1 Introduction

1.1 Topics

This course covers the mathematical topics most directly related to computer science. Topics include: logic, basic set theory, proof techniques, number theory, mathematical induction, recursion, recurrence relations, counting, probability and graph theory. Emphasis will be placed on providing a context for the application of the mathematics within computer science. Some of these applications are listed below.

- The analysis of algorithms requires the ability to count the number of operations in an algorithm. Recursive algorithms in particular depend on the solution to a recurrence equation, and a proof of correctness by mathematical induction.

- Software engineering uses sets, graphs, trees and other data structures.

- Number theory is at the heart of secure messaging systems and cryptography.

- Logic is used in AI research in theorem proving and in database query systems.

- Proofs by induction and the more general notions of mathematical proof are ubiquitous in theory of computation, compiler design and formal grammars.

1.2 Lectures and Textbook

Lectures will be 80-minutes every Tuesdays and Thursdays from 12:55pm-2:15pm.
The text book is *Discrete Mathematics with Applications, $4^{th}Ed$* by Susanna S. Epp.
The course website: http://cs.brynmawr.edu/Courses/cs231/spring2015/. Required readings from the text will be posted on the syllabus.

1.3 Grading

There are three exams. Homeworks consist of problem sets. All homeworks and exams will receive a numerical score. At the end of the semester, a total score (to which the corresponding final grade is assigned) will be calculated from a weighted average of all scores according to the following weights: Homeworks: 25%, Exams: 25% each, Total: 100%

2 What is Discrete Mathematics?

Study of mathematical structures that are discrete rather than continuous, therefore it excludes topics in "continuous mathematics" such as calculus. Topics that we are about to cover includes logic, set theory, elementary number theory, counting and probability, and graph theory (if time allows).

Some examples of Discrete Math:

Example 1 *Counting examples.*

- *How many different ways to choose a pair of people from a group of n people?*

- *How many pins are there in a bowling alley with n lanes?*

- *How many unique squares are there in an $n \times n$ grid?*

Example 2 *The logician Raymond Smullyan describes an island containing two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who speak to you as follows:*

> *A says: B is a knight.*
> *B says: A and I are of opposite type.*

What are A and B?

3   What is a proof?

Mathematics deals with various questions, not only problems solving equitions or computing answers numbers (which you are familiar with from high school math), but also problems involving sets, functions and many other mathmatical objects (e.g., graphs). When mathematicians solve all kinds of math problems, they often justify their conclusions with *deductive reasoning* in the form of a *proof*.

Here are some examples:

- **Theorem 1** $\sqrt{2}$ *is irrational.*

- **Theorem 2** *There are infinitely many prime numbers.*

- How many pairs from a group of n? How big is the nth triangle number?
  Make a list, pair them up: $(n-1) + (n-2) + ... + 2 + 1$. This is also the number of appropriate pins in an alley with $n-1$ lanes. These numbers are called *triangle numbers*.

  As you may know, *square numbers* are $0, 1, 4, 9, ...$ since they are squares. Therefore, the formular for square numbers are n^2.

  To get a formula to compute triangle numbers, there are different ways:

  - Counting argument:
  - Geometric argument: [Draw graph] Notation matters!
  - The Gauss' Pairing idea: