Recursively Defined Sequences

CS231

Dianna Xu

1

Recurrence Relation

- It is often more convenient to define a sequence using previous terms:
  - $a_n = 2a_{n-1}$
    $a_0 = 1$
  - $a_n = a_{n-1} + a_{n-2}$
    $a_0 = 1$
    $a_1 = 1$

2

Recursive Definitions of Sums and Product

- Sum $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n-1} a_i + a_n \quad \sum_{i=1}^{1} a_i = a_1$
  - sum(n) = sum(n-1)+n, n>1
  - sum(1) = 1
- Product $\prod_{i=1}^{n} a_i = \prod_{i=1}^{n-1} a_i \times a_n \quad \prod_{i=1}^{1} a_i = a_1$
  - product(n) = product(n-1)xn, n>1
  - product(1) = 1

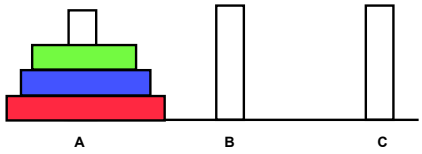
3

The Towers of Hanoi

- Move all rings from one peg to another, without ever placing a larger ring on top of a smaller one

4

The Towers of Hanoi

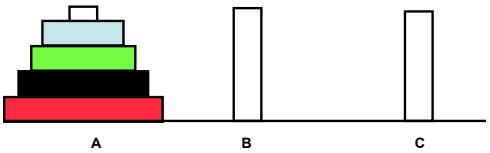


Goal: Move stack of rings to another peg

- Rule 1: May move only 1 ring at a time
- Rule 2: May never have larger ring on top of smaller one

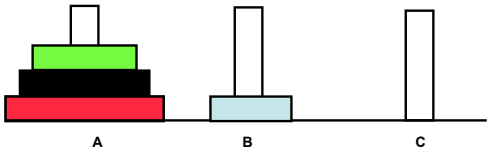
5

The Towers of Hanoi



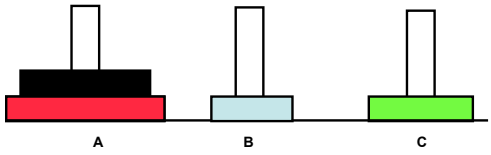
6

The Towers of Hanoi



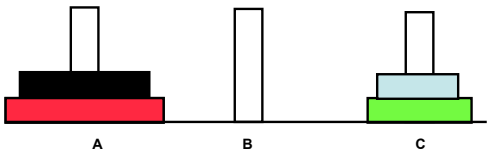
7

The Towers of Hanoi



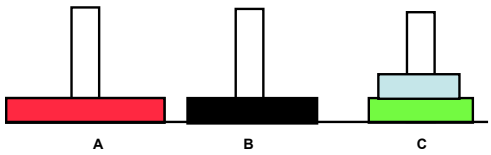
8

The Towers of Hanoi



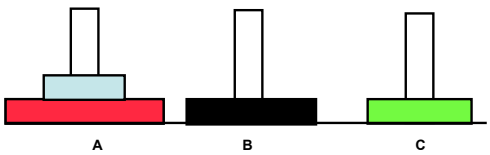
9

The Towers of Hanoi



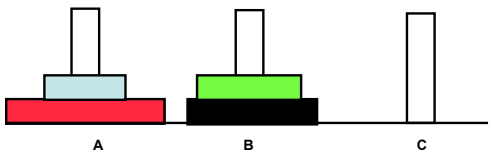
10

The Towers of Hanoi



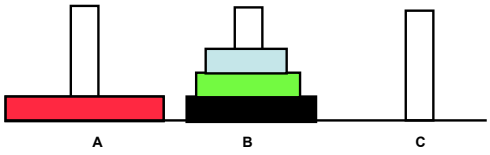
11

The Towers of Hanoi



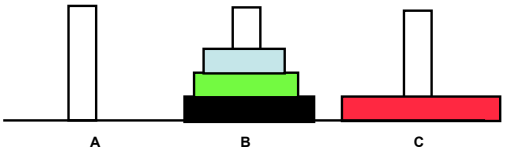
12

The Towers of Hanoi



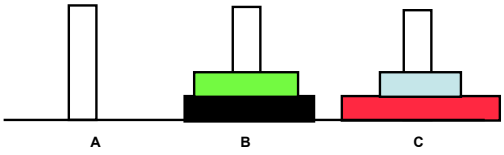
13

The Towers of Hanoi



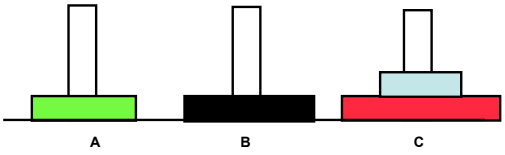
14

The Towers of Hanoi



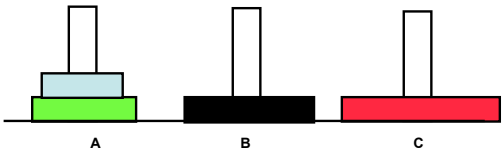
15

The Towers of Hanoi



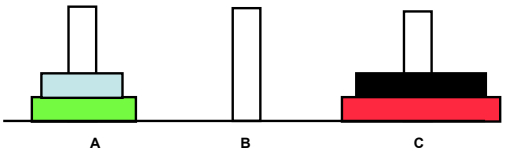
16

The Towers of Hanoi



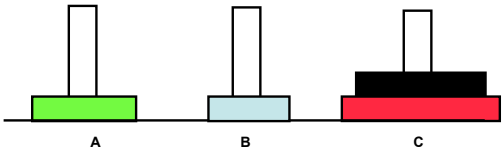
17

The Towers of Hanoi



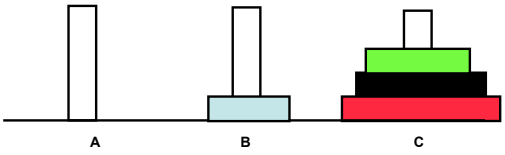
18

The Towers of Hanoi



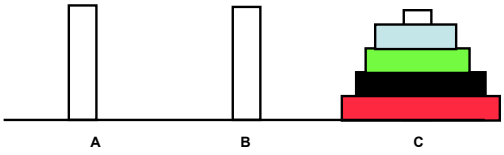
19

The Towers of Hanoi



20

The Towers of Hanoi



21

The Tower of Hanoi

- How many moves does it take to move 64 disks?
- $m_k = m_{k-1} + 1 + m_{k-1} = 2m_{k-1} + 1, \; m_1 = 1$

$m_2 = 2 \times 1 + 1 = 3 = 2^2 - 1$

$m_3 = 2 \times 3 + 1 = 7 = 2^3 - 1$

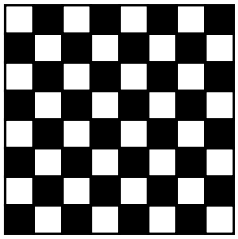
$m_4 = 2 \times 7 + 1 = 15 = 2^4 - 1$

$m_5 = 2 \times 15 + 1 = 31 = 2^5 - 1$

22

The Power of Exponents

A rich king and a wise peasant…



23

The Wise Peasant's Pay

Day(n)	Pieces of Grain
1	2
2	4
3	8
4	16
…	
63	9,223,000,000,000,000,000
64	18,450,000,000,000,000,000

2^N

24

How Bad is 2^n?

- Imagine being able to grow a billion (1,000,000,000) pieces of grain a second…
- It would take
  - **585 years** to grow enough grain just for the 64th day
  - Over a thousand years to fulfill the peasant's request**!**

25

So the King cut off the peasant's head.

26