

### Basic logic gates

- Not  $x \rightarrow \bar{x}$
- And  $\begin{matrix} x \\ y \end{matrix} \rightarrow xy$       $\begin{matrix} x \\ y \\ z \end{matrix} \rightarrow xyz$
- Or  $\begin{matrix} x \\ y \end{matrix} \rightarrow x+y$       $\begin{matrix} x \\ y \\ z \end{matrix} \rightarrow x+y+z$
- Nand  $\begin{matrix} x \\ y \end{matrix} \rightarrow \overline{xy}$
- Nor  $\begin{matrix} x \\ y \end{matrix} \rightarrow \overline{x+y}$
- Xor  $\begin{matrix} x \\ y \end{matrix} \rightarrow x \oplus y$

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### The full adder

• The "HA" boxes are half-adders

x	y	c	s <sub>1</sub>	c <sub>1</sub>	c <sub>2</sub>	carry	sum
1	1	1	0	1	0	1	1
1	1	0	0	1	0	1	0
1	0	1	1	0	1	1	0
1	0	0	1	0	0	0	1
0	1	1	1	0	1	1	0
0	1	0	1	0	0	0	1
0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	0

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### Number System Conversion

- $A9BF_{16} = ?_2$
- $1010100110111111_2$
- $456_{10} = ?_2$
- $111001000_2$
- $456_{10} = ?_7$
- $1221_7$

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### Two's Complement

- Given a positive integer  $a$ , the two's complement of  $a$  is the  $n$ -bit representation of  $2^n - a$
- $2^8 - 35 = 256 - 35 = 221 = 11011101_2$
- $a$ 's two's complement represents  $-a$
- Always relative to a fixed bit length
- Bit length of 32 and 64 are most commonly used

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### One's Complement

- An easier way to calculate two's complement
- $2^8 - a = (2^8 - 1) - a + 1$
- $2^8 - 1 = 11111111_2$
- Subtracting any binary number from all 1's is equivalent to negating all bits, i.e. taking the one's complement

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### Example

$$2^8 - 35 = (2^8 - 1) - 35 + 1 =$$

$$11111111_2$$

$$- 00100011_2$$

$$11011100_2 + 1 = 11011101_2 = 221$$

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## Two's Complement Again

- To find the two's complement of a positive integer  $a$ :
  - Write the  $n$ -bit binary representation for  $a$
  - Negate all bits
  - Add 1 to the resulting binary notation

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## 8-bit Representations

Integer	8-bit Binary	2's complement
127	01111111	
126	01111110	
...	...	
2	00000010	
1	00000001	
0	00000000	
-1	11111111	$2^8 - 1$
-2	11111110	$2^8 - 2$
-3	11111101	$2^8 - 3$
...	...	
-127	10000001	$2^8 - 127$
-128	10000000	$2^8 - 128$

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## Addition with Negative Numbers

$$64 - 15 = 64 + (-15) =$$

$$01000000_2 + ((11111111_2 - 00001111_2) + 1_2) =$$

$$01000000_2 + 11110001_2$$

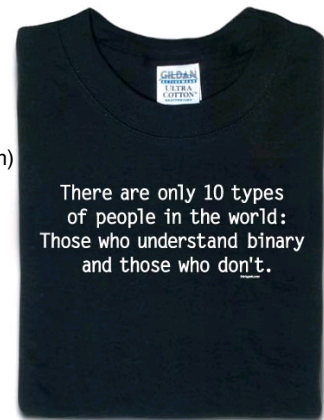
$$01000000_2$$

$$11110001_2$$

$$00110001_2 = 49$$

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There are only 10 types  
of people in the world:  
Those who understand binary  
and those who don't.