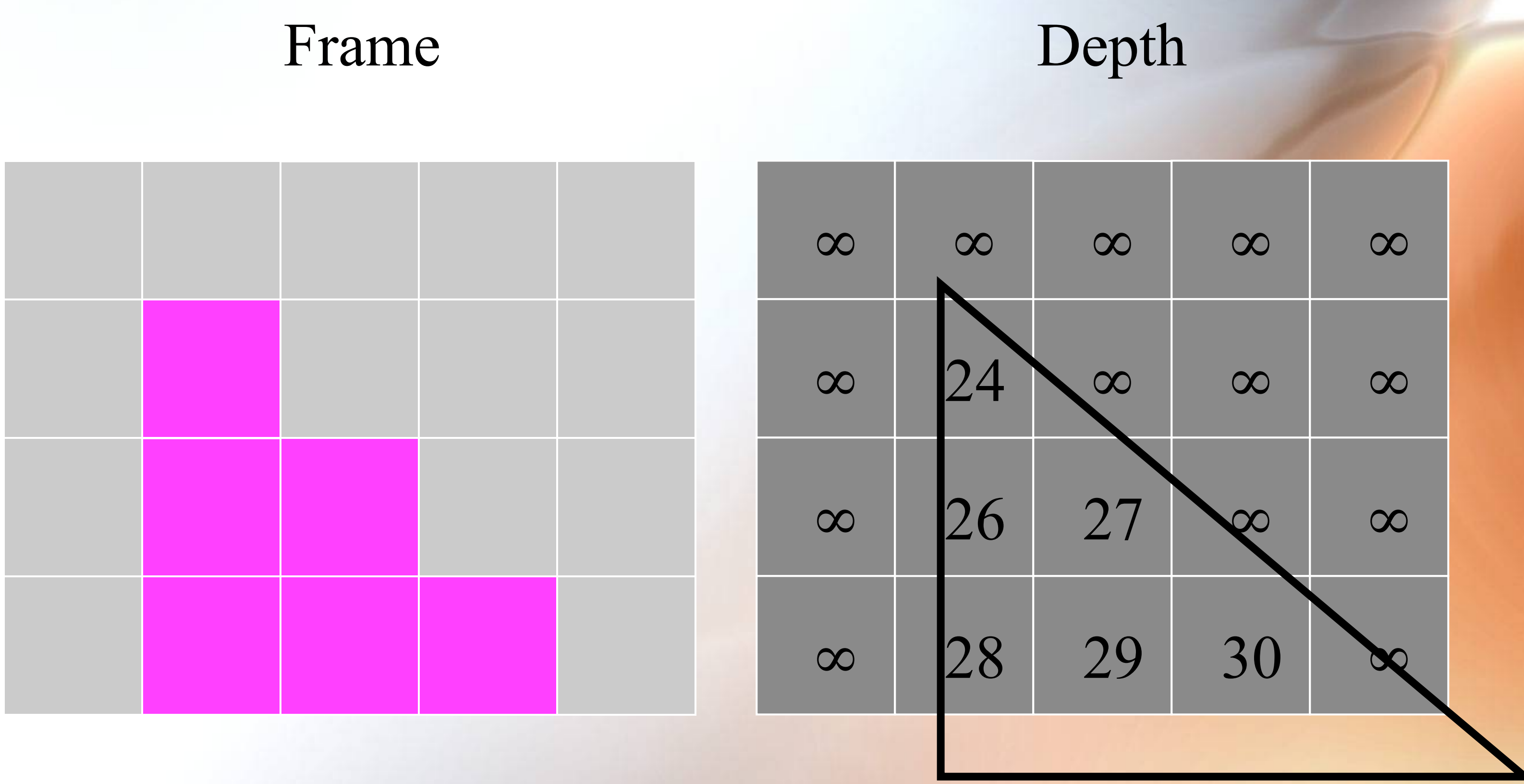
Initialize (“New Frame”)

Depth Buffer Operation – First Polygon



Pink Triangle -- depths computed at pixel centers

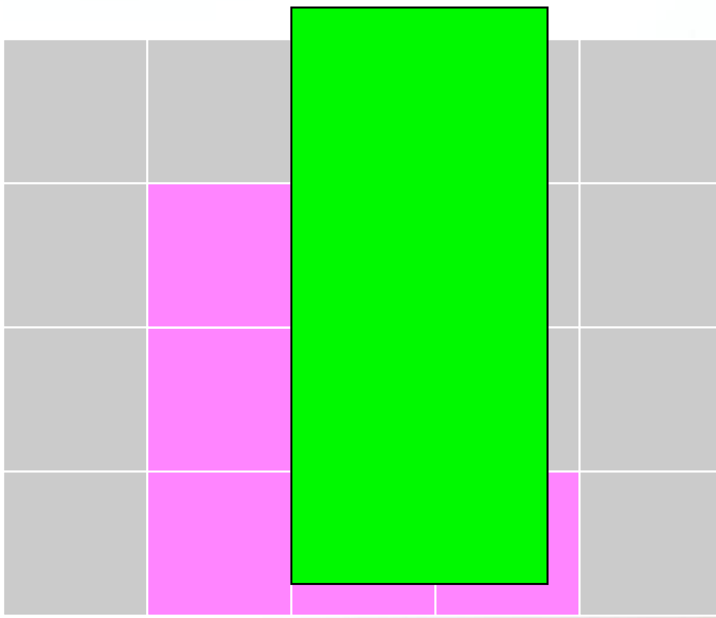
Depth Buffer Operation -- First Polygon



Pink Triangle -- pixel values assigned

Depth Buffer Operation – Second Polygon

Frame



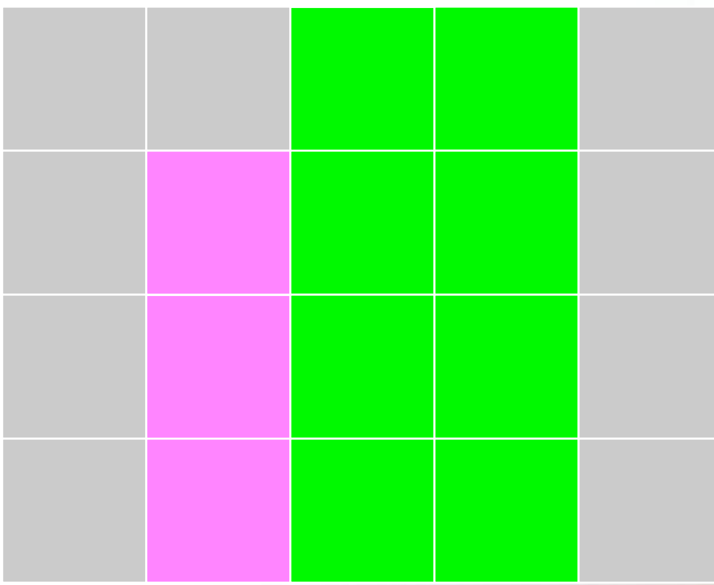
Depth

∞	∞	10	9	∞
∞	24	9	8	∞
∞	26	8	7	∞
∞	28	7	6	∞

Green Rectangle -- depths computed at pixel centers

Depth Buffer Operation – Second Polygon

Frame



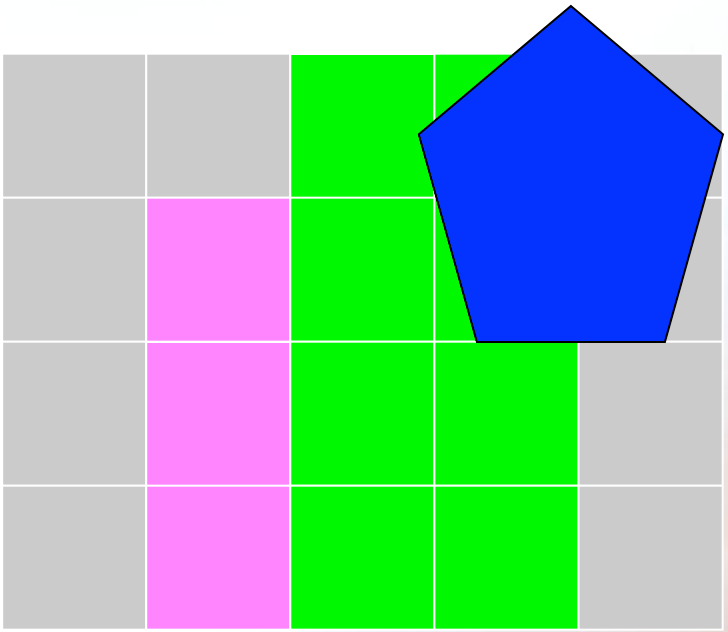
Depth

∞	∞	10	9	∞
∞	24	9	8	∞
∞	26	8	7	∞
∞	28	7	6	∞

Green Rectangle -- pixel values assigned: NOTE REPLACEMENTS!

Depth Buffer Operation – Third Polygon

Frame



Depth

68 not stored

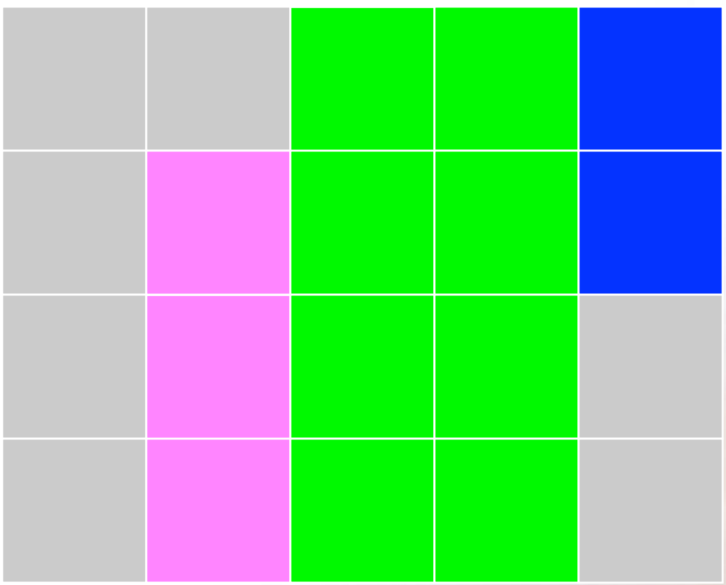
∞	∞	10	9	72
∞	24	9	8	75
∞	26	8	7	∞
∞	28	7	6	∞

71 not stored

Blue Pentagon -- depths computed at pixel centers

Depth Buffer Operation – Third Polygon

Frame



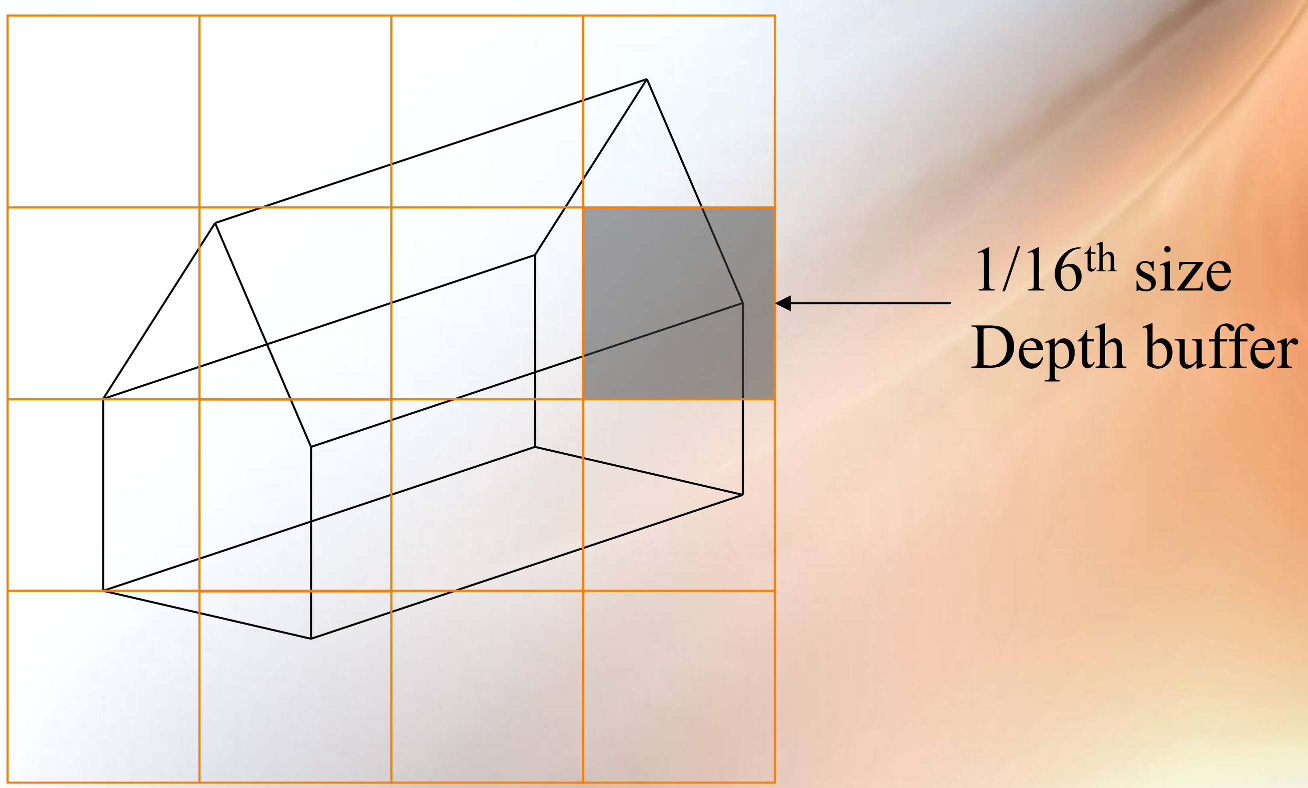
Depth

∞	∞	10	9	72
∞	24	9	8	75
∞	26	8	7	∞
∞	28	7	6	∞

Blue Pentagon -- pixel values assigned: NOTE ‘GOES BEHIND’!

Static Screen Subdivision

- **Use smaller depth buffer and repeat multiple times**



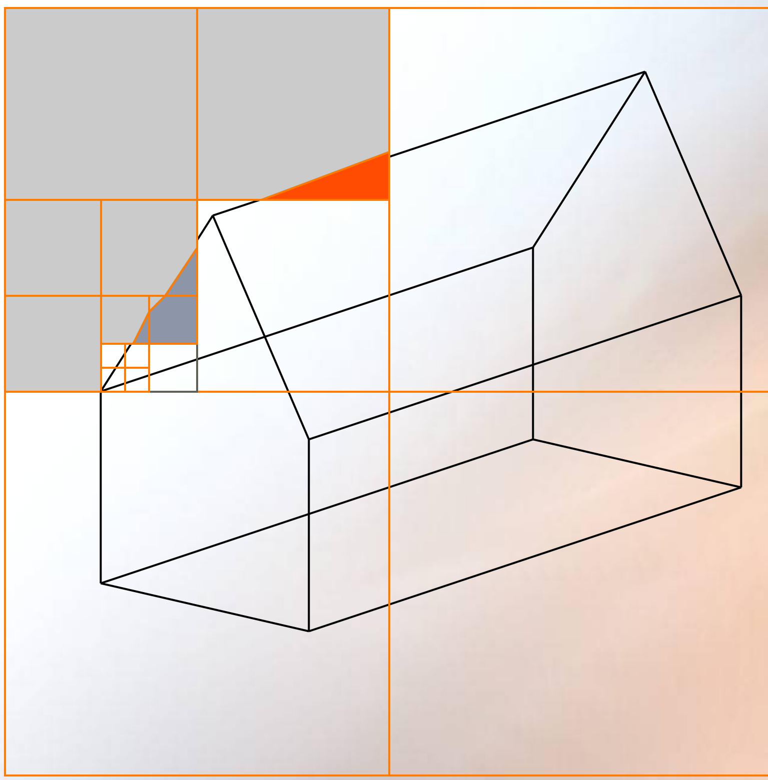
Adaptive Screen Subdivision

- Subdivide frame buffer into smaller chunks in detail areas.
- Implement as a recursive algorithm -- Warnock.
  - If frame area is simple, then just draw it.
  - If complex, then subdivide into quadrants and recurse.
- Simple = all background

  covered entirely by one polygon

  split into two regions by one polygon edge

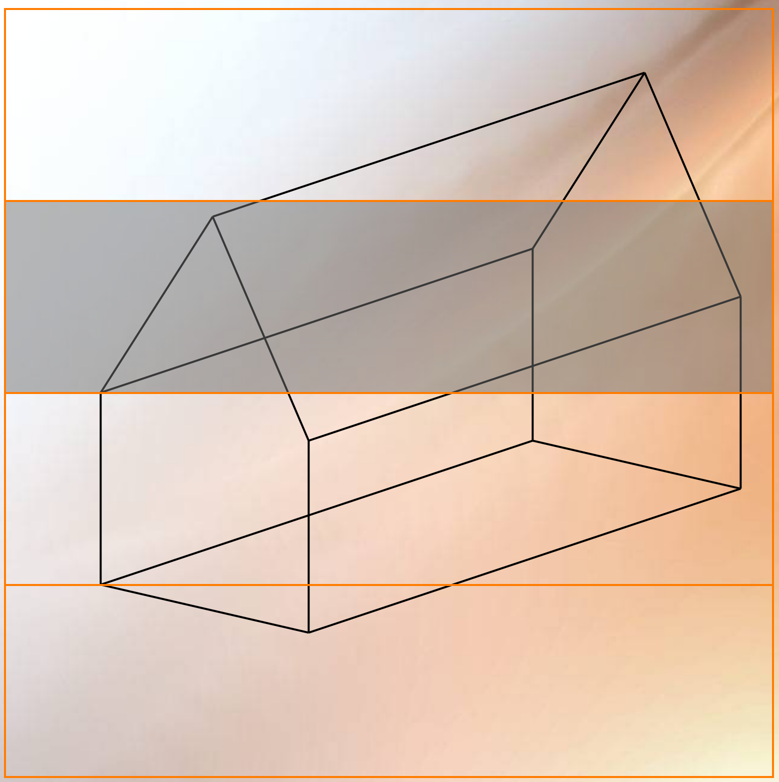
Warnock's Algorithm: Adaptive Screen Subdivision



Neat algorithm, but slow because recursion gets deep at many edges.

Static Screen Subdivision: Strips

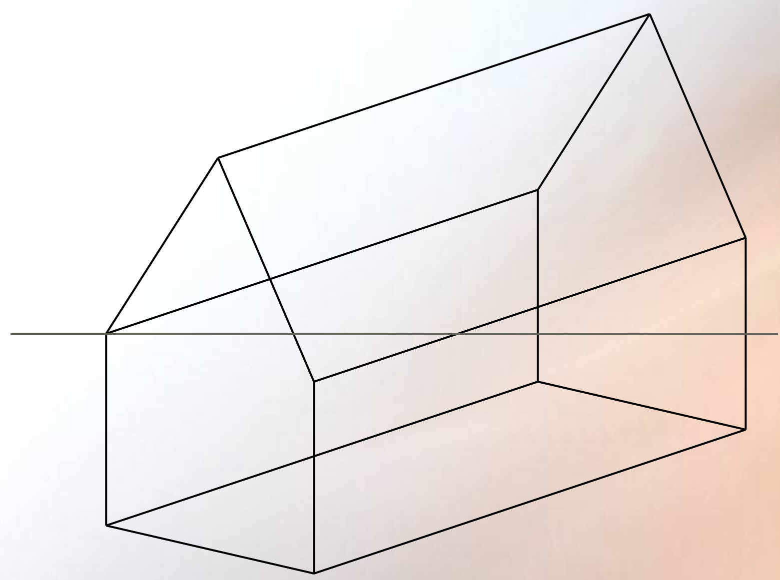
- **Use depth buffer consisting of a number of scan lines:**



Advantage is that image is created in full width strips, top to bottom.

Static Screen Subdivision: Scan-Lines

- In the limit, the strip can be a single scan-line.
- All polygons need be processed for each scan-line!

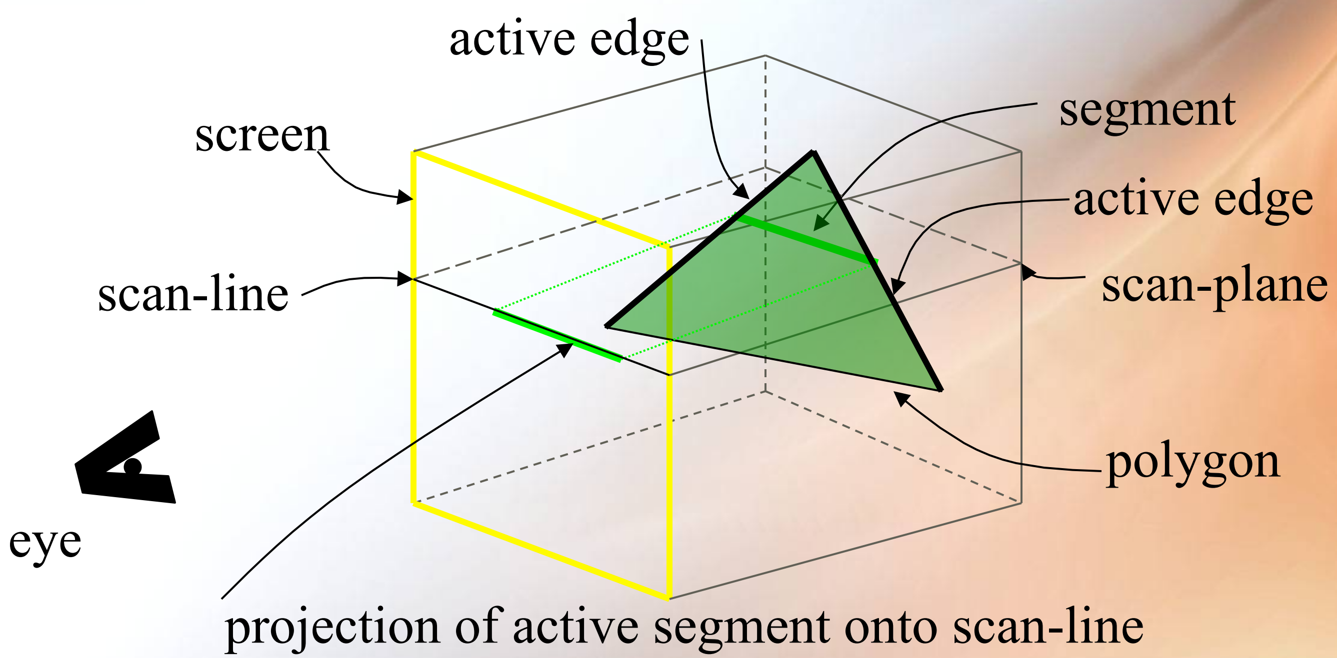


Watkins came up with a data structure that avoided this overhead.

Scan-Line Algorithm Definitions

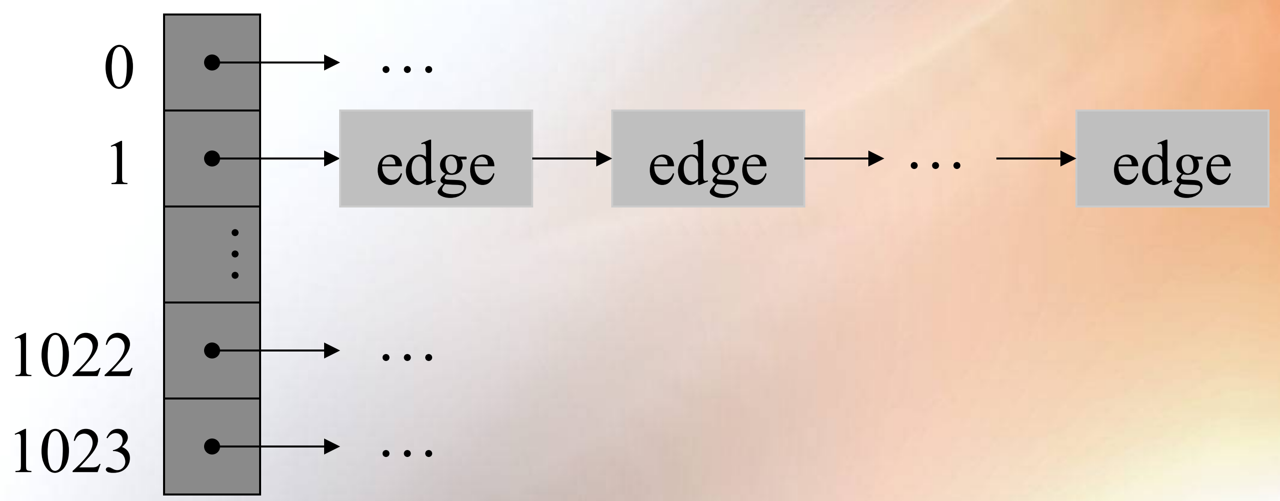
- <u>**Scan-Plane**</u> **: The projection of the scan line into the world.**
- <u>**Edge**</u> **: Line between two polygon vertices.**
- <u>**Active Edge**</u> **: An edge intersected by the scan-plane.**
- <u>**Segment**</u> **: Portion of a polygon between two active edges.**

Scan-Line Algorithm Definitions



Scan-Line Algorithm Overview

- **For each new image: vertically sort all polygon edges by y_s coordinate. Use a bucket sort with one bucket per scan-line of vertical resolution. Within the edge list, sort by x_s :**

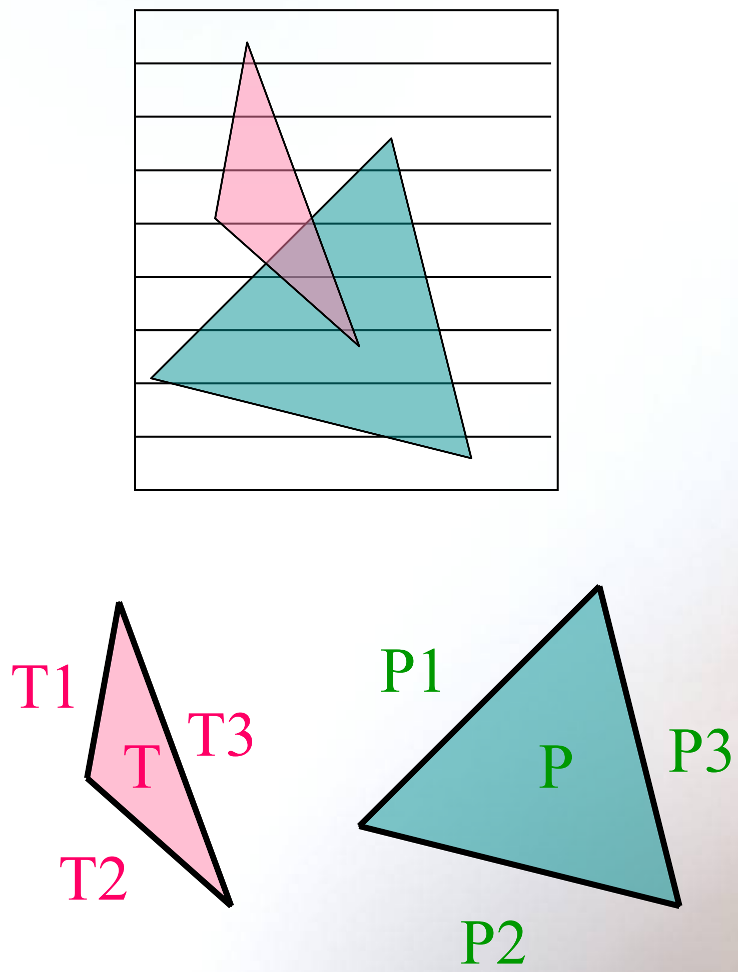


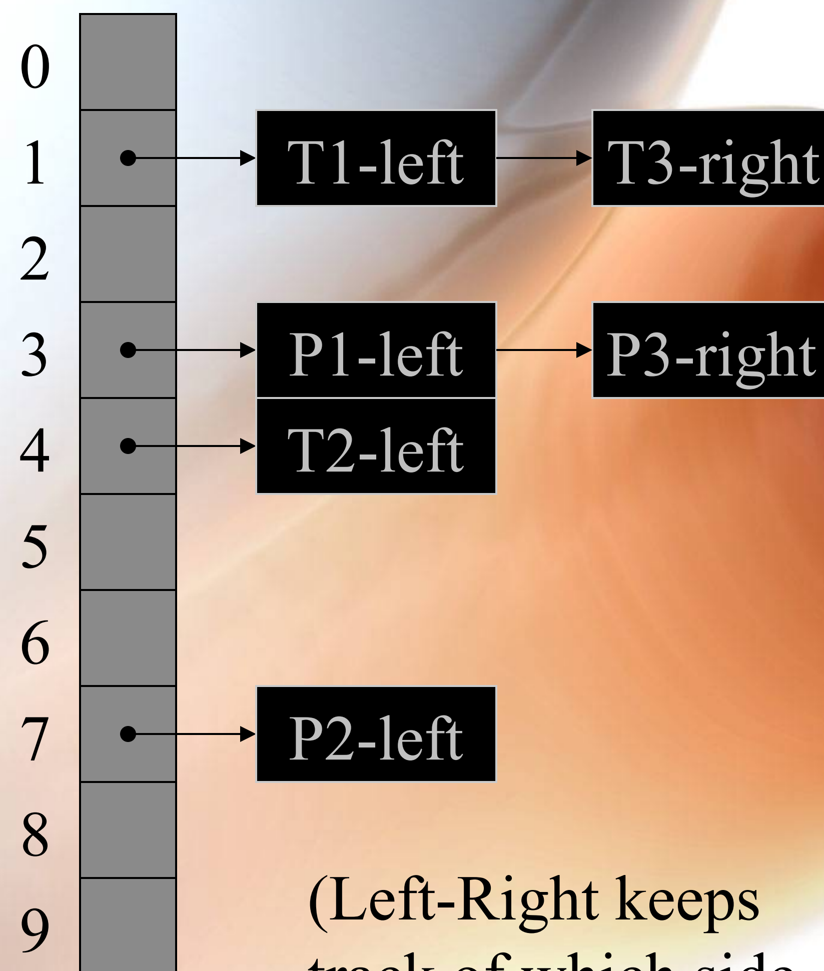
For each new image…

- For each new scan-line: Advance the active scan-line downward from the top. At each scan-line generate the active edge list based on additions, deletions, and modifications to the edge blocks already stored in that bucket.

- Let’s do an example with 10 scan-lines and just 2 triangle polygons called T and P:
  - T has three edges T1, T2, and T3
  - P has three edges P1, P2, and P3

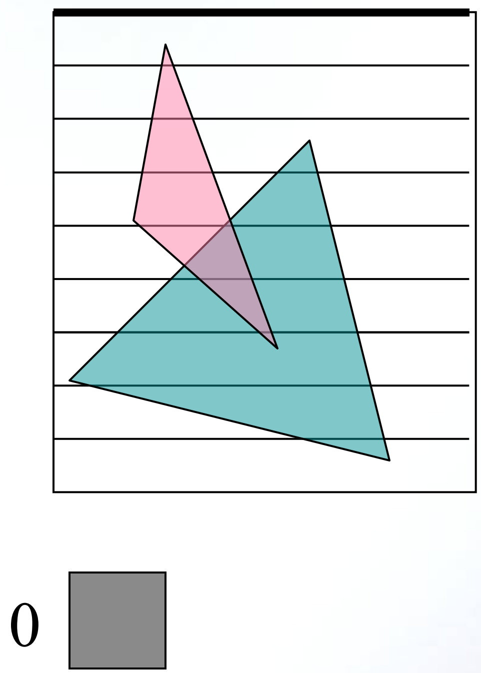
Initial State of Scan-Line Buckets (y-x sort)





(Left-Right keeps track of which side the edge bounds.)

For each Scan-Line, Build the Active Edge List

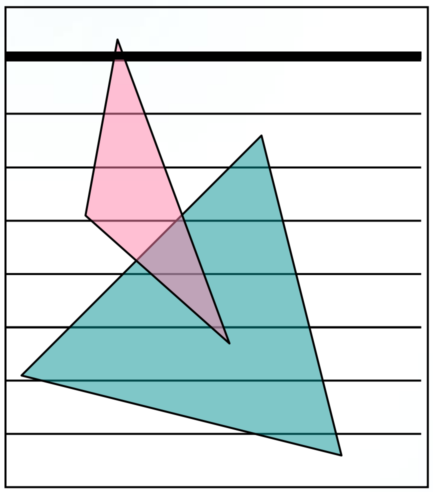


Scan-line 0

- **No additions**
- **No deletions**
- **No updates**

For each Scan-Line, Build the Active Edge List

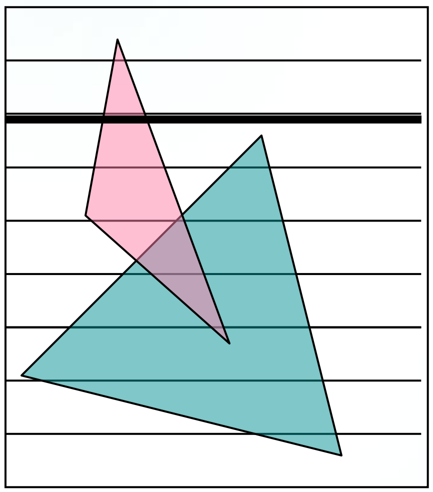
Scan-line 1



- **2 additions**
- **No deletions**
- **No updates**

For each Scan-Line, Build the Active Edge List

Scan-line 2

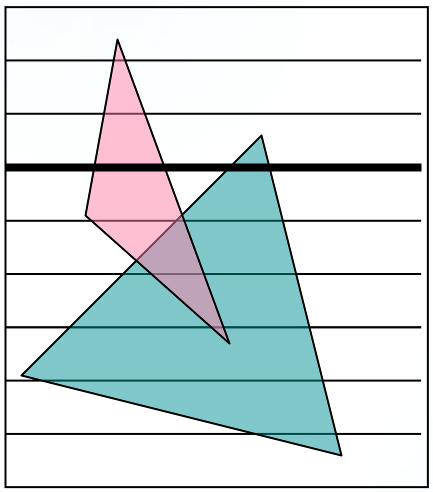


- **No additions**
- **No deletions**
- **2 updates**

2 → T1-left → T3-right

Update x   Update x

For each Scan-Line, Build the Active Edge List

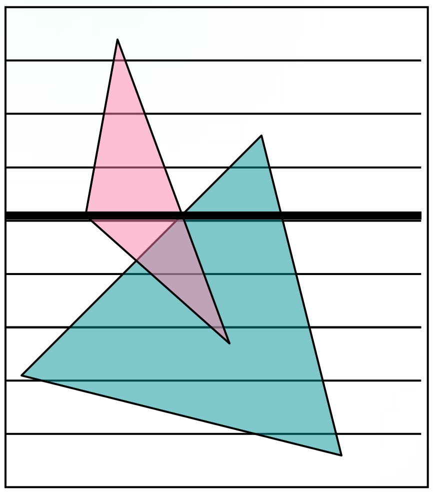


Scan-line 3

- **2 additions**
- **No deletions**
- **2 updates**

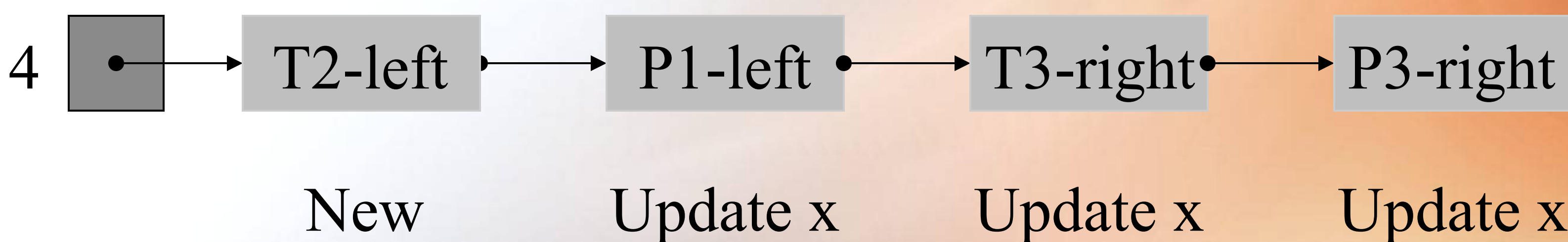
3	T1-left	T3-right	P1-left	P3-right
	Update x	Update x	New	New

For each Scan-Line, Build the Active Edge List



Scan-line 4

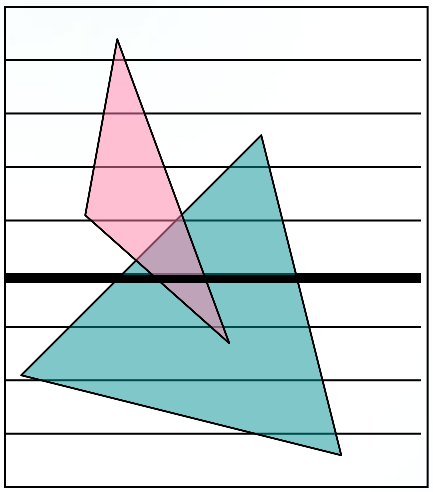
- **1 addition**
- **1 deletion**
- **3 updates**



Note that these two blocks are re-sorted to maintain x order.

For each Scan-Line, Build the Active Edge List

Scan-line 5



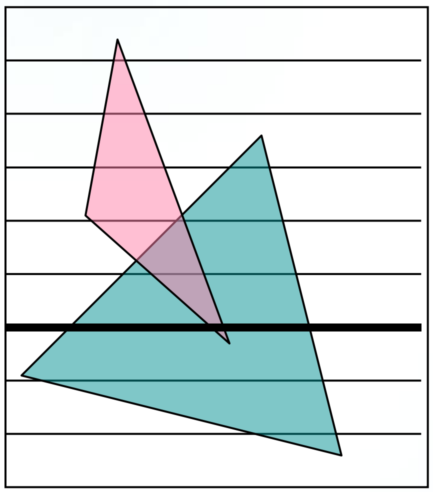
- **No additions**
- **No deletions**
- **4 updates**

5 → P1-left → T2-left → T3-right → P3-right

Update x | Update x | Update x | Update x

Note that these two blocks are re-sorted to maintain x order.

For each Scan-Line, Build the Active Edge List



Scan-line 6

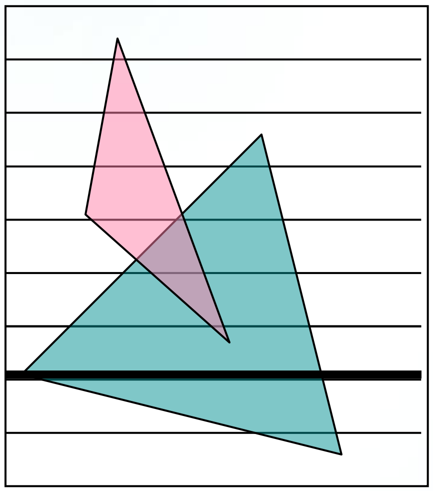
- **No additions**
- **No deletions**
- **4 updates**

6 → P1-left → T2-left → T3-right → P3-right

P1-left	T2-left	T3-right	P3-right
Update x	Update x	Update x	Update x

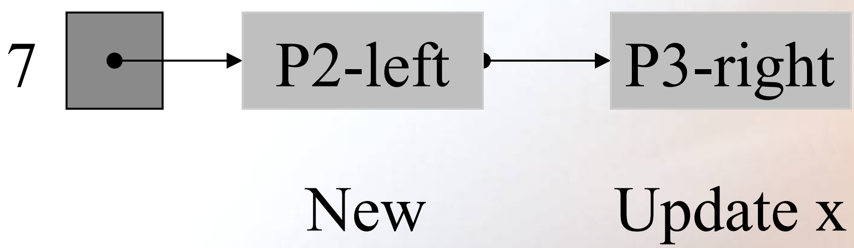
No re-sorting is needed.

For each Scan-Line, Build the Active Edge List



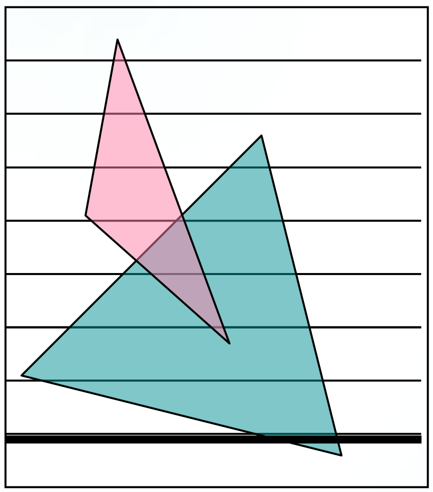
Scan-line 7

- **1 addition**
- **3 deletions**
- **1 update**



For each Scan-Line, Build the Active Edge List

Scan-line 8

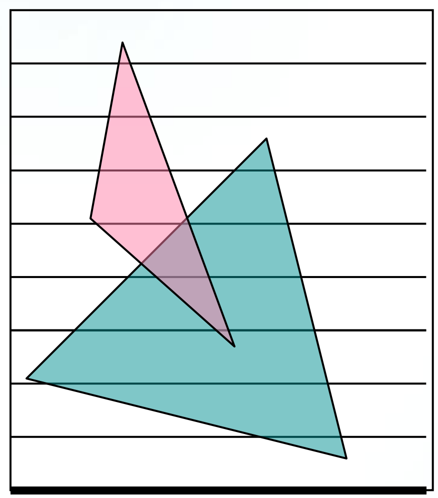


- **No additions**
- **No deletions**
- **2 updates**

8 [•] → P2-left → P3-right

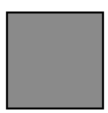
Update x   Update x

For each Scan-Line, Build the Active Edge List



Scan-line 9

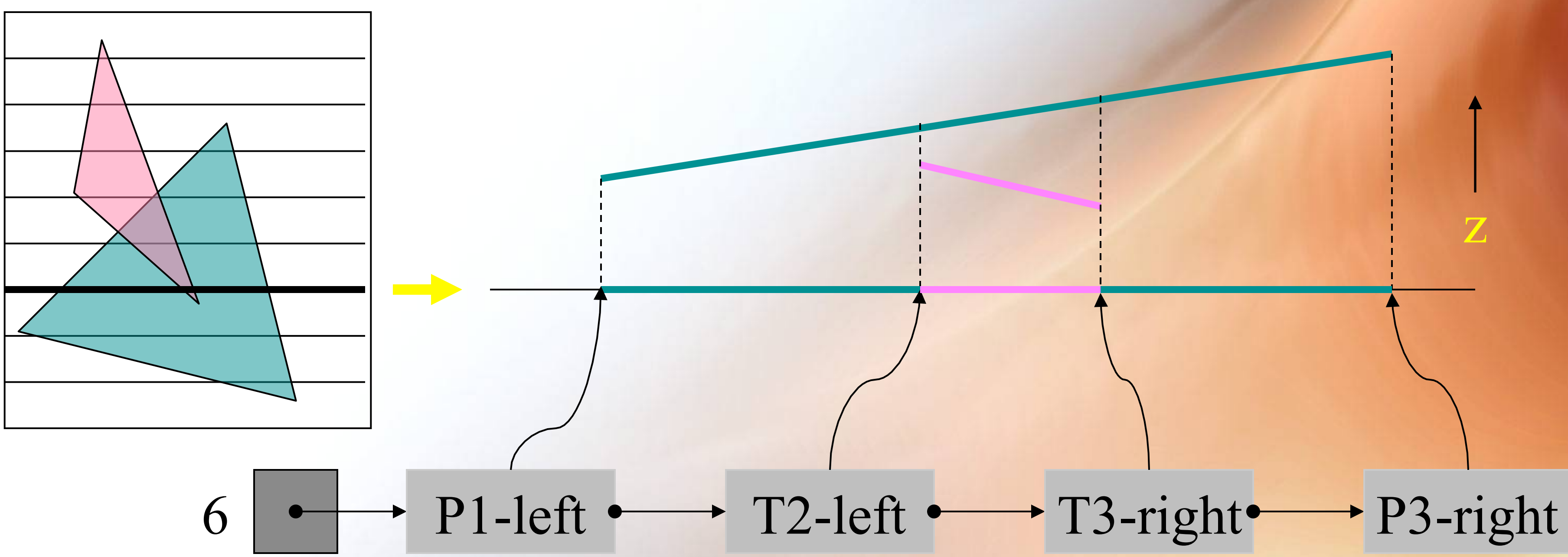
- **No additions**
- **2 deletions**
- **No updates**

9 

Generate the Segment List

- For each scan line: scan the active scan-line left to right to determine visible segments or segment fragments, based on depth (smallest z) comparisons.

For each scan-line...

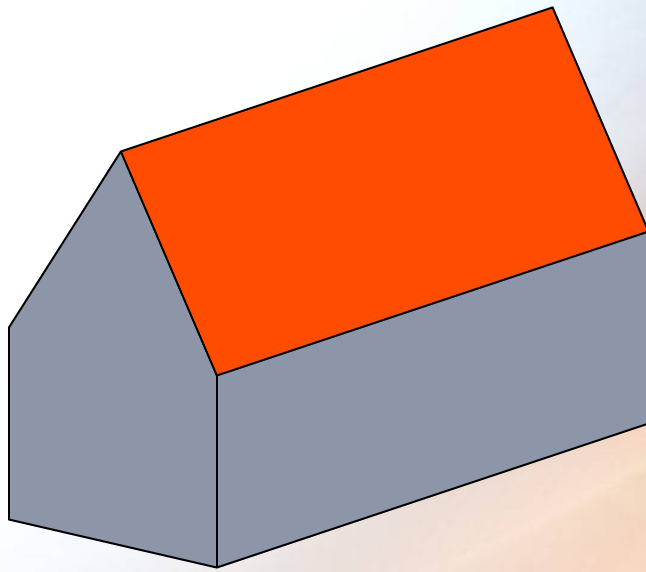


Painter’s Algorithm

- Sort polygons on distance from viewer.
- Rasterize polygons into frame buffer in sorted order from furthest to closest.

- Doesn’t always work, why do it?
  – Sorting is done prior to rendering.
  – No extra depth buffer memory or pixel depth checking (i.e., no special hardware needed –this was before OpenGL cards…)

Painter’s Example -- House



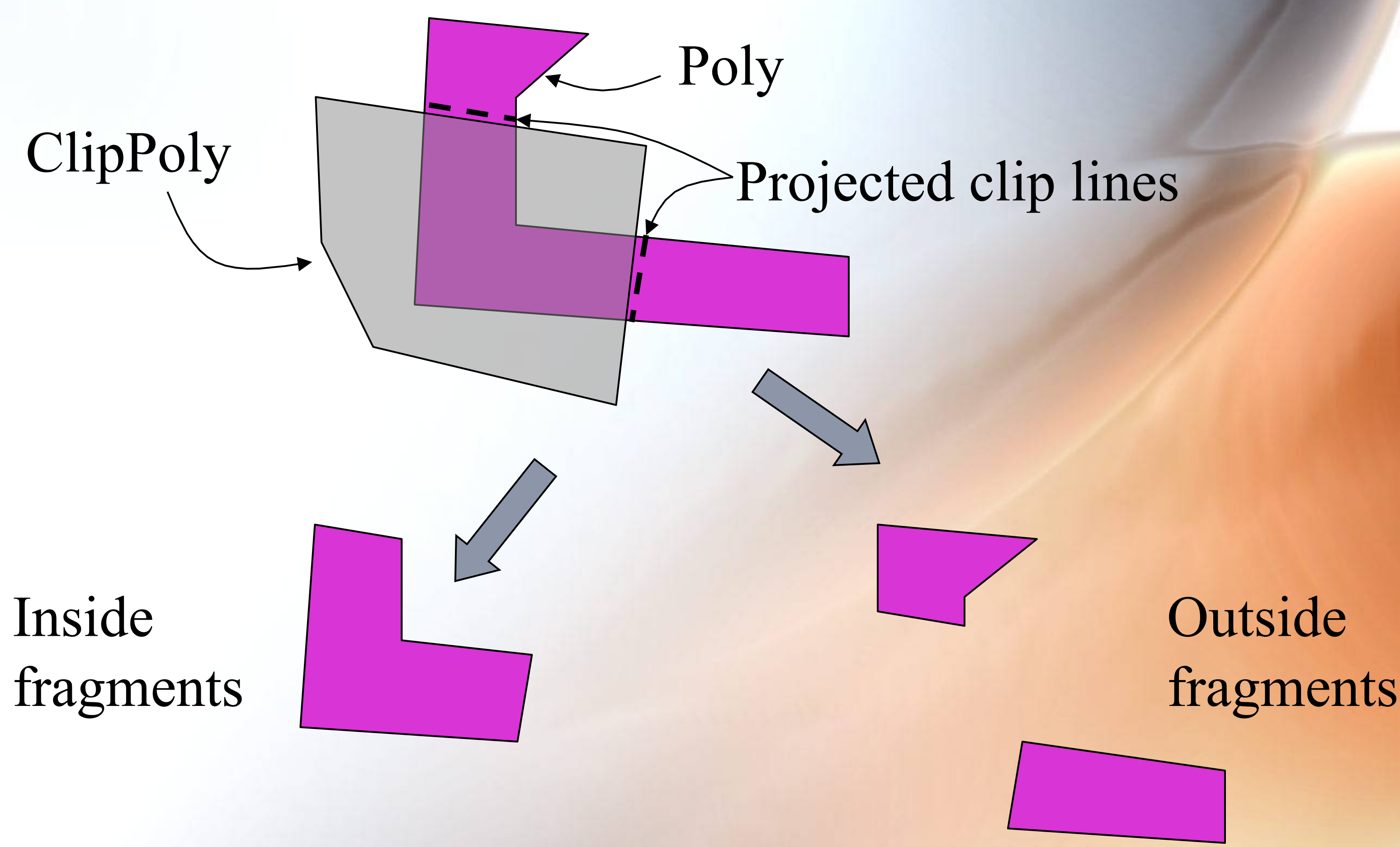
Exact Algorithm -- Atherton and Weiler

- Screen subdivision by projected polygon outlines.
- Uses each polygon as a *cookie-cutter* on remainder of scene.
- Within each cookie-cutter polygon:

  if remainder of scene is behind cookie-cutter polygon, then draw the cookie-cutter polygon

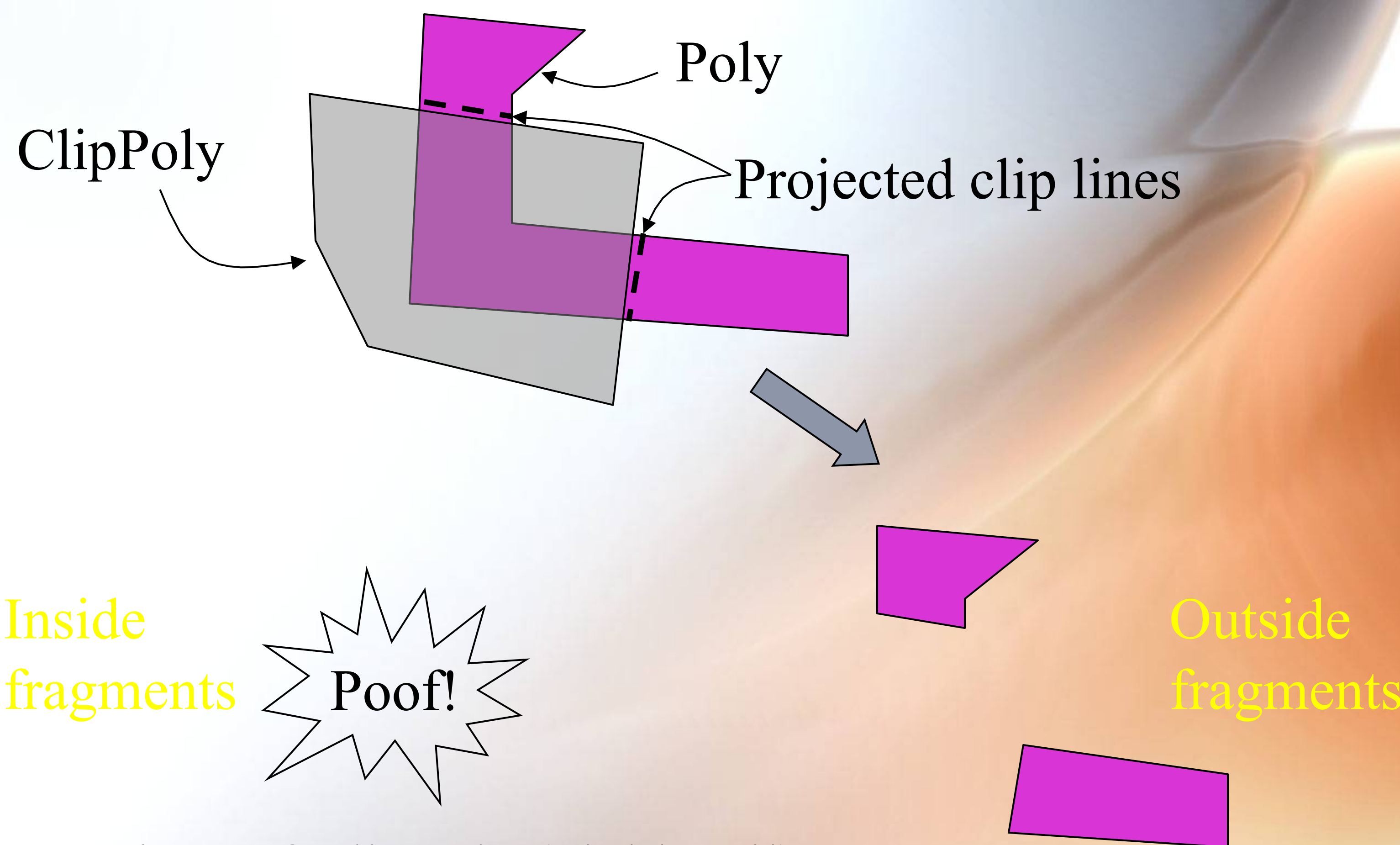
  else re-enter the algorithm with another polygon as the cookie-cutter.

Clipping a Polygon with ClipPoly



Append these to OutList

Clipping a Polygon with ClipPoly

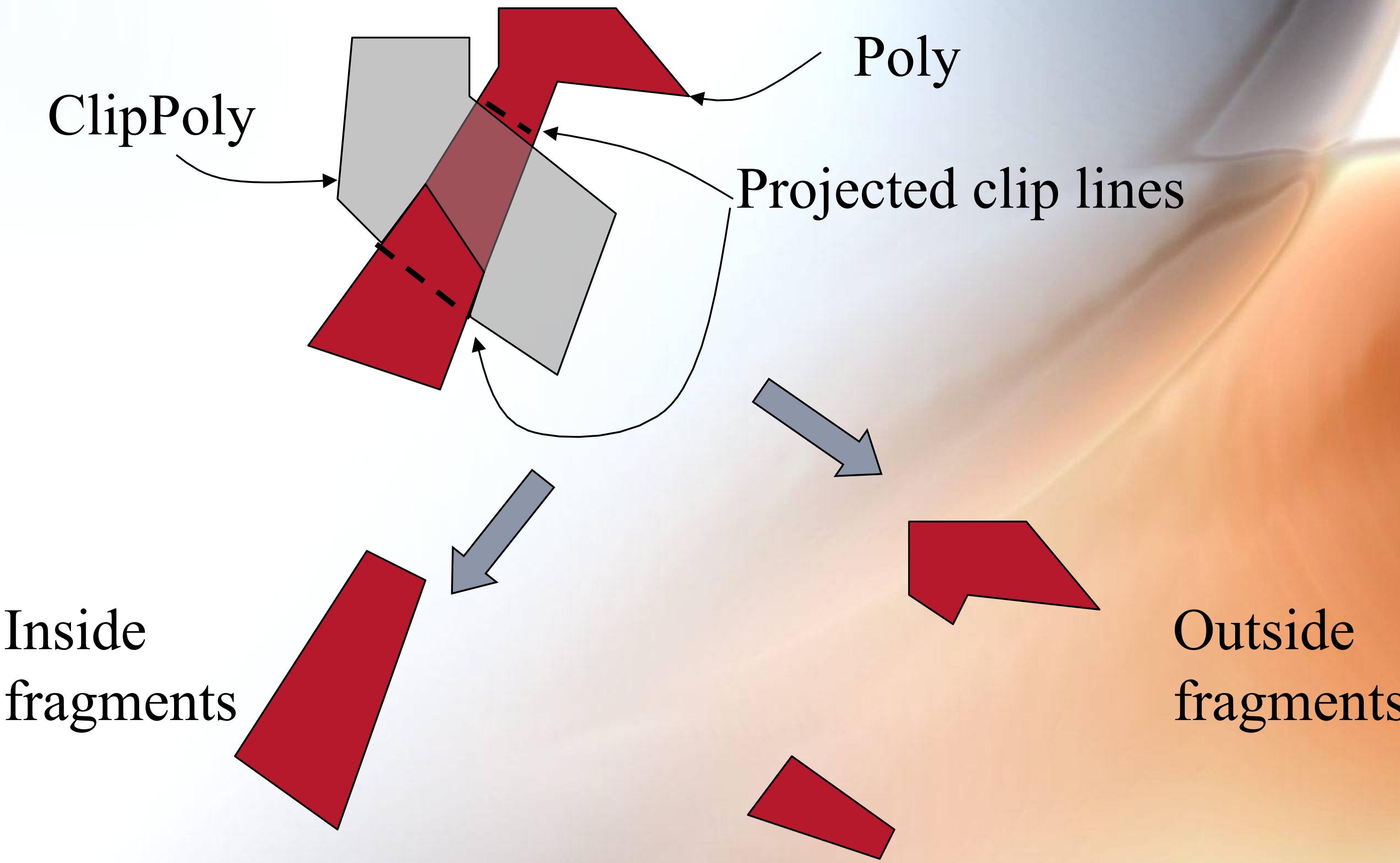


Clip to plane of ClipPoly (yields Nil)

<u>Append to InList (Nil)</u>

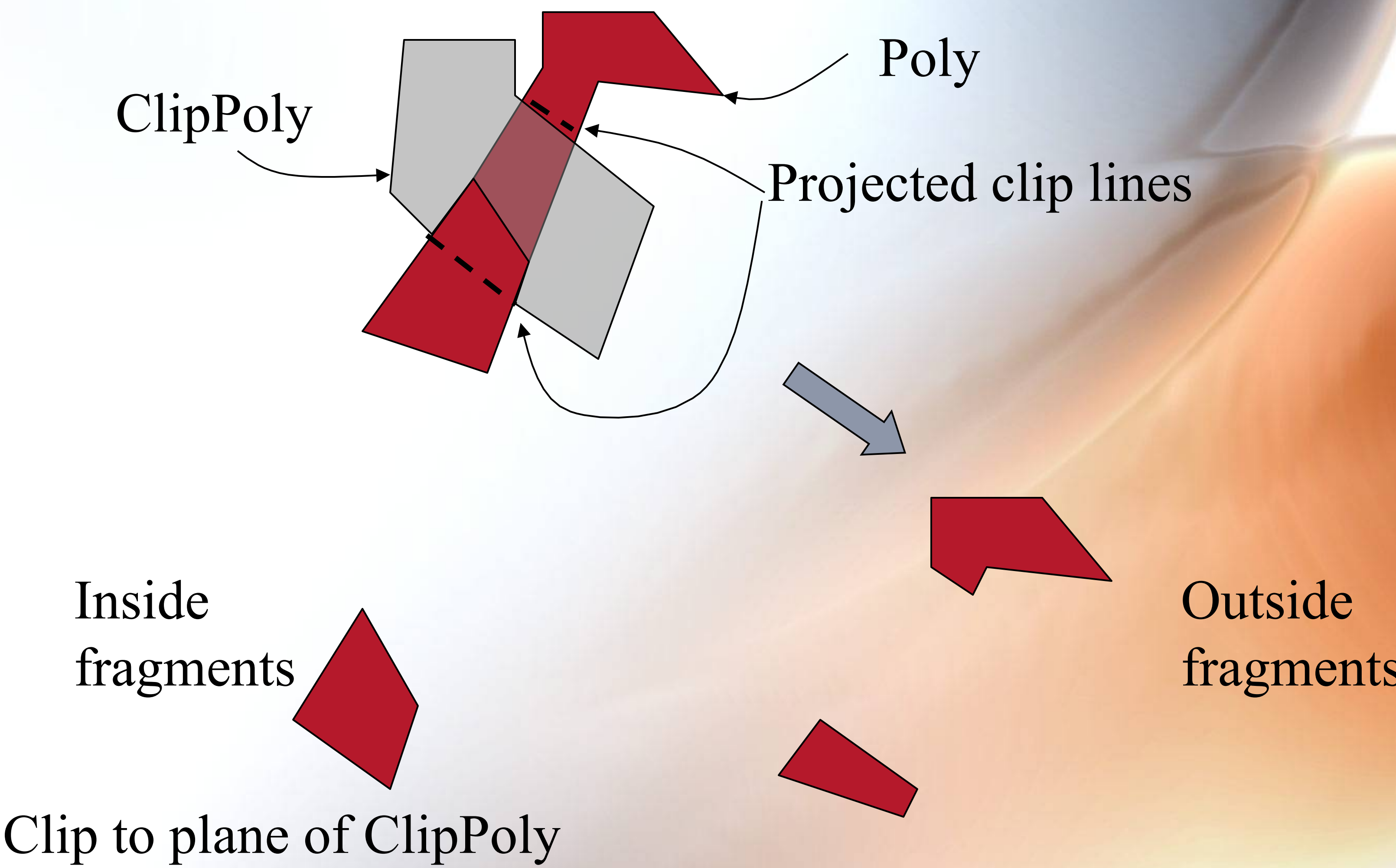
<u>Append these to OutList</u>

Clipping a Penetrating Polygon with ClipPoly



Append these to OutList

Clipping a Penetrating Polygon with ClipPoly

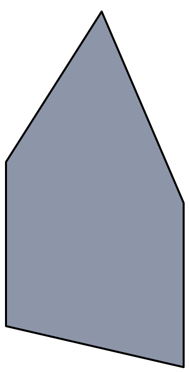


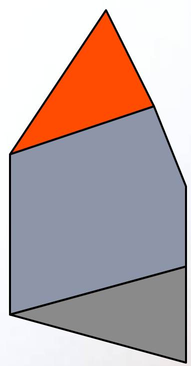
<u>Append this fragment to InList</u>

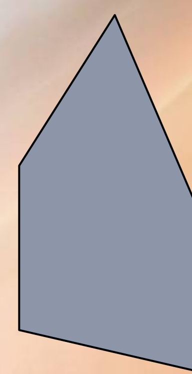
<u>Append these to OutList</u>

House Example

ClipPoly | Inside fragments | Draw result

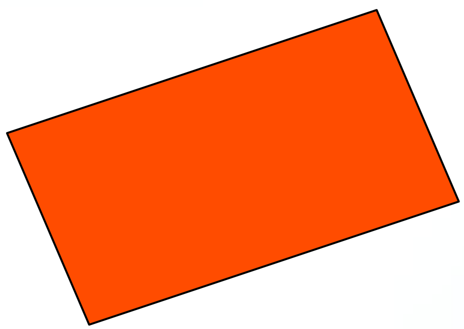




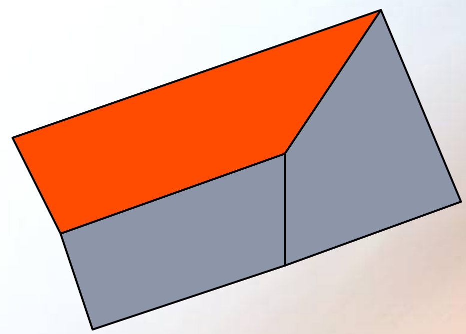


House Example

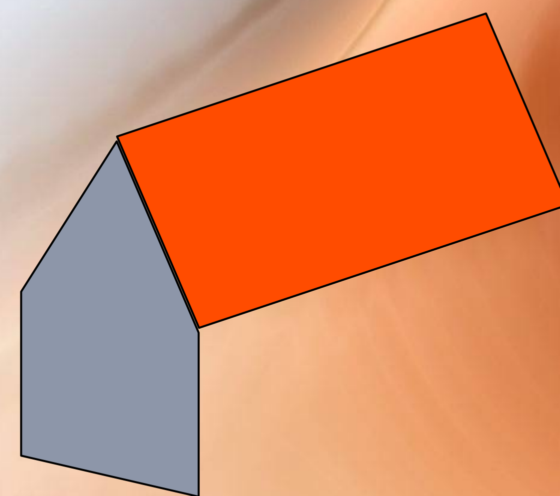
ClipPoly



Inside fragments

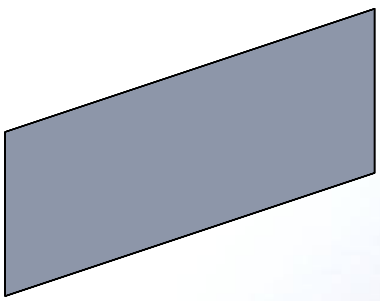


Draw result

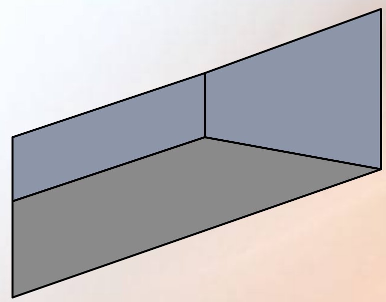


House Example

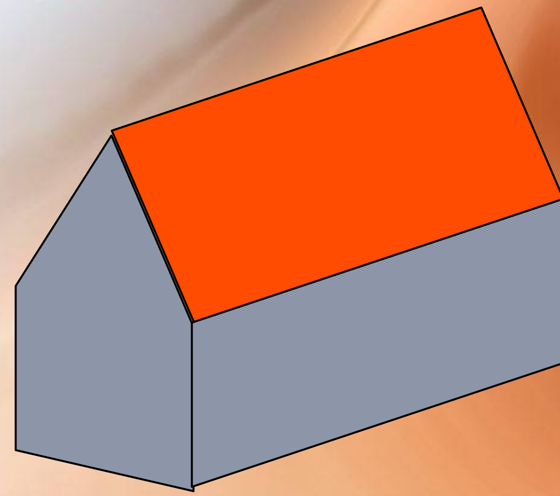
ClipPoly



Inside fragments



Draw result



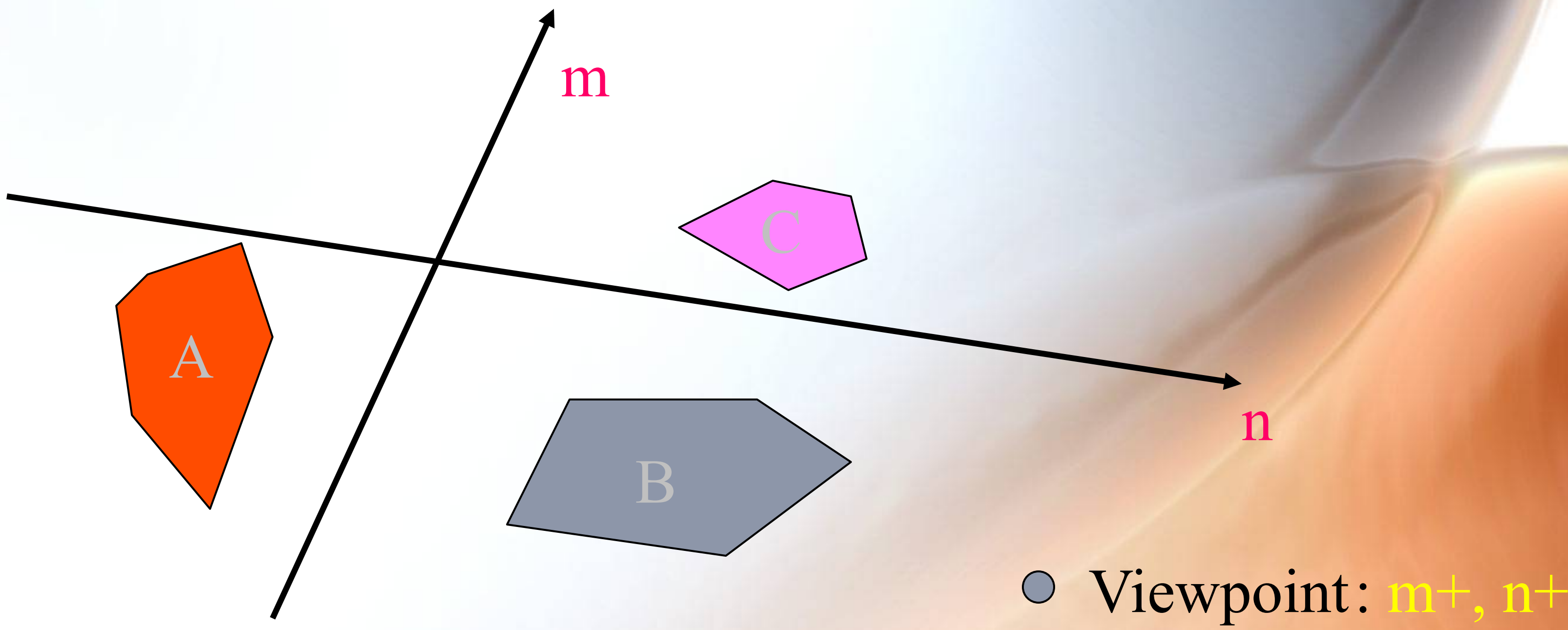
Pre-Visibility Culling

- A family of techniques the attempt to cull as many invisible polygons BEFORE they are even sent into the rendering pipeline.
- Enhanced rendering performance, e.g., for games.
- Often combined with binary space partitioning

Binary Space Partitioning

- There exist scenes in which visibility can be predetermined and is independent of view (camera) viewpoint.
- Main requirement is *linear separability*: polygons are either on one side of a separating plane or another.
- Basic idea: compute visibility in advance, then use this structure to pre-define the display ordering (back to front).
- Data structure built is called a *binary space partition* tree or *BSP-tree*.

Separating Planes and the Viewpoint



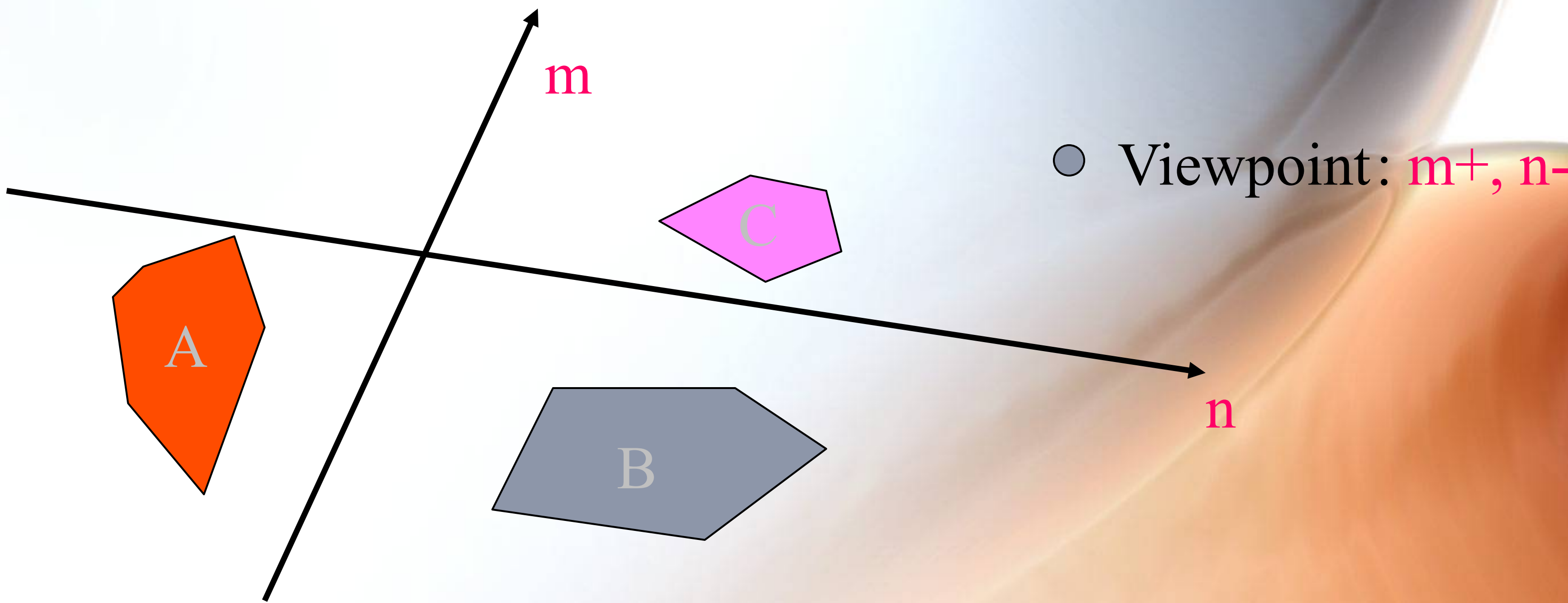
- **Find separating planes (m, n) such that each object (A, B, C) is in its own region of space.**

(B, C) > A
wrt m

B > C
wrt n

Therefore B > C > A
wrt Viewpoint: m+, n+

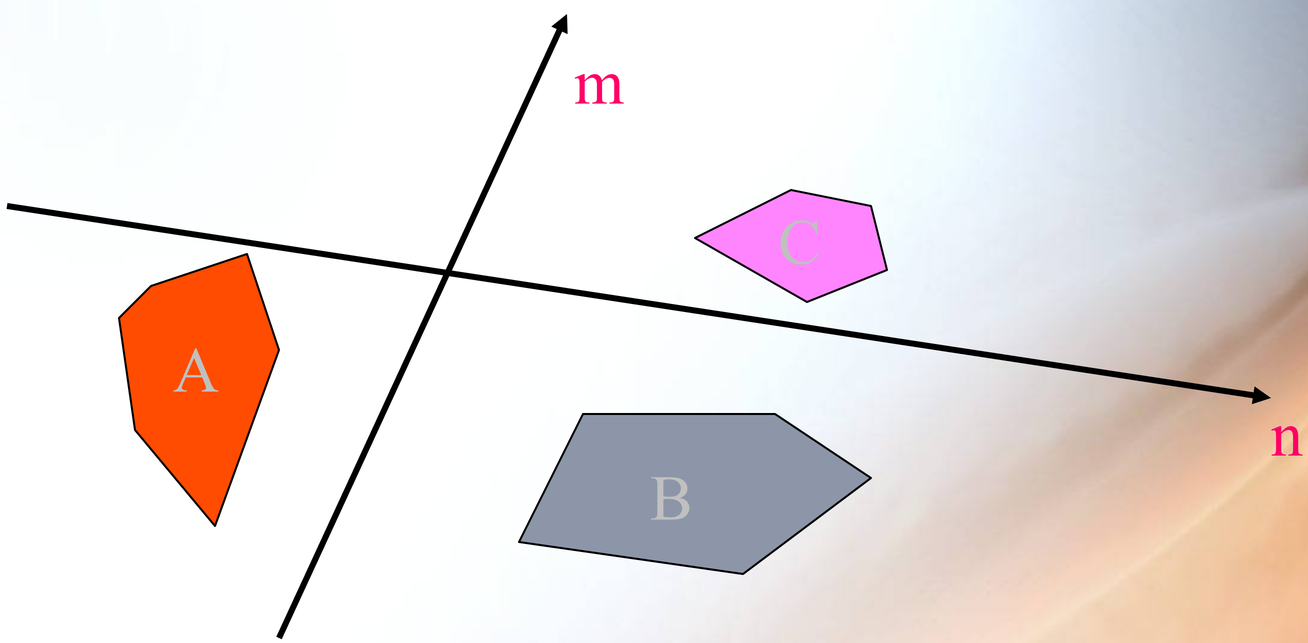
Repeat for each Region in which Viewpoint may Lie



(B, C) > A
wrt m

C > B
wrt n

Therefore C > B > A
wrt Viewpoint: m+, n-



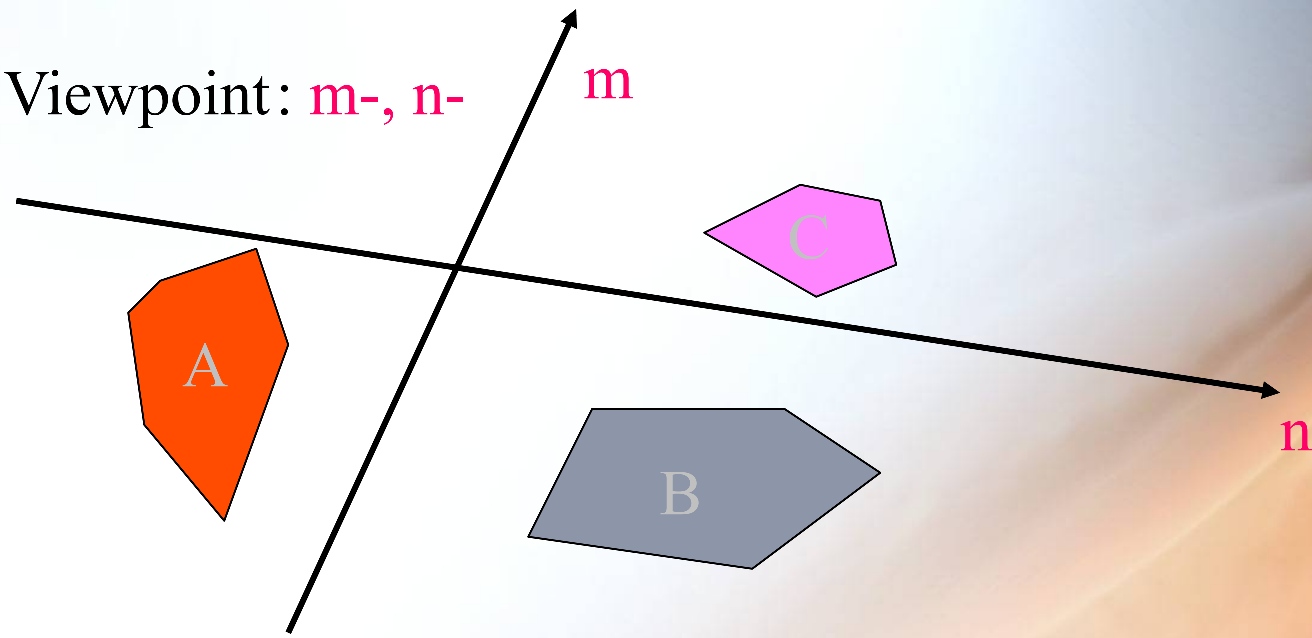
- Viewpoint : m-, n+

A > (B, C)
wrt m

B > C
wrt n

Therefore A > B > C
wrt Viewpoint: m-, n+

- Viewpoint: m-, n-

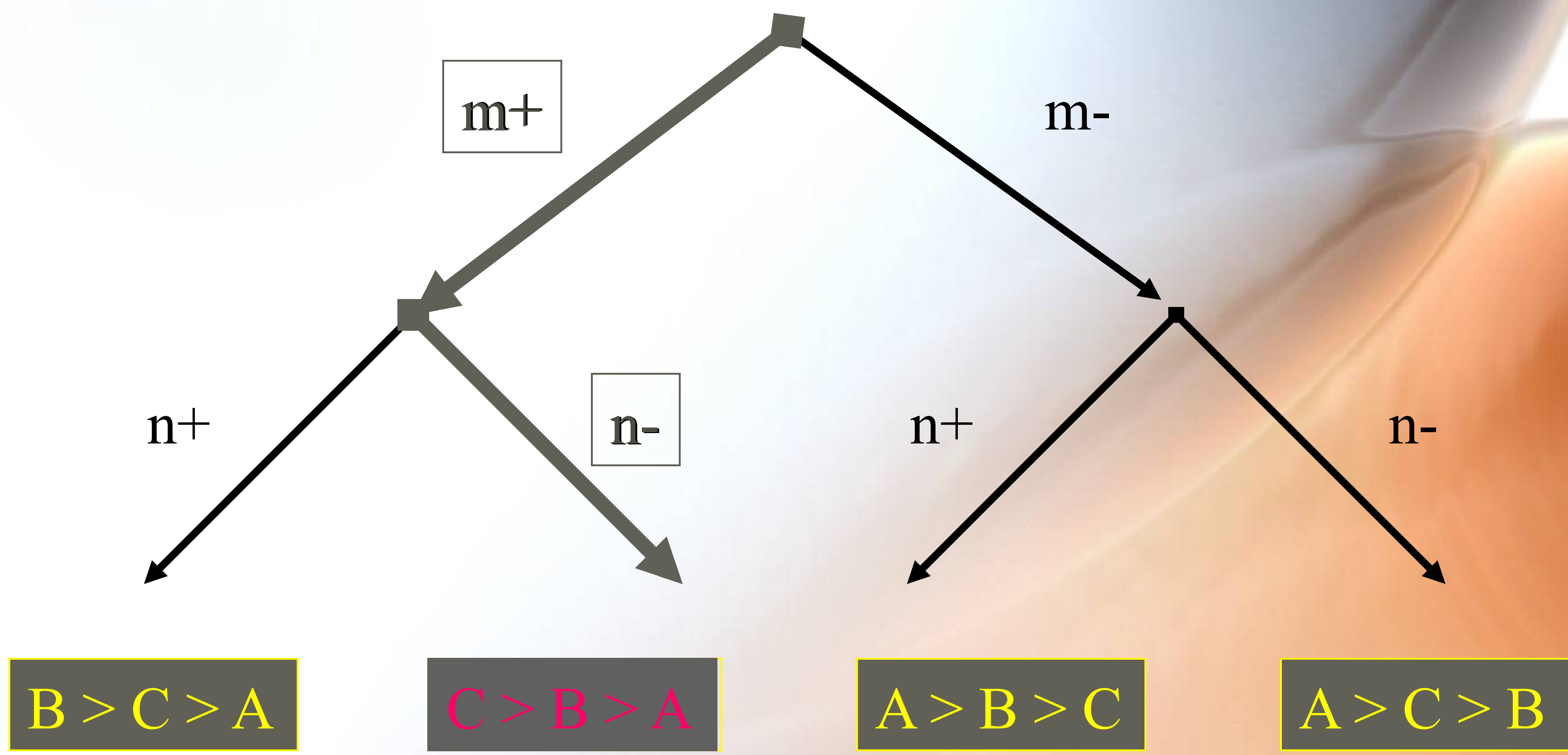


A > (B, C)
wrt m

C > B
wrt n

Therefore A > C > B
wrt Viewpoint: m-, n-

Combine all into a Binary Space Partition (BSP) Tree

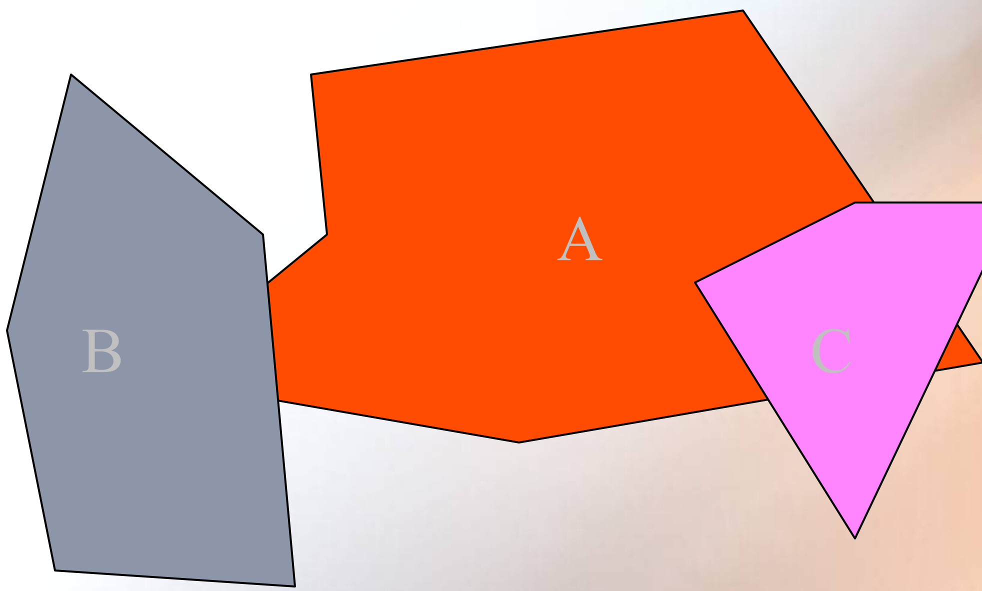


So as soon as we compute what sides of the separating planes the viewpoint is on, we immediately know the object rendering order that guarantees correct visibility.

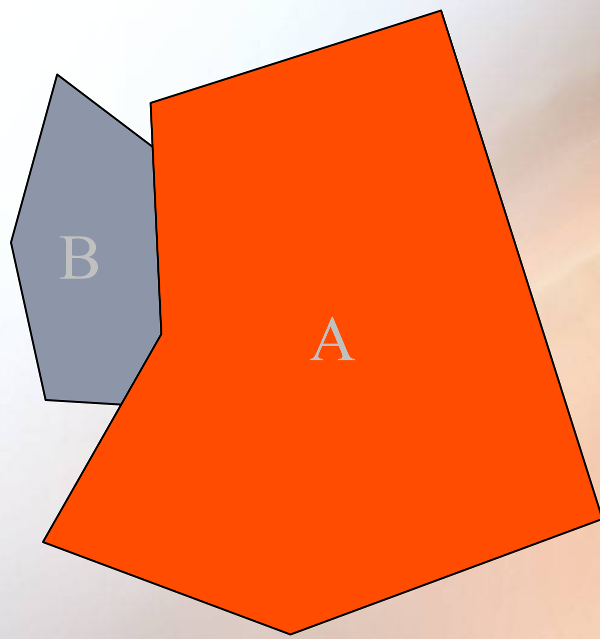
Display in Back-to-Front Order

Order: C > B > A

Assume that back faces are culled;
A, B, C may even be *convex clusters* of polygons



Order: A > C > B



This is Basically the “DOOM” Graphics Engine!

- Extend to 3D polygons.
- Complex environments are pre-processed to create the BSP tree.
- In practice, slightly more complicated trees are build to allow crossing features (walls).
- Use nice textures on surfaces (see this later).

http://symbolcraft.com/pjl/graphics/bsp/

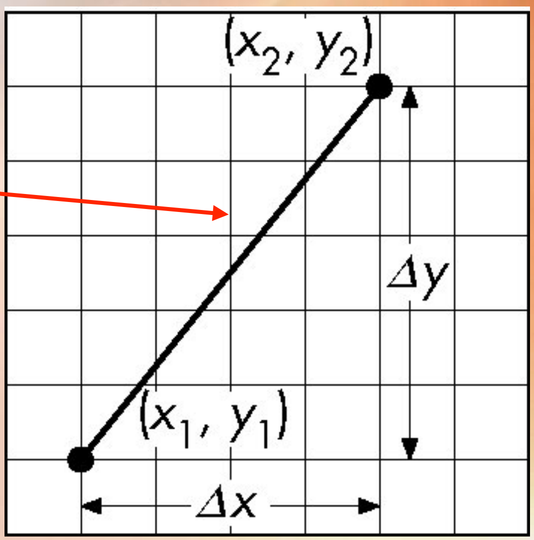
Rasterization

- Rasterization (scan conversion)
  - Shade pixels that are inside object specified by a set of vertices
    - Line segments
    - Polygons: scan conversion = fill
- Shades determined by color, texture, shading model
- Here we study algorithms for determining the correct pixels starting with the vertices

Scan Conversion of Line Segments

- Start with line segment in window coordinates with integer values for endpoints
- Assume implementation has a `write_pixel` function

y = mx + h



$$m = \frac{\Delta y}{\Delta x}$$

DDA Algorithm

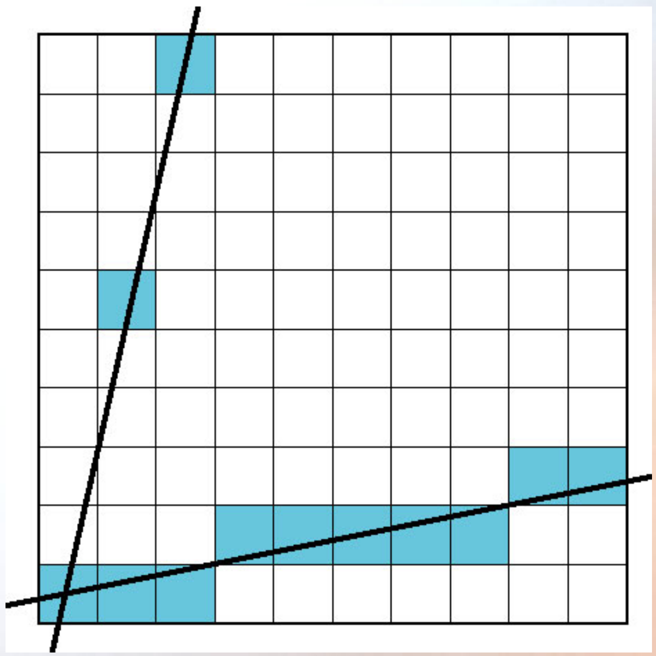
- Digital Differential Analyzer
  - DDA was a mechanical device for numerical solution of differential equations
  - Line $y=mx+h$ satisfies differential equation

    $dy/dx = m = \Delta y/\Delta x = y_2-y_1/x_2-x_1$
- Along scan line $\Delta x = 1$

```
for(x=x1; x<=x2; x++) {
    y+=m;
   write_pixel(x, round(y), line_color)
}
```

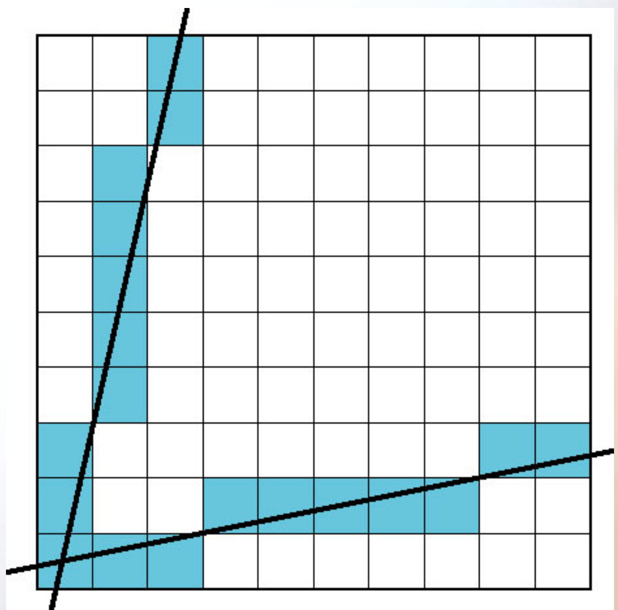
Problem

- DDA = for each x plot pixel at closest y
  - Problems for steep lines



Using Symmetry

- Use for $1 \geq m \geq 0$
- For $m > 1$, swap role of x and y
  - For each y, plot closest x

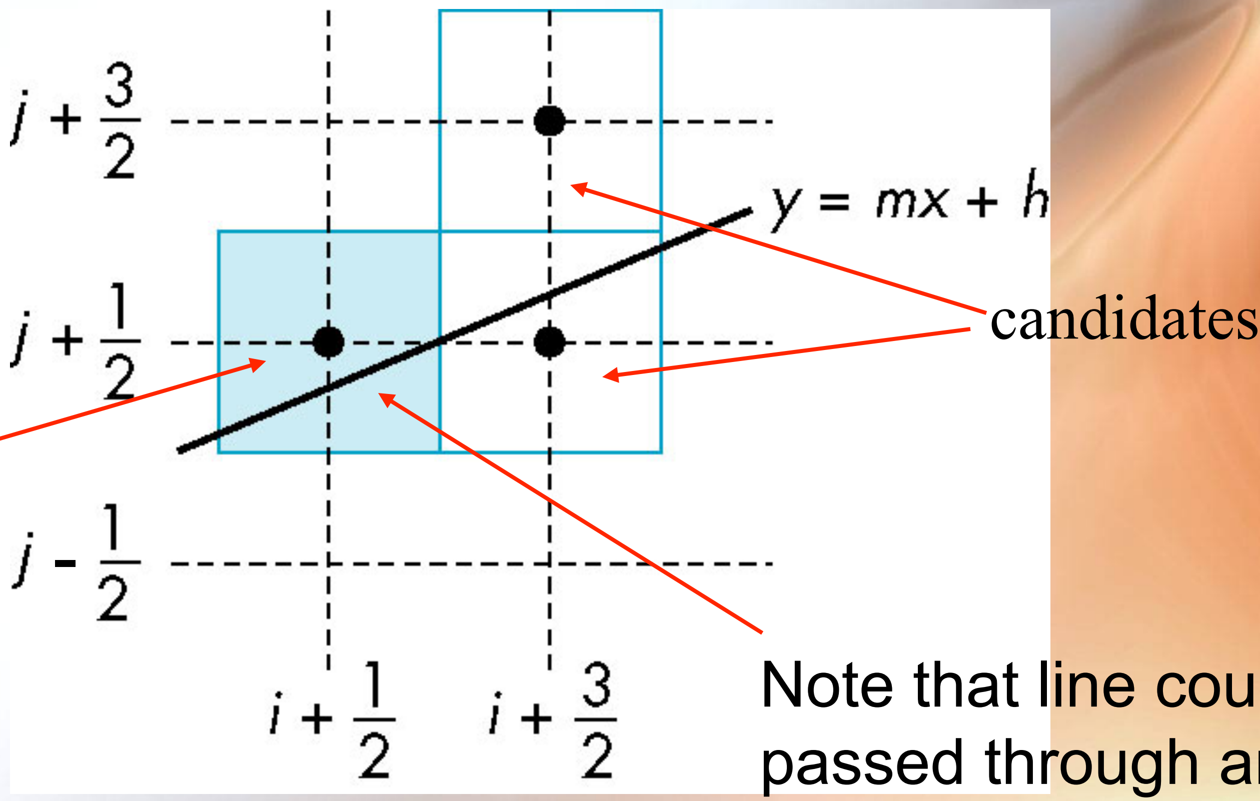


Bresenham's Algorithm

- DDA requires one floating point addition per step
- We can eliminate all fp through Bresenham's algorithm
- Consider only $1 \geq m \geq 0$
  - Other cases by symmetry
- Assume pixel centers are at half integers
- If we start at a pixel that has been written, there are only two candidates for the next pixel to be written into the frame buffer

Candidate Pixels

$1 \geq m \geq 0$



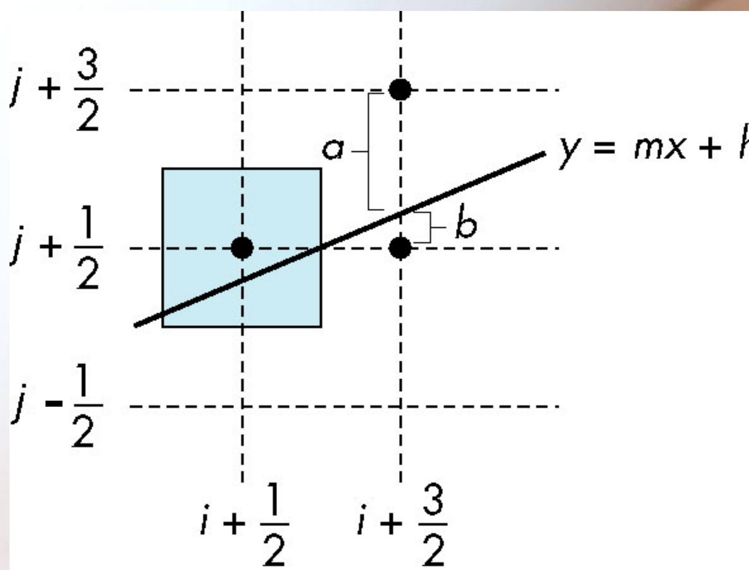
last pixel

Note that line could have passed through any part of this pixel

Decision Variable

$d = \Delta x(a-b)$

d is an integer
$d < 0$ use upper pixel
$d > 0$ use lower pixel



Incremental Form

- **More efficient if we look at d_k, the value of the decision variable at $x = k$**

$$d_{k+1} = d_k - 2\Delta y, \quad \text{if } d_k > 0$$
$$d_{k+1} = d_k - 2(\Delta y - \Delta x), \quad \text{otherwise}$$

- For each x, we need do only an integer addition and a test
- Single instruction on graphics chips

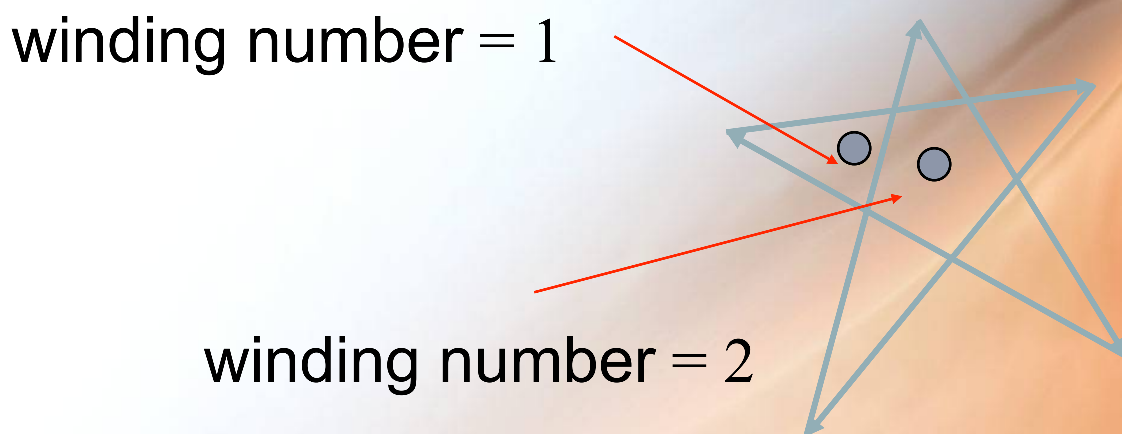
Polygon Scan Conversion

- Scan Conversion = Fill
- How to tell inside from outside
  - Convex easy
  - Nonsimple difficult
  - Odd even test
    - Count edge crossings
  - Winding number

odd-even fill

Winding Number

- Count clockwise encirclements of point



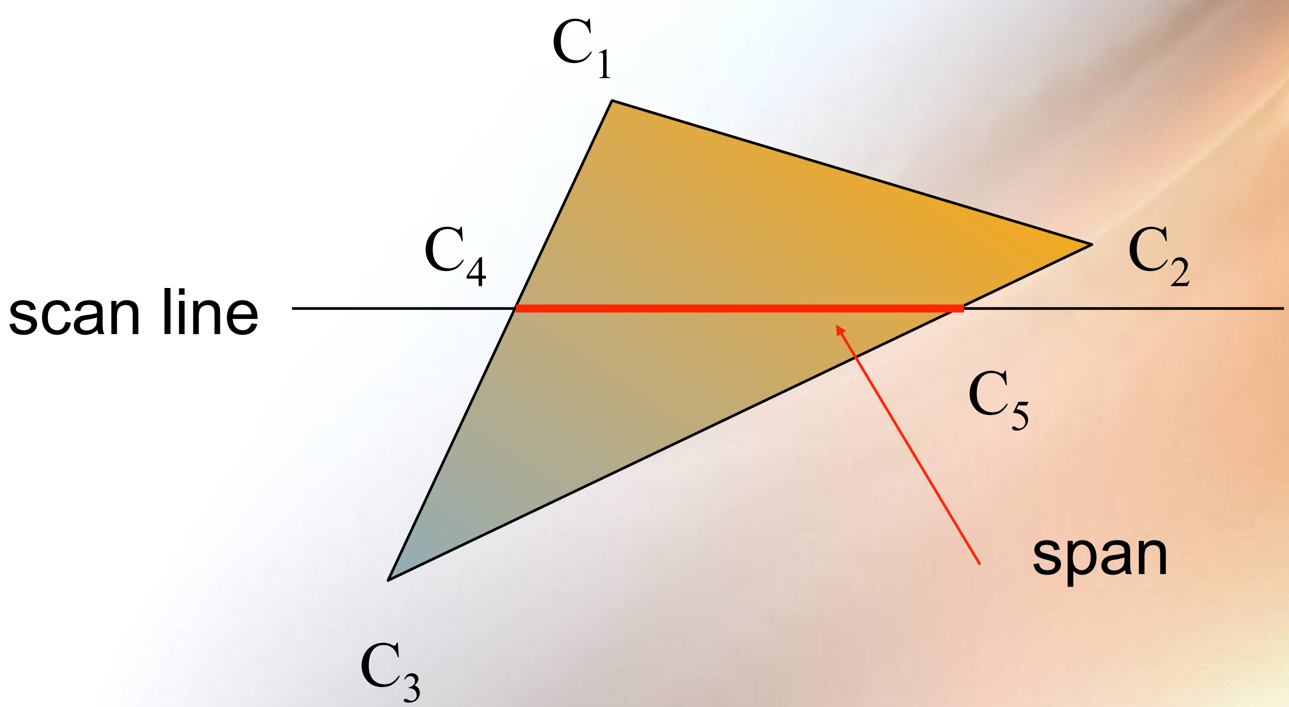
- Alternate definition of inside: inside if winding number ≠ 0

Filling in the Frame Buffer

- Fill at end of pipeline
  - Convex Polygons only
  - Nonconvex polygons assumed to have been tessellated
  - Shades (colors) have been computed for vertices (Gouraud shading)
  - Combine with z-buffer algorithm
    - March across scan lines interpolating shades
    - Incremental work small

Using Interpolation

C_1 C_2 C_3 specified by `glColor` or by vertex shading
C_4 determined by interpolating between C_1 and C_2
C_5 determined by interpolating between C_2 and C_3
interpolate between C_4 and C_5 along span



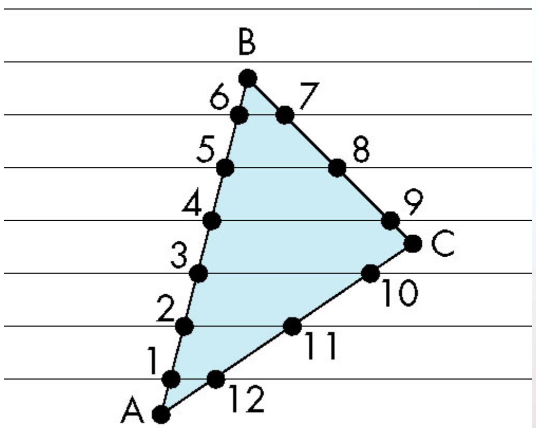
Flood Fill

- **Fill can be done recursively if we know a seed point located inside (WHITE)**
- **Scan convert edges into buffer in edge/inside color (BLACK)**

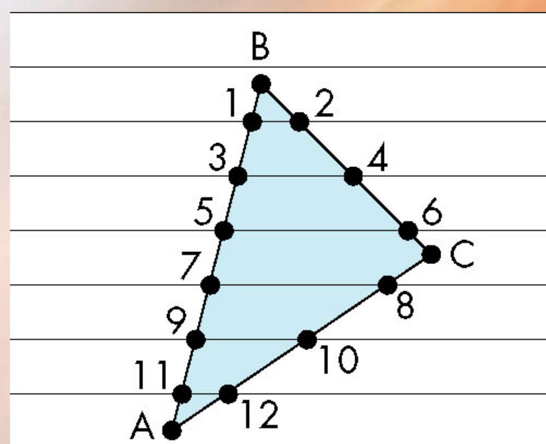
```
flood_fill(int x, int y) {
    if(read_pixel(x,y)= = WHITE)
{
        write_pixel(x,y,BLACK);
        flood_fill(x-1, y);
        flood_fill(x+1, y);
        flood_fill(x, y+1);
        flood_fill(x, y-1);
    }
}
```

Scan Line Fill

- Can also fill by maintaining a data structure of all intersections of polygons with scan lines
  - –Sort by scan line
  - –Fill each span

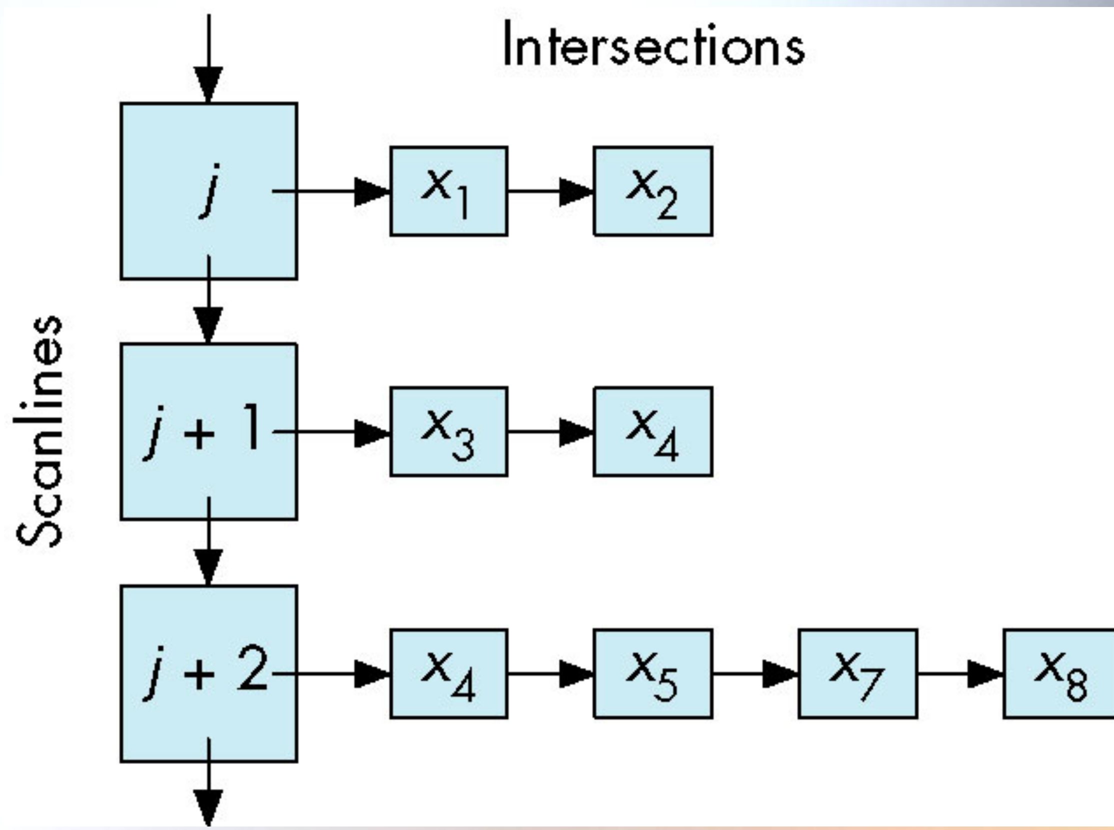


vertex order generated by vertex list



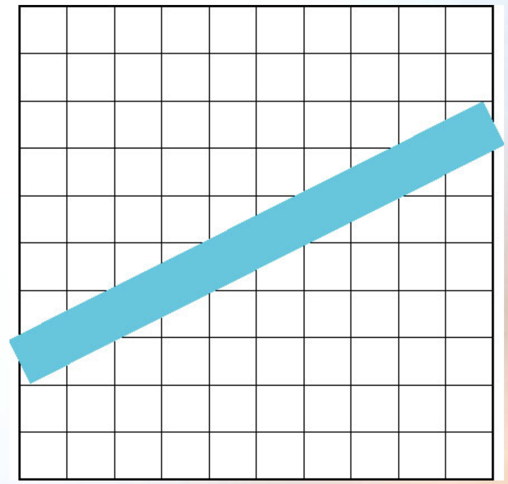
desired order

Data Structure



Aliasing

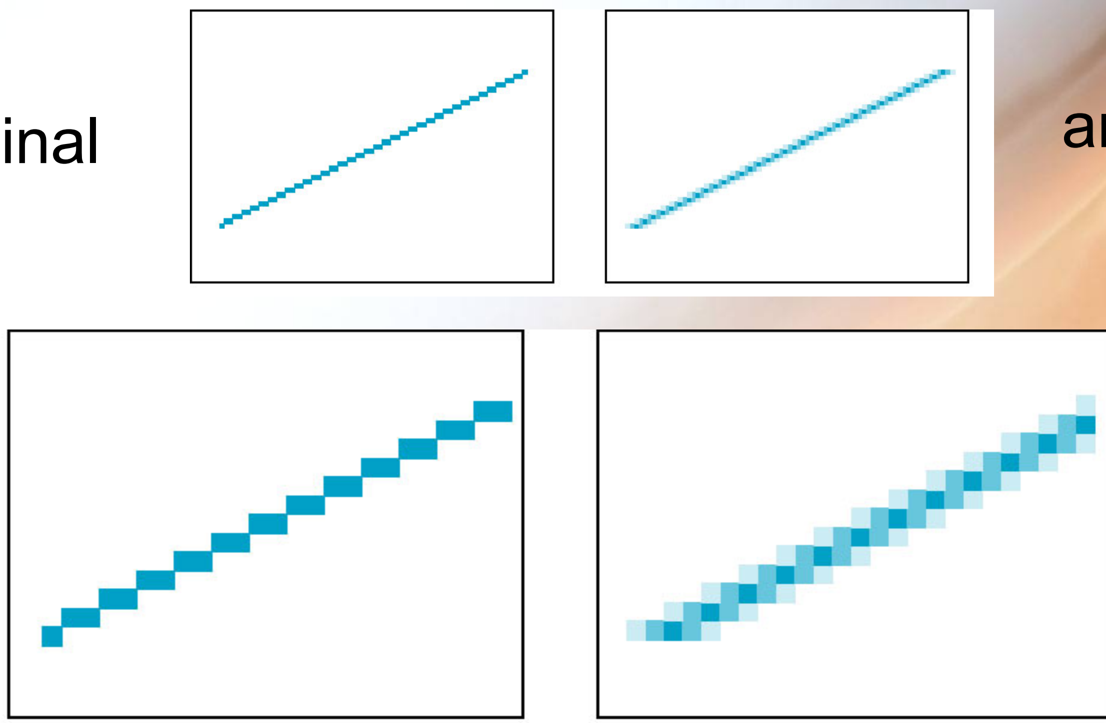
- Ideal rasterized line should be 1 pixel wide



- Choosing best y for each x (or visa versa) produces aliased raster lines

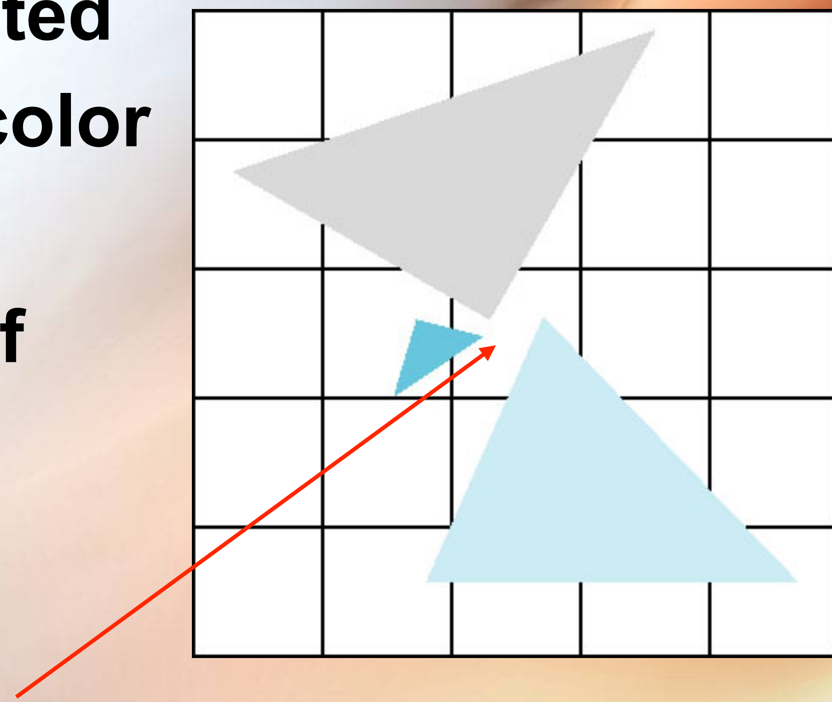
Antialiasing by Area Averaging

- Color multiple pixels for each x depending on coverage by ideal line



Polygon Aliasing

- Aliasing problems can be serious for polygons
  - Jaggedness of edges
  - Small polygons neglected
  - Need compositing so color of one polygon does not totally determine color of pixel



All three polygons should contribute to color