

CS311 Computational Geometry Spring 2023

Assignment: 2

Due Date: 2/7/23

Office: Park 203

Professor: Dianna Xu

E-mail: dxu@brynmawr.edu

URL: <http://cs.brynmawr.edu/cs311>

Assignment 2

1. Let P be a polygon with n vertices v_0, \dots, v_{n-1} , $v_i = (x_i, y_i)$ ordered counterclockwise in the plane. Prove that the area of P is:

$$\frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i)$$

2. Textbook Exercise 1.21. Obviously, you should not list figures for all, but describe how your figure/construction generalizes.
3. Textbook Exercise 1.22.
4. Prove or disprove: every polygon with an even number of vertices may be partitioned by diagonals into convex quadrilaterals. What about just quadrilaterals, not necessarily convex?
5. The dual graph of any triangulation of a simple polygon is a free tree. A free tree is a *path* if it has no vertices of degree greater than two. Keep in mind that a polygon may have many different triangulations (see Figure 1). Prove or disprove: Every monotone polygon has a triangulation whose dual graph is a path. (If you prove this statement, you will need to show that you can always find a triangulation whose dual is a path. If you disprove it, you will need to show an example of a monotone polygon that has no triangulation whose dual is a path.)

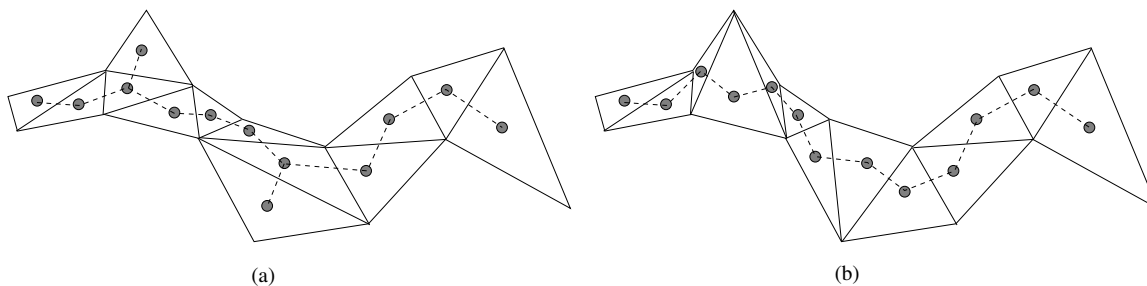


Figure 1: Two triangulations of the same polygon. (a) Dual is not a path. (b) Dual is a path.

6. Given n points in the plane, sorted by x -coordinates, describe an algorithm which constructs a simple polygon whose vertices are those points. There may be many such polygons; your algorithm needs to output just one. Your input is a list of n points and your output should also be a list of n points, describing the polygon in ccw order. Your algorithm should run in $O(n)$ time. Argue that it does.
7. There is a two-piece dissection of the 18×18 square to the left, and the 23×15 rectangle to the right with a centrally placed 7×3 hole, see Figure 2. Each is composed of 324 unit squares. Try to find the dissection!
8. Show that a regular tetrahedron can not be scissors congruent with a cube.

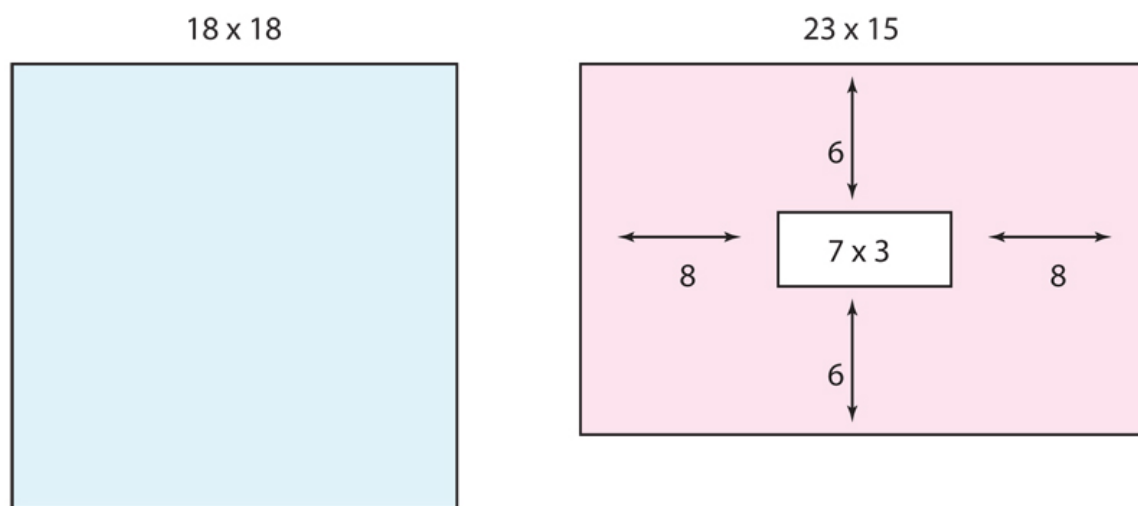


Figure 2: There is a two-piece dissection that takes one figure to the other