Valid and Invalid Arguments

CS 231
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Review

- $p \rightarrow q \equiv$
- $\sim p \vee q$
- Contrapositive:
- $\sim q \rightarrow \sim p$
- Inverse:
- $\sim p \rightarrow \sim q$
- Converse:
- $q \rightarrow p$
- $p \oplus q \equiv$
- $(p \vee q) \wedge \sim(p \wedge q)$
- $\sim(p \wedge q) \equiv$
- $\sim p \vee \sim q$
- $\sim(p \vee q) \equiv$
- $\sim p \wedge \sim q$
- p is sufficient for q
- $p \rightarrow q$
- p is necessary for q
- $\sim p \rightarrow \sim q$

Review

- Associative Law:
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- Distributive Law:
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
- Absorption Law:
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$

Definitions

- An argument is a sequence of statements (statement forms).
- All statements in an argument except for the last one, are called premises. (assumptions, hypotheses)
- The final statement is the conclusion.
- A valid argument means the conclusion is true if the premises are all true, with all combinations of variable truth values.

Examples

- All Greeks are human and all humans are mortal; therefore, all Greeks are mortal.
- Some men are athletes and some athletes are rich; therefore, some men are rich.
- Some men are swimmers and some swimmers are fish; therefore, some men are fish.

Modus Ponens

$$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$$

Modus Ponens example

- Assume you are given the following two statements:
  - "you are in this class"
  - "If you are in this class, you are a student"

$$\begin{array}{l} p \\ \underline{p \to q} \\ \therefore q \end{array}$$

- Let p = "you are in this class"
- Let q = "you are a student"

- By Modus Ponens, you can conclude that you are a student.

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Modus Ponens

- Consider $(p \wedge (p \to q)) \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$$\begin{array}{l} p \\ \underline{p \to q} \\ \therefore q \end{array}$$

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Modus Tollens

- Assume that we know: $\sim q$ and $p \to q$
  - Recall that $p \to q \equiv \sim q \to \sim p$
- Thus, we know $\sim q$ and $\sim q \to \sim p$
- We can conclude $\sim p$

$$\begin{array}{l} \sim q \\ \underline{p \to q} \\ \therefore \sim p \end{array}$$

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Modus Tollens example

- Assume you are given the following two statements:
  - "you are not a student"
  - "if you are in this class, you are a student"

$$\begin{array}{l} \sim q \\ \underline{p \to q} \\ \therefore \sim p \end{array}$$

- Let p = "you are in this class"
- Let q = "you are a student"

- By Modus Tollens, you can conclude that you are not in this class

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Generalization & Specialization

- Generalization: If you know that p is true, then $p \vee q$ will ALWAYS be true

$$\frac{p}{\therefore p \vee q} \qquad \frac{q}{\therefore p \vee q}$$

- Specialization: If $p \wedge q$ is true, then p will ALWAYS be true

$$\frac{p \wedge q}{\therefore p} \qquad \frac{p \wedge q}{\therefore q}$$

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Example of proof

- We have the hypotheses:
  - "It is not sunny this afternoon and it is colder than yesterday"
  - "We will go swimming only if it is sunny"
  - "If we do not go swimming, then we will take a canoe trip"
  - "If we take a canoe trip, then we will be home by sunset"
- Does this imply that "we will be home by sunset"?

p
q
r
s
t

$$\begin{array}{l} \sim p \wedge q \\ r \to p \\ \sim r \to s \\ \underline{s \to t} \\ t \end{array}$$

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Example of proof

1. ~p ∧ q — 1st hypothesis
2. ~p — Specialization using step 1
3. r → p — 2nd hypothesis
4. ~r — Modus tollens using steps 2 & 3
5. ~r → s — 3rd hypothesis
6. s — Modus ponens using steps 4 & 5
7. s → t — 4th hypothesis
8. t — Modus ponens using steps 6 & 7

$$\frac{p \wedge q}{\therefore p} \qquad \frac{p \quad p \to q}{\therefore q} \qquad \frac{\sim q \quad p \to q}{\therefore \sim p}$$

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More rules of inference

- Conjunction: if *p* and *q* are true separately, then *p*∧*q* is true $\qquad \frac{p \quad q}{\therefore p \wedge q}$
- Elimination: If *p*∨*q* is true, and *p* is false, then *q* must be true $\qquad \frac{p \vee q \quad \sim p}{\therefore q} \quad \frac{p \vee q \quad \sim q}{\therefore p}$
- Transitivity: If *p*→*q* is true, and *q*→*r* is true, then *p*→*r* must be true $\qquad \frac{p \to q \quad q \to r}{\therefore p \to r}$

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Even more rules of inference

- Proof by division into cases: if at least one of *p* or *q* is true, then *r* must be true $\qquad \frac{p \vee q \quad p \to r \quad q \to r}{\therefore r}$
- Contradiction rule: If ~*p*→**c** is true, we can conclude *p* (via the contra-positive) $\qquad \frac{\sim p \to c}{\therefore p}$
- Resolution: If *p*∨*q* is true, and ~*p*∨*r* is true, then *q*∨*r* must be true $\qquad \frac{p \vee q \quad \sim p \vee r}{\therefore q \vee r}$
  - Not in the textbook

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Example of proof

- Given the hypotheses:
  - "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on" — $(\sim r \vee \sim f) \to (s \wedge l)$
  - "If the sailing race is held, then the trophy will be awarded" — $s \to t$
  - "The trophy was not awarded" — $\sim t$
- Can you conclude: "It rained"? — r

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Example of proof

1. $\sim t$ — 3rd hypothesis
2. $s \to t$ — 2nd hypothesis
3. $\sim s$ — Modus tollens using steps 1 & 2
4. $(\sim r \vee \sim f) \to (s \wedge l)$ — 1st hypothesis
5. $\sim(s \wedge l) \to \sim(\sim r \vee \sim f)$ — Contrapositive of step 4
6. $(\sim s \vee \sim l) \to (r \wedge f)$ — DeMorgan's law and double negation law
7. $\sim s \vee \sim l$ — Generalization using step 3
8. $r \wedge f$ — Modus ponens using steps 6 & 7
9. r — Specialization using step 8

$$\frac{p \quad p \to q}{\therefore q} \qquad \frac{p}{\therefore p \vee q} \qquad \frac{p \wedge q}{\therefore p} \qquad \frac{\sim q \quad p \to q}{\therefore \sim p}$$

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~~Modus Badus~~

Fallacy of the converse — Fallacy of affirming the conclusion

- Consider the following: $\qquad \frac{q \quad p \to q}{\therefore p} \qquad \frac{q \quad \sim q \to \sim p}{\therefore p}$
- Is this true?

p	q	p→q	q∧(p→q)	(q∧(p→q)) → p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Not a valid rule!

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Modus Badus example

- Assume you are given the following two statements:
  - "you are a student" q
  - "If you are in this class, you are a student" $\underline{p \rightarrow q}$

$\therefore p$

- Let p = "you are in this class"
- Let q = "you are a student"

- It is clearly wrong to conclude that if you are a student, you must be in this class

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Fallacy of the inverse

~~Modus Badus~~

Fallacy of denying the hypothesis

- Consider the following:

$$\begin{array}{l} \sim p \\ \underline{p \rightarrow q} \\ \therefore \sim q \end{array}$$

- Is this true?

p	q	$p \rightarrow q$	$\sim p \wedge (p \rightarrow q))$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	$\boxed{F}$
F	F	T	T	T

Not a valid rule!

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Modus Badus example

- Assume you are given the following two statements:
  - "you are not in this class" $\sim p$
  - "if you are in this class, you are a student" $\underline{p \rightarrow q}$

$\therefore \sim q$

- Let p = "you are in this class"
- Let q = "you are a student"

- You CANNOT conclude that you are not a student just because you are not taking Discrete Math

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