

Recursively Defined Sequences

CS231
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Recurrence Relation

- It is often more convenient to define a sequence using previous terms:

$$- a_n = 2a_{n-1}$$

$$a_0 = 1$$

$$- a_n = a_{n-1} + a_{n-2}$$

$$a_0 = 1$$

$$a_1 = 1$$

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Recursive Definitions of Sums and Product

- Sum $\sum_{i=1}^n a_i = \sum_{i=1}^{n-1} a_i + a_n$ $\sum_{i=1}^1 a_i = a_1$

$$- \text{sum}(n) = \text{sum}(n-1) + n, n > 1$$

$$- \text{sum}(1) = 1$$

- Product $\prod_{i=1}^n a_i = \prod_{i=1}^{n-1} a_i \times a_n$ $\prod_{i=1}^1 a_i = a_1$

$$- \text{product}(n) = \text{product}(n-1) \times n, n > 1$$

$$- \text{product}(1) = 1$$

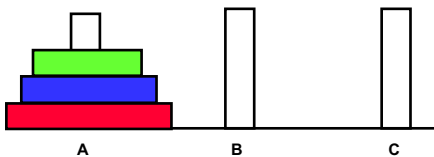
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The Towers of Hanoi

- Move all rings from one peg to another, without ever placing a larger ring on top of a smaller one

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The Towers of Hanoi

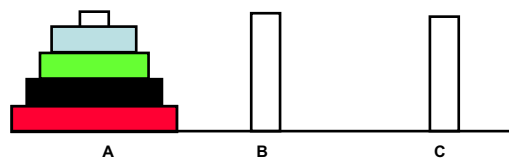


Goal: Move stack of rings to another peg

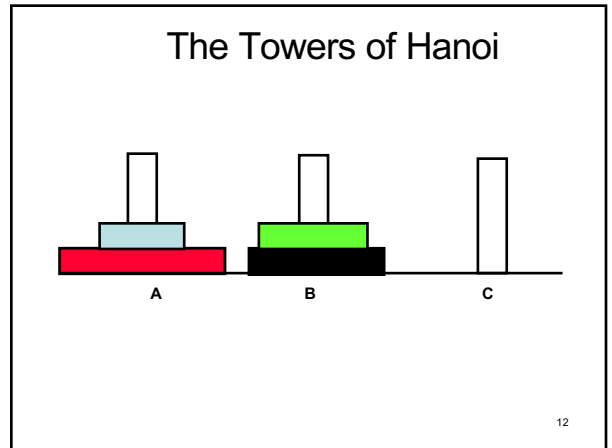
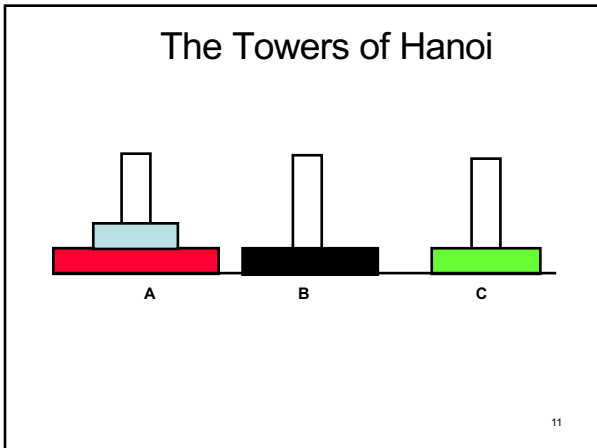
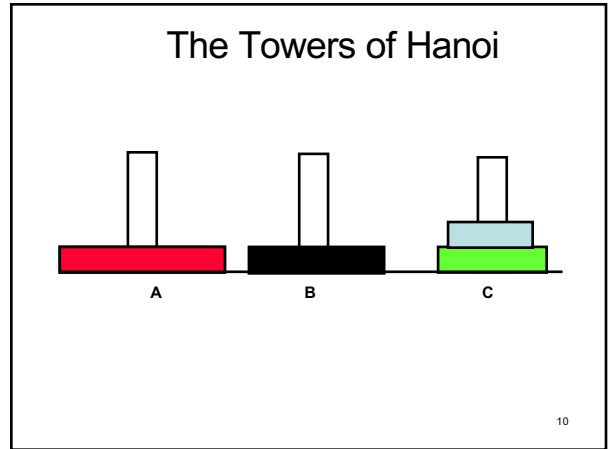
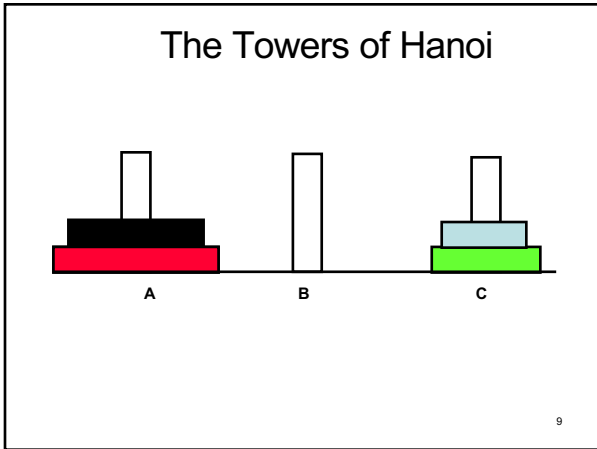
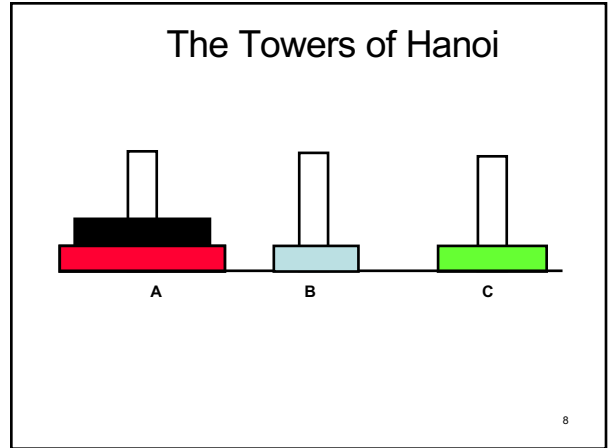
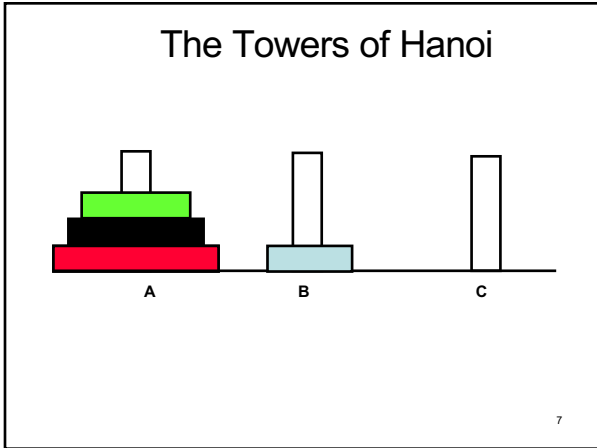
- Rule 1: May move only 1 ring at a time
- Rule 2: May never have larger ring on top of smaller one

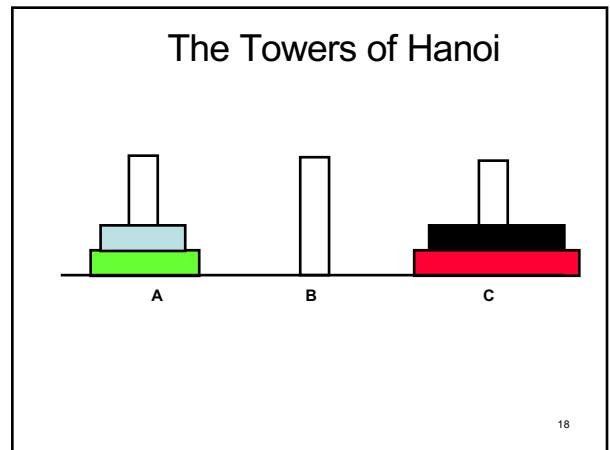
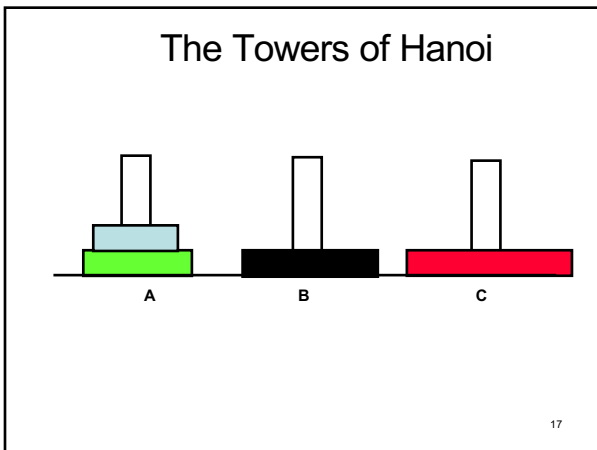
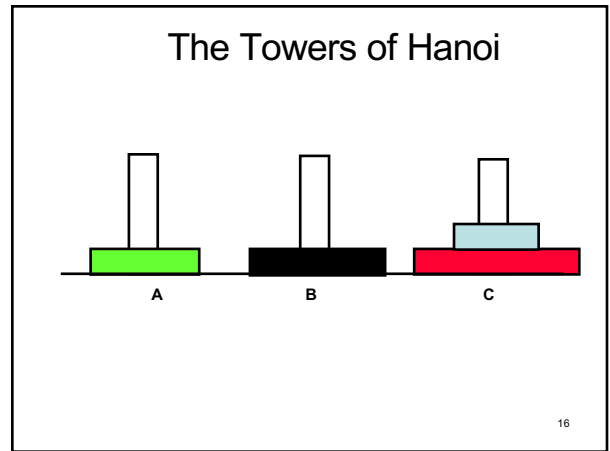
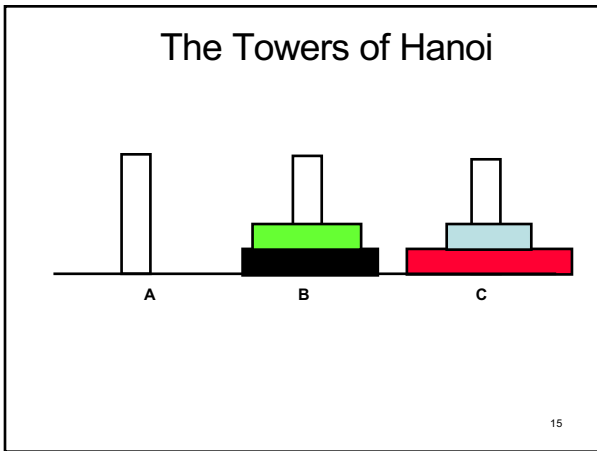
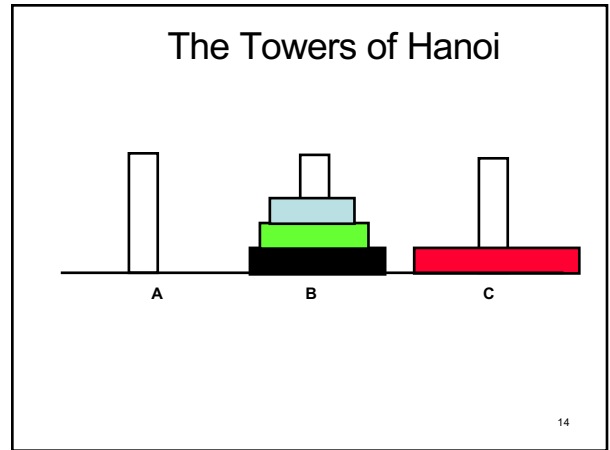
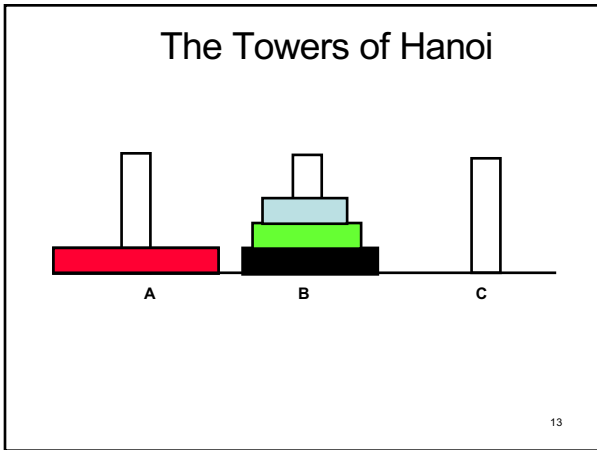
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The Towers of Hanoi



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The Towers of Hanoi

A B C

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The Towers of Hanoi

A B C

20

The Towers of Hanoi

A B C

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The Tower of Hanoi

- How many moves does it take to move 64 disks?
- $m_k = m_{k-1} + 1 + m_{k-1} = 2m_{k-1} + 1, m_1 = 1$
- $m_2 = 2 \times 1 + 1 = 3 = 2^2 - 1$
- $m_3 = 2 \times 3 + 1 = 7 = 2^3 - 1$
- $m_4 = 2 \times 7 + 1 = 15 = 2^4 - 1$
- $m_5 = 2 \times 15 + 1 = 31 = 2^5 - 1$

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The Power of Exponents

A rich king and a wise peasant...

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The Wise Peasant's Pay

<u>Day(n)</u>	<u>Pieces of Grain</u>
1	2
2	4
3	8
4	16
...	
63	9,223,000,000,000,000
64	18,450,000,000,000,000

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How Bad is 2^n ?

- Imagine being able to grow a billion (1,000,000,000) pieces of grain a second...
- It would take
 - **585 years** to grow enough grain just for the 64th day
 - Over a thousand years to fulfill the peasant's request!

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So the King cut off the
peasant's head.

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