Probability Trees and the Multiplication Rule

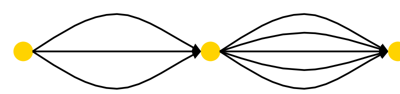
CS231

Dianna Xu

1

The multiplication rule

- Also called the product rule
- If there are n_1 ways to do task 1, and n_2 ways to do task 2
  - Then there are $n_1 x n_2$ ways to do both tasks in sequence
  - We must make one choice AND a second choice



2

Product rule example

- Sample question
  - There are 18 MATH majors and 17 CS majors
  - How many ways are there to pick one math major and one CS major?
- Total is 17 x 18 = 306

3

Product rule example

- How many strings of 4 decimal digits…
- Do not contain the same digit twice?
  - We want to chose a digit, then another that is not the same, then another…
    - First digit: 10 possibilities
    - Second digit: 9 possibilities (all but first digit)
    - Third digit: 8 possibilities
    - Fourth digit: 7 possibilities
  - Total = 10x9x8x7 = 5040
- End with an even digit?
  - First three digits have 10 possibilities
  - Last digit has 5 possibilities
  - Total = 10x10x10x5 = 5000

4

When the product rule is difficult to apply

- President, treasurer and secretary are to be chosen among A, B, C, D. A can not be president and either C or D must be secretary.
- Naïve application of the product rule:
  - President: 3
  - Treasurer: 3
  - Secretary: 2
  - Total = 18

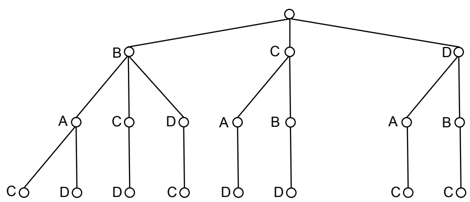
5

Tree diagrams

- We can use tree diagrams to enumerate the possible choices
- Once the tree is laid out, the result is the number of (valid) leaves

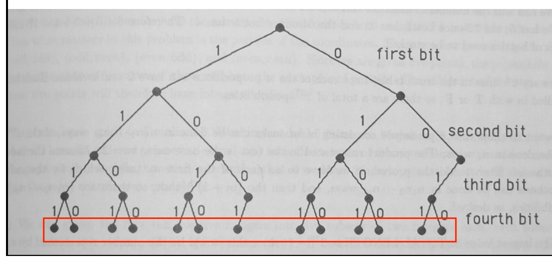
6

Only 8 choices

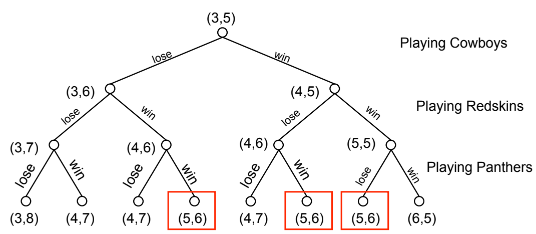


Tree diagrams example

- Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s



How many ways can the Eagles get to 5-6 in the next 3 games?



Permutations

- Given a set of n elements, its permutations can be counted this way:
  - Choose one element for first position: n
  - Choose next element for second position: n-1
  - …
  - Total: nx(n-1)x…x1 = n!

r-permutation

- An r-permutation of a set of n elements is an ordered selection of r elements from the n elements.
  - A♦, 5♥, 7♣, 10♠, K♠ is a 5-permutation of the set of cards
- The notation for the number of *r*-permutations: *P*(*n*,*r*)
  - The poker hand is one of P(52,5) permutations

r-permutations

- Number of poker hands (5 cards):
  - *P*(52,5) = 52x51x50x49x48 = 311,875,200
- Number of (initial) blackjack hands (2 cards):
  - *P*(52,2) = 52x51 = 2,652

$$P(n,r) = n(n-1)(n-2)...(n-r+1)$$

$$= \frac{n!}{(n-r)!}$$

$$= \prod_{i=n-r+1}^{n} i!$$

r-permutation Formula

- There are n ways to choose the first element
  - n-1 ways to choose the second
  - n-2 ways to choose the third
  - …
  - n-r+1 ways to choose the r^{th} element
- By the product rule, that gives us: $P(n,r) = n(n-1)(n-2)\ldots(n-r+1)$

13

r-permutations example

- How many ways are there for 3 students in this class to sit together?
- There are 50 students in the class
  - P(50,3) = 50x49x48 = 117,600
  - Note that the positions they take do matter

14

Permutations vs. *r*-permutations

- *r*-permutations: Choosing an ordered 5 card hand is $P(52,5)$
  - When people say "permutations", they almost always mean *r*-permutations
    - But the name can refer to both
- Permutations: Choosing an order for all 52 cards is $P(52,52) = 52!$
  - Thus, $P(n,n) = n!$

15

Sample question

- How many permutations of {a, b, c, d, e, f, g} end with a?
  - Note that the set has 7 elements
  - The last character must be a
  - The rest can be in any order
- Thus, we want a 6-permutation on the set {b, c, d, e, f, g}
- P(6,6) = 6! = 720
- Why is it not P(7,6)?

16