Rooted Trees

CS231
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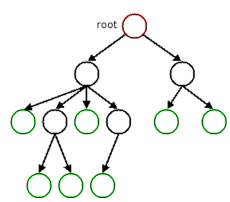
Rooted Tree

- One vertex in a tree is distinguished and known as root. Traditionally drawn on top.
- The level of a vertex is the number of edges between it and the root.
- The height of a tree is the maximum level of any vertex of the tree.

Terminology

- Children
- Parent
- Siblings
- Ancestor
- Descendent

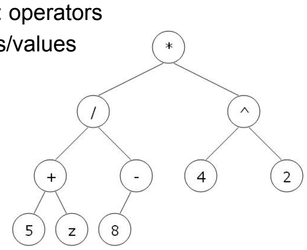


Binary Tree

- A rooted tree with every vertex having at most two children is known as a binary tree.
- Each child is either the left or right child.
- A binary tree in which each parent has exactly two children is known as a full binary tree.
- A left (resp. right) subtree is the tree rooted in the left (resp right) child.

Representations

- Algebraic expressions
  - internal vertices: operators
  - leaves: variables/values



Characteristics

- If T is a full binary tree with k internal vertices (k>0), then T has a total of 2k+1 vertices and k+1 leaves.
- Proof
  - each internal vertex has exactly 2 children
  - total # of children: 2k
  - root

Height

- If T is any binary tree of height h with t leaves (h>0), then $t \leq 2^h$
- Or $\log_2 t \leq h$
- Proof by strong induction on h
  - P(0): T has only a root → 1 leaf: $1 \leq 2^0$
  - Assume P(i), $0 \leq i \leq k$: All binary trees with height less than or equal to k has at most 2^k leaves
  - P(k+1): T is a binary tree of height k+1

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Induction Cont.

- $k \geq 0$ → $k+1 \geq 1$, root has at least one child
- Root has exactly one child c:
  - The subtree rooted at c, T_c is of height k
  - By the inductive hypothesis, T_c has at most 2^k leaves
- Root has two children c_1 and c_2:
  - One of the subtrees (say T_{c1}) is of height k and T_{c2} is of any height between 0 and k
  - By the inductive hypothesis, both have at most 2^k leaves, which gives a total of at most 2^{k+1}.

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