Trees

CS231

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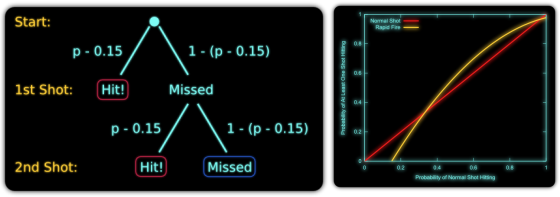
Tree

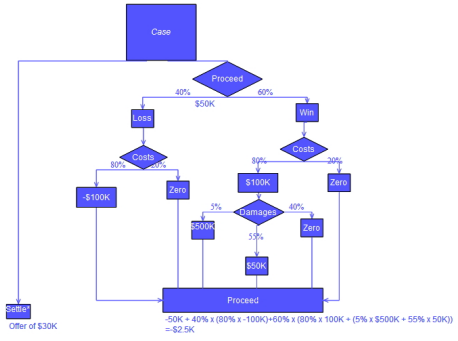
- A tree is a connected graph with no circuits.
- A loop is a circuit, so are parallel edges.
- A tree is a simple graph.
- A trivial tree has one vertex and no edges.
- A forest is a circuit-free graph that is not connected, i.e. it has trees as connected components.

Examples of Trees

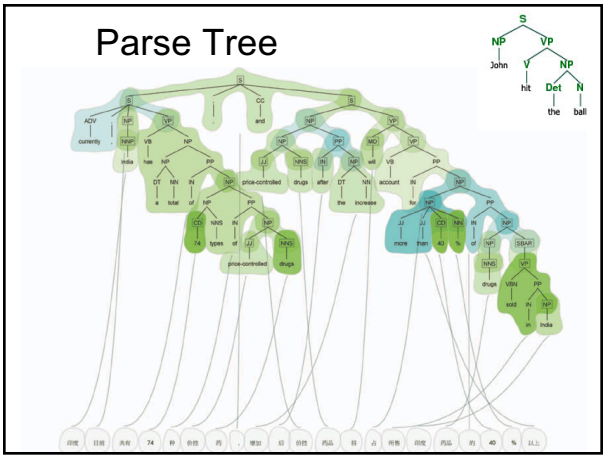
- Probability tree



Decision Tree

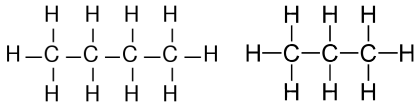


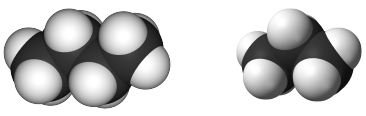
Parse Tree



Molecular Diagrams

- Butane (C_4H_{10}) and Propane (C_3H_8)





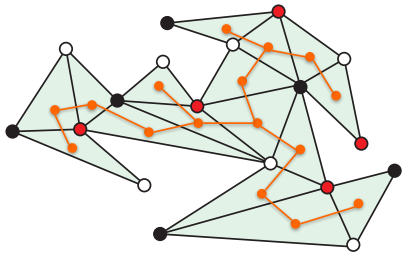
Leaves

- Lemma: Any nontrivial tree has at least 1 vertex of degree 1.
- Constructive Proof:
  - *T* is an arbitrary and particularly chosen tree
  - Pick a vertex *v* of *T* and let *e* be an edge incident on *v*
  - …
- A tree vertex of degree 1 is called a leaf.
- Others are called internal vertices.

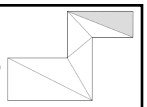
Characteristics

- A tree with *n* vertices has *n*-1 edges.
- Any connected graph with *n* vertices and *n*-1 edges is a tree.

Dual Graph



Meister's Two Ears



- Every polygon with *n*>3 vertices has at least two ears.
- Dual graph version: A tree of two or more nodes must have at least two leaves.
- Proof by induction
- Proof by contradiction

Every Planar Graph Can be 4-colored