Graphs

CS231

Dianna Xu

1

Definition

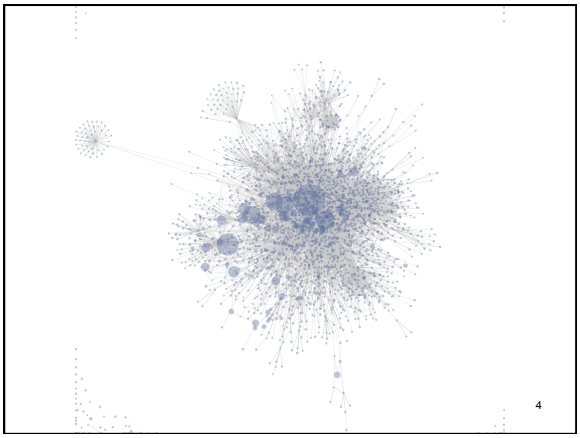
- A Graph G consists of a nonempty set V of vertices and a set E of edges.
  - loop
  - parallel edges
  - an edge is *incident* on its endpoints
  - endpoints are *adjacent* to the edge
  - isolated vertex

2

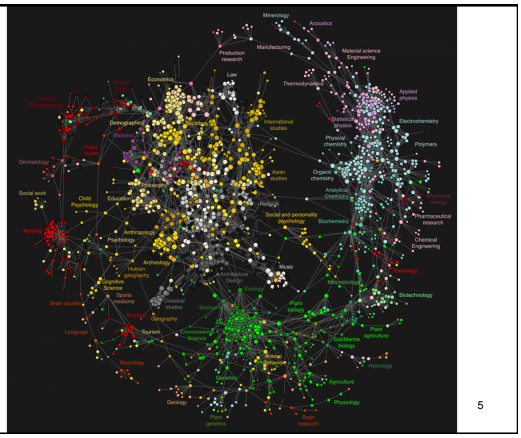
Using Graphs to Represent a Network

- Edges are drawn between nodes that relate to each other in some way
- Hub-and-spoke
  - telephone, power, gas
  - air transit, train and highways
  - WWW
  - social networks
  - preference maps/recommender systems

3

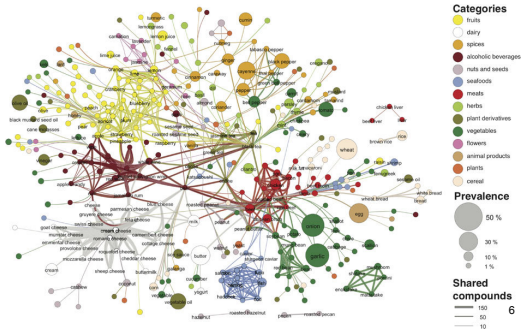


4



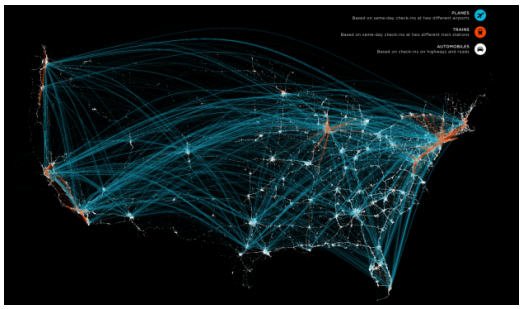
5

Flavor Network

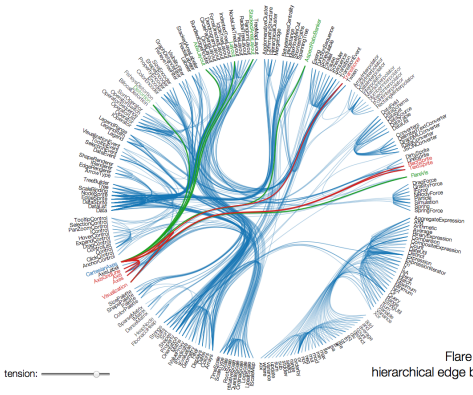


6

Thanksgiving Travels

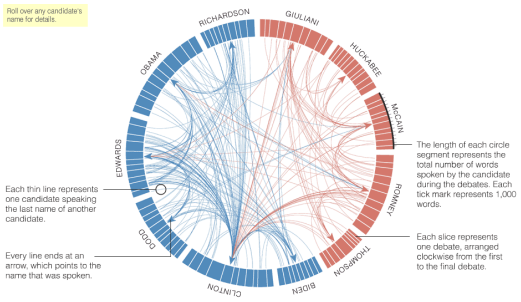


7



8

Naming Names



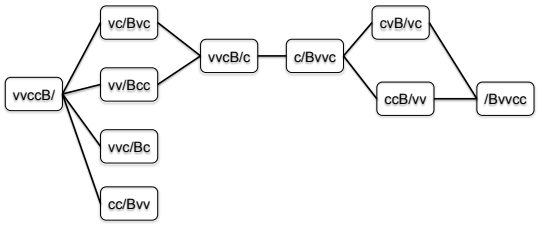
9

Vegetarians and Cannibals

- An island with two types of people, vegetarians (Vs) or cannibals (Cs)
- Initially 2 Vs and 2 Cs are on the bank of a river
- A boat that hold a max of 2
- # of Vs can not be less than # of Cs at any time

10

Graph Solution



11

Special Graphs

- A *simple* graph does not contain any loops or parallel edges.
- A *complete* graph K_n, is a simple graph with *n* vertices and one edge connecting each pair of distinct vertices.
- A graph H is said to be a *subgraph* of G, iff every vertex in H is also a vertex in G and every edge in H is also an edge in G with the same endpoints. G is then a *supergraph* of H.

12

Bipartite

- A complete bipartite graph $K_{m,n}$, is a simple graph with vertices v_1 ...v_m and w_1 ... w_n such that:
  - There is an edge from each v_i to each w_j.
  - There is no edge from any v_i to any other v_k
  - There is no edge from any w_j to any other w_l

$K_{2,3}$

$K_{4,5}$

13

Degrees

- Let G be a graph and v a vertex of G.
- The degree of v, deg(v), equals the number of edges incident on v.
- A loop contributes twice to its incident vertex's degree
- The total degree of G is the sum of the degrees of all vertices in G

14

Handshake Lemma

- The total degree of a graph G is twice the number of edges in G.
- Given a graph G and its vertex set V and edge set E, |V| = n,
  - $\deg(v_1) + \deg(v_2) + ... + \deg(v_n) = 2\times|E|$
- The total degree of a graph is even.

15

Vertices of Odd Degree

- In any graph there is an even number of vertices of odd degree.
  - total degree of all vertices is even
  - sum of even-degree vertices is even
  - sum of odd-degree vertices is even

16