Algorithms

CS 231
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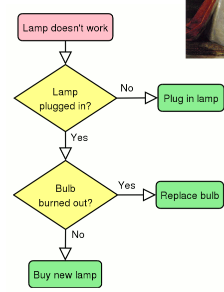
Algorithm

- Step-by-step method for performing some action
- An algorithm is a sequence of finite instructions, often used for calculation (Wikipedia)

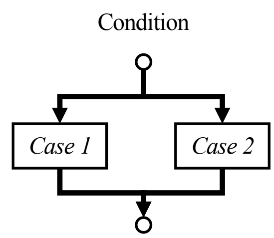


Pseudo code

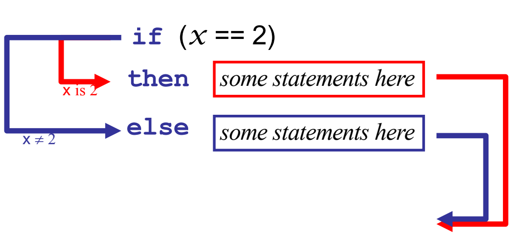
- Variables
  - Memory storage
  - Size and type
- Assignment statement
  - x := expr

Conditional

- **if** (condition) **then** s1 **else** s2

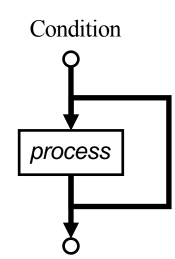


Structure



while loop

```
while (condition)
    body
end while
```



`for` loop

```
for (i:= initial expr to final expr )
      body
next i
```


Greatest common divisor

- a, $b \in \mathbb{Z}$, $a \neq 0$, $b \neq 0$, gcd(a, b) is the integer d with the following properties:
  - $d \mid a$ and $d \mid b$
  - $\forall\, c \in \mathbb{Z}$, if $c \mid a$ and $c \mid b$, then $c \leq d$

Lemmas

- Lemma 1:
  - If r is a positive integer, then gcd(r, 0) = r
- Lemma 2:
  - Given a, $b \in \mathbb{Z}$, with $b \neq 0$, and q, $r \in \mathbb{Z}$ such that: $a = bq + r$
  - Then gcd(a, b) = gcd(b, r)
    - Show gcd(a, b) ≤ gcd(b, r)
    - And gcd(b, r) ≤ gcd(a, b)

The Euclidean Algorithm

- By hand: gcd(123, 456)
  - 456 = 123*3 + 87 → gcd(456, 123) = gcd(123, 87)
  - 123 = 87*1 + 36 → gcd(123, 87) = gcd(87, 36)
  - 87 = 36*2 + 15 → gcd(87, 36) = gcd(36, 15)
  - 36 = 15*2 + 6 → gcd(36, 15) = gcd(15, 6)
  - 15 = 6*2 + 3 → gcd(15, 6) = gcd(6, 3)
  - 6 = 2*3 → gcd(6, 3) = 3

Algorithm: Euclidean

Input: A, B (A, B in $\mathbb{Z}$, A > B ≥ 0)
Algorithm Body:
  a:= A, b:= B, r:= B
  while ($b \neq 0$)
    r:=a mod b
    a:=b
    b:=r
  end while
Output: gcd := a

Properties of an Algorithm

- Input
- Output
- Definiteness
- Correctness
- Finiteness
- Effectiveness
- Generality

Finding the Max Element

- Give an algorithm for finding the largest value in a finite sequence of integers

Input: $a_1, a_2 \ldots a_n \in \mathbb{Z}$

Algorithm Body:

$max := a_1$

for $(i := 2$ **to** $n)$

  if $(max < a_i)$ **then** $max := a_i$

next i

Output: max

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Searching

- Locating an element in an ordered list
- 1 2 3 5 6 7 8 10 12 13 15 16 18 19 20 22
- Linear Search – Sequential Search
- Binary Search
  - 1 2 3 5 6 7 8 10   12 13 15 16 18 19 20 22
  - 12 13 15 16   18 19 20 22
  - 18 19   20 22
  - 18   19

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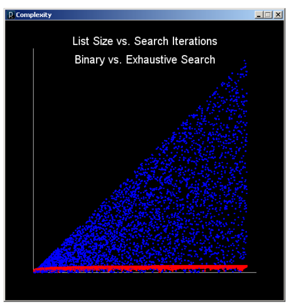
Running Time



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Sorting

- Ordering the elements of a list
- Bubble Sort
  - compare each pair and swap if necessary
- Insertion Sort
  - the front of the list is kept in order
  - the sorted list starts with 1 element, the first
  - each successive element is compared and inserted into the correct position in the sorted list

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Greedy Algorithm

- Optimization problems – to find a solution to the given problem that either maximizes or minimizes the value of some parameter
- The simplest approach – greedy
  - select the best available choice at each step
  - does not consider consequences of all sequences
  - solution is not always optimal

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