2

Conditional Probability

CS231 Dianna Xu

Boy or Girl?

• A couple has two children, one of them is a girl. What is the probability that the other one is also a girl? Assuming 50/50 chances of conceiving boys and girls.

Conditional Probability

Let A and B be events in a sample space
 S. If P(A) ≠ 0, then the conditional probability of B given A (P(B|A)) is:

 $P(B|A) = P(A \cap B) / P(A)$

 $P(A) = P(A \cap B) / P(B|A)$ $P(A \cap B) = P(B|A) \times P(A)$

Example

- Two cards are drawn from a well-shuffled deck. What is the probability that:
 both are kings?
 second draw is a king?
 A = 1st draw is king, B = 2nd draw is king
- $P(A) = 4/52, P(A^c) = 48/52$
- $P(B|A) = 3/51, P(B|A^c) = 4/51$
- P(A∩B) = 4/52×3/51 = 12/2652
- P(A∩B) + P(A^c∩B) = 4/52×3/51 + 48/52×4/51

Example

- If the experiment of drawing a pair is repeated over time, what would be the expected value of the number of of kings?
- 2 kings: P(A∩B) = 4/52×3/51 = 12/2652
- 1 king: = P(A°∩B)+P(A∩B°) = 48/52×4/51 + 4/52×48/51 = 384/2652
- Expected value of # of kings: 2×12/2652 + 1×384/2652 ≈ 0.154

5

3

Example

- 5% of manufactured components are defective in general.
- The method for screening out defective items is not totally reliable. The test rejects good parts as defective in 1% of the cases and accepts defective parts as good ones in 10% of the cases.
- Given that the test indicates that an item is good, what is the probability that this item is, in fact, defective?

6

Definitions

- T = A component tested good
- *D* = A component is defective
- T^c = A component tested defective
- G = A component is good ($G = D^c$)
- Want to solve: P(D|T)

P(D|T)

- P(D) = 0.05, P(G) = 0.95
- $P(T^c|G) = 0.01$ (false positive) P(T|G) = 0.99
- $P(T \cap G) = P(T|G) \times P(G) = 0.99 \times 0.95 = 0.9405$
- P(T|D) = 0.1 (false negative)
- $P(T \cap D) = P(T|D) \times P(D) = 0.1 \times 0.05 = 0.005$
- $T = (T \cap G) \cup (T \cap D)$

7

9

11

- P(T) = 0.9405 + 0.005 = 0.9455
- $P(D|T) = P(T \cap D)/P(T) = 0.005/0.9455 = 0.0052882$

Medical Screening

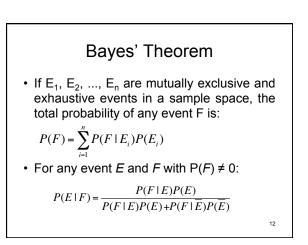
- 1% of population suffer from a certain disease.
- The method for screening is not totally reliable. The test reports false positive in 5% of the cases and false negative in 10% of the cases.
- Given that a person has a negative test result, what is the probability that this person is, in fact, sick?
- Given that a person has a positive test result, what is the probability that this person is, in fact, sick?

Definitions

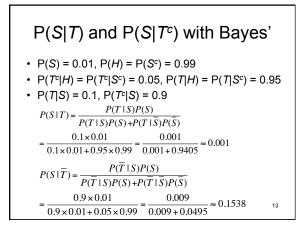
- T = A person cleared the test (negative)
- S = A person is sick
- T^c = A person did not clear the test (positive)
- H = A person is healthy ($H = S^c$)
- Want to solve: P(S|T) and P(S|T^c)

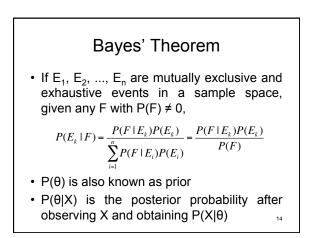
P(S|T)

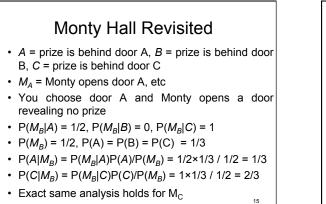
- P(S) = 0.01, P(H) = 0.99
- $P(T^c|H) = 0.05$ (false positive) P(T|H) = 0.95
- $P(T \cap H) = P(T|H) \times P(H) = 0.95 \times 0.99 = 0.9405$
- P(*T*|*S*) = 0.1 (false negative)
- $P(T \cap S) = P(T|S) \times P(S) = 0.1 \times 0.01 = 0.001$
- $T = (T \cap H) \cup (T \cap S)$
- P(T) = 0.9405 + 0.001 = 0.9415
- $P(S|T) = P(T \cap S)/P(T) = 0.001/0.9415 \approx 0.001$

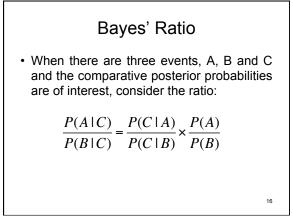


10



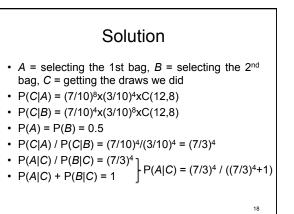






Example

- Two bags, one contains 70 red and 30 blue balls, and the other 30 red and 70 blue balls.
- Choose one bag randomly and draw with replacement.
- 8 red and 4 blue balls are drawn in 12 tries.
- What is the probability that it was the predominantly red bag that was chosen?



Dramatic Taxicab

- A cab was involved in a hit-and-run at night.
- Two cab companies operate in the city, with green and blue cabs, respectively.
- 85% of the cabs are green.
- A witness identified the cab as blue.
- The witness correctly identified the two colors 80% of the time under night-time testing.
- What is the probability that the witness was right?

19

<section-header><list-item><list-item><list-item><list-item><list-item><list-item><list-item><list-item></table-row><list-item></table-row></table-row></table-row></table-row></table-row></table-row></table-row></table-row>

P(*A*∩*B*^c)

- If *A* and *B* are independent events, so are *A* and *B*^c.
- From set theory:
 - $-\left(A\cap B\right) \cup\left(A\cap B^{c}\right) =A$
 - $-(A\cap B)\cap (A\cap B^c)=\emptyset$
- $P((A \cap B)U(A \cap B^c)) = P(A \cap B) + P(A \cap B^c) = P(A)$
- $P(A \cap B^c) = P(A) P(A \cap B) = P(A) P(A)P(B)$
- $= P(A)(1 P(B)) = P(A)P(B^{c})$

21

Loaded Coin

- A coin is loaded so that the probability of heads is 0.6. After 10 tosses, what is the probability of obtaining 8 heads?
- Consider HHHHHHHHTT
- $P(HHHHHHHHTT) = 0.6^8 \times 0.4^2$
- How many ways can you get 8 heads with 10 tosses? – C(10, 8)
- P(8 heads) = C(10, 8)× 0.6^8 × 0.4^2 ≈ 0.12

22