Logic Gates

CS 231

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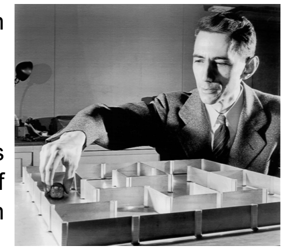
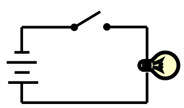
Claude Shannon

- Father of information theory
- His master thesis was the foundation of digital circuit design theory.

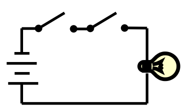


Circuits

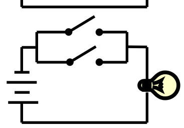
- Single switch



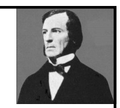
- Switches in series



- Switches in parallel



Boolean algebra



- Boolean – a variable that is either true or false
- Boolean algebra – logical calculus of truth values
  - Very similar to Boolean logic
  - Variables can only be 1 or 0
    - Instead of true / false

Overview of Boolean algebra

- Not is a horizontal bar above the number
  - $\bar{0} = 1$
  - $\bar{1} = 0$
- Or is a plus
  - 0+0 = 0
  - 0+1 = 1
  - 1+0 = 1
  - 1+1 = 1
- And is multiplication
  - 0*0 = 0
  - 0*1 = 0
  - 1*0 = 0
  - 1*1 = 1

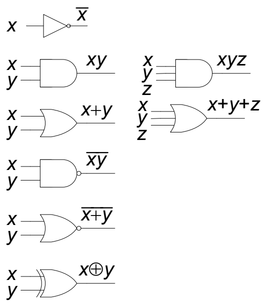
Overview of Boolean algebra

- Example: translate $(x+y+z)(\bar{x}\bar{y}\bar{z})$ to a Boolean logic expression
  - $(x \vee y \vee z) \wedge (\sim x \wedge \sim y \wedge \sim z)$
- We can define a Boolean function:
  - $F(x,y) = (x+y)(\bar{x}\bar{y}) = (x \vee y) \wedge (\sim x \wedge \sim y)$
- And then write a "truth table" for it:

x	*y*	*F(x,y)*
1	1	0
1	0	0
0	1	0
0	0	0

Basic logic gates

- Not
- And
- Or
- Nand
- Nor
- Xor



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Combinational circuit rules

- In general, gates can be combined into combinational circuits
- Rules:
  - Never combine two input wires
  - A single input wire can be split partway and used as input for two separate gates
  - An output wire can be used as input
  - No output of a gate can eventually feed back into that same gate

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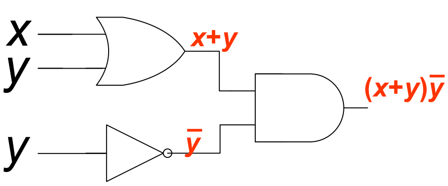
Converting between circuits and Boolean expressions

- Find the output of the following circuit



- Answer: $(x+y)\overline{y}$
  - Or $(x \vee y) \wedge \sim y$

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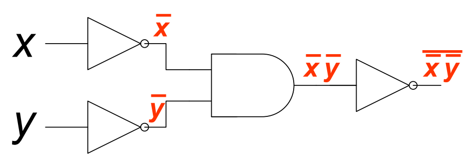
Converting between circuits and equations

- Find the output of the following circuit



- Answer: $\overline{\overline{x}\,\overline{y}}$
  - Or $\sim(\sim x \wedge \sim y) \equiv x \vee y$

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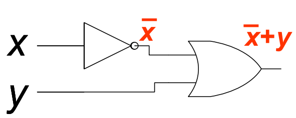
Converting between circuits and equations

- Write the circuits for the following Boolean algebraic expressions

a) $\overline{x}+y$



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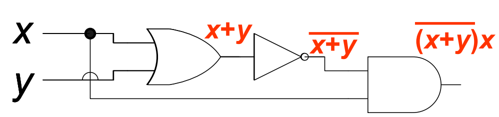
Converting between circuits and equations

- Write the circuits for the following Boolean algebraic expressions

b) $\overline{(x+y)}x$



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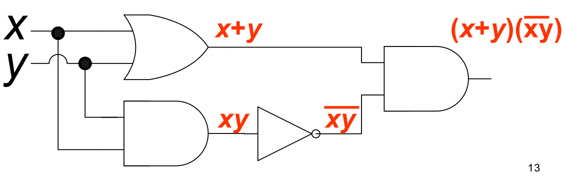
Writing xor using and/or/not

- $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$
- $x \oplus y \equiv (x + y)(\overline{xy})$

x	y	$x \oplus y$
1	1	0
1	0	1
0	1	1
0	0	0



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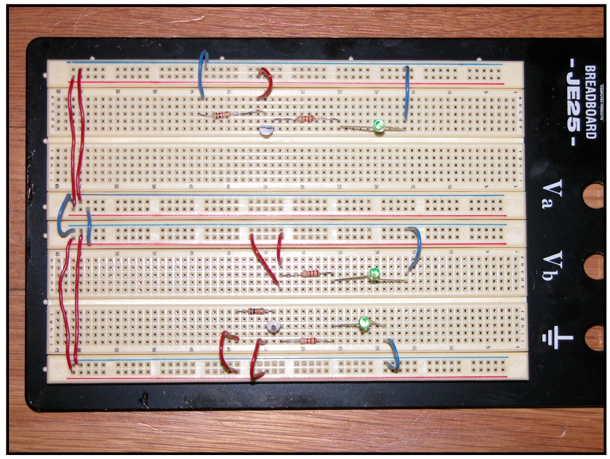
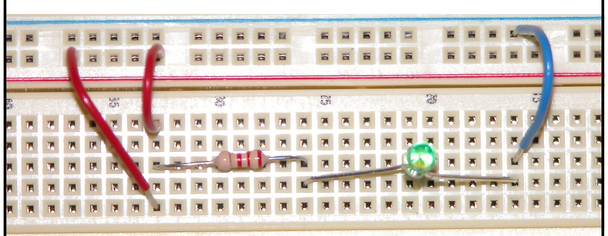


OR gate



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Integrated Circuits

- Very advanced (and miniaturized) electronic circuits
- Least expensive way to make logic gates in large volumes
- Mainly consist of semiconductor devices
  - Transistors – on/off/amplify
- Integrate a large number of transistors onto a tiny chip (die) …

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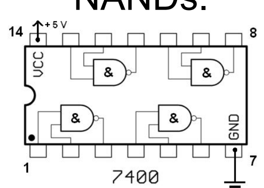
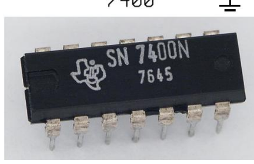
The 7400 chip, containing four NANDs.





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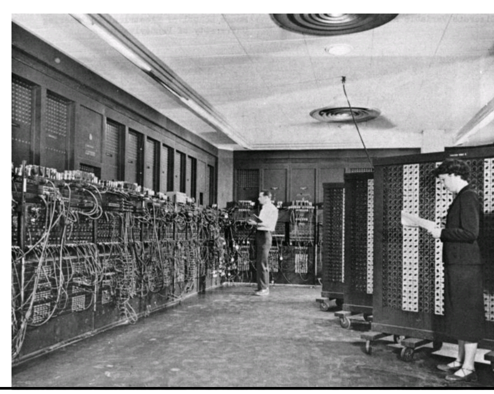
Eniac: the first computer



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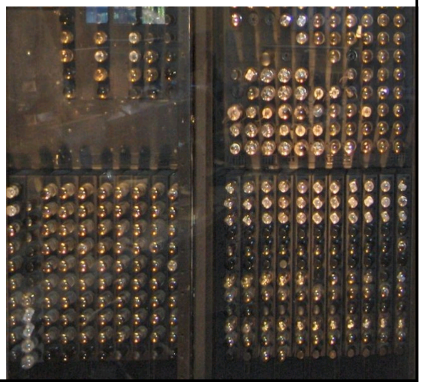
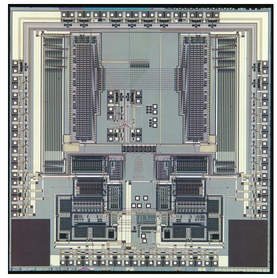
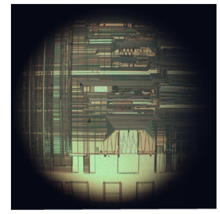
Eniac's vacuum tubes

A back panel of Eniac, showing the vacuum tubes



Integrated Circuits – Chips





Upper interconnect layers on an Intel 80486 DX2 microprocessor die.

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Moore's Law

- The complexity of an integrated circuit, with respect to minimum component cost, doubles every 18 months.
- True since the early 1970s
- Current leading developmental constraint?

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