

Trees

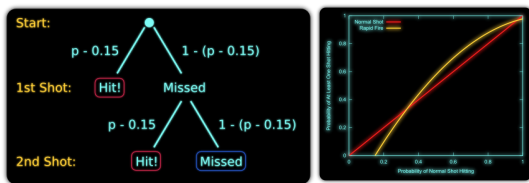
CS231
Dianna Xu

Tree

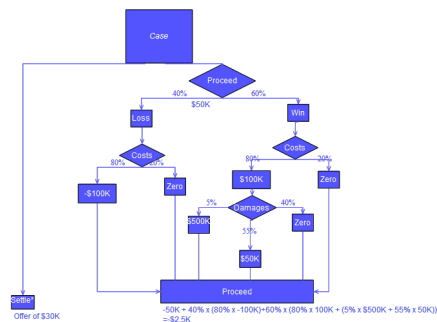
- A tree is a connected graph with no circuits.
- A loop is a circuit, so are parallel edges.
- A tree is a simple graph.
- A trivial tree has one vertex and no edges.
- A forest is a circuit-free graph that is not connected, i.e. it has trees as connected components.

Examples of Trees

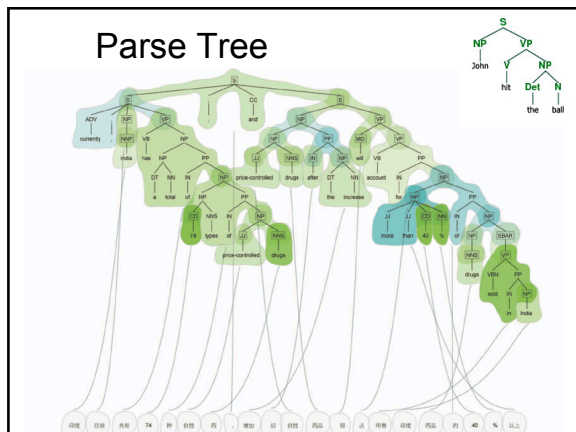
- Probability tree



Decision Tree

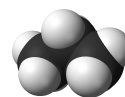
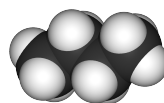
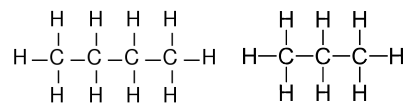


Parse Tree



Molecular Diagrams

- Butane (C_4H_{10}) and Propane (C_3H_8)



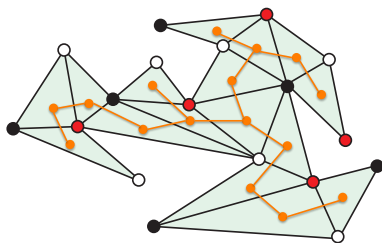
Leaves

- Lemma: Any nontrivial tree has at least 1 vertex of degree 1.
- Constructive Proof:
 - T is an arbitrary and particularly chosen tree
 - Pick a vertex v of T and let e be an edge incident on v
 - ...
- A tree vertex of degree 1 is called a leaf.
- Others are called internal vertices.

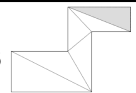
Characteristics

- A tree with n vertices has $n-1$ edges.
- Any connected graph with n vertices and $n-1$ edges is a tree.

Dual Graph



Meister's Two Ears



- Every polygon with $n > 3$ vertices has at least two ears.
- Dual graph version: A tree of two or more nodes must have at least two leaves.
- Proof by induction
- Proof by contradiction

Every Planar Graph Can be 4-colored

