1. Determine whether each of the following is a **statement**, a **predicate**, or neither. Give truth values to **statements** when possible.

   (a) $\sqrt{2} > 1$

   (b) $\sin^2 x + \cos^2 x = 1$

   (c) When is the square of a number greater than one?

   (d) Get some rest.

   (e) a is an even number.

   (f) $\forall x \in \mathbb{R}, (x^2 > 3 \rightarrow x > 1)$

2. Give the **converse, inverse, contrapositive**, and (non-trivial!) **negation** of the following statements.

   (a) $\sim p \rightarrow r$

3. Simplify the following statement, citing laws used at every step:

$$\sim ((p \vee q) \rightarrow (p \wedge q))$$

4. Convert the following binary number to hexadecimal (base-16): 1101001110011010_2

5. Convert the following base-5 number to decimal: 1234_5

6. Find the decimal equivalent of the following 16-bit 2's complement number:1001110000000101_2

7. For the following table,

   (a) construct a Boolean expression having the table as its truth table

   (b) design a **minimal** circuit having the table as its input/output table

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

8. Rewrite the following statements formally (by defining predicates and using quantifiers and variables, in other words, no English in the final statement).You may assume the domain is $\mathbb{R}$, and use the following predicate definitions only:

   - $Integer(x)$: x is an integer
   - $Even(x)$: x is even (an even integer)

   (a) Every integer is either even or odd.

   (b) The sum of two odd integers is an even integer.

   (c) There is an integer whose multiplicative inverse is also an integer.

9. Write the non-trivial **negation** of the following statements:

(a) There exist a real number x such that for all real numbers y, $xy > y$.

(b) $\forall n \in \mathbb{Z}, (n \text{ even} \rightarrow n+1 \text{ odd})$.

10. Consider the following statements:

    (a) All hummingbirds are richly colored.

    (b) No large birds live on honey.

    (c) Birds that do not live on honey are dull in color.

    (d) Hummingbirds are small.

10.1. Express each of the the statements using quantifiers and predicates (no English). Assume the domain is all birds. Clearly define the predicates you are using.

10.2. Can you conclude (d), i.e. "Hummingbirds are small", from the first three? Prove this using logical equivalences and rules of inference. Clearly label which rule you are using on each step.