Expected Values

CS231
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Probability Axioms

• Let $E$ be an event in a sample space $S$. The probability of the complement of $E$ is:
  $$P(\bar{E}) = 1 - P(E)$$

• The probability of the union of two events $E_1$ and $E_2$ is:
  $$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
Example

• If you choose a number between 1 and 100, what is the probability that it is divisible by 2 or 5 or both?

• Let $n$ be the number chosen
  - $p(2|n) = 50/100$ (all the even numbers)
  - $p(5|n) = 20/100$
  - $p(2|n)$ and $p(5|n) = p(10|n) = 10/100$
  - $p(2|n)$ or $p(5|n) = p(2|n) + p(5|n) - p(10|n)$
    - $= 50/100 + 20/100 - 10/100$
    - $= 3/5$
When is gambling worth it?

• This is a *statistical* analysis, not a moral/ethical discussion

• What if you gamble $1, and have a $\frac{1}{2}$ probability to win $10$?

• What if you gamble $1$ and have a $1/100$ probability to win $10$?

• One way to determine if gambling is worth it:
  – probability of winning * payout $\geq$ amount spent per play
Expected Value

- The expected value of a process with outcomes of values $a_1, a_2, ..., a_n$ which occur with probabilities $p_1, p_2, ..., p_n$ is:

$$\sum_{i=1}^{n} a_i p_i$$
Expected values of gambling

• Gamble $1, and have a \( \frac{1}{2} \) probability to win $10
  \[(10-1)*0.5+(-1)*0.5 = 4\]

• Gamble $1 and have a \( \frac{1}{100} \) probability to win $10?
  \[(10-1)*0.01+(-1)*0.99 = -0.9\]

• Another way to determine if gambling is worth it: Expected value > 0
When is lotto worth it?

• In many older lotto games (Pick-6) you have to choose 6 numbers from 1 to 48
  – Total possible choices (order does not matter) are $C(48,6) = 12,271,512$
  – Total possible winning numbers is $C(6,6) = 1$
  – Probability of winning is $0.0000000814$
    • Or 1 in 12.3 million

• If you invest $1 per ticket, it is only statistically worth it if the payout is $> 12.3$ million
Powerball lottery

• Modern powerball lottery: you pick 5 numbers from 1-55
  – Total possibilities: $\binom{55}{5} = 3,478,761$
• You then pick one number from 1-42 (the powerball)
  – Total possibilities: $\binom{42}{1} = 42$
• You need to do both - apply the product rule,
  – Total possibilities are $3,478,761 \times 42 = 146,107,962$
• The probability for the jackpot is about 1 in 146 million
• If you count in the other (sub)prizes, then you will “break even” if the jackpot is $121M