CS 231 Review Questions 11/5/13

1. Prove that if m is an even integer, then $m + 7$ is odd. Do this proof in three ways: direct proof, proof by contraposition and proof by contradition.
2. Using proof by contradiction, prove that $\forall n \in \mathbb{Z}, 4 \nmid (n^2 + 2)$
3. Using induction, prove that for all integers $n \geq 1, 2^{2n} - 1$ is divisible by 3, i.e. $3|2^{2n} - 1$
4. Given sets A, B, and C in the same universe, determine if each of the following statements is true or false. If it is true, then prove it. If it is false, then give a counter example.
   (a) $C \subseteq A \wedge C \subseteq B \rightarrow C \subseteq A \cup B$
   (b) $C \subseteq A \cup B \rightarrow C \subseteq A \wedge C \subseteq B$
   (c) $\overline{A} \cap (A \cup B) = B \setminus A$
5. Prove that give a set S, the cardinality of its power set is $2^{|S|}$.
6. Prove that if $a_1, a_2, ..., a_n$ are n distinct real numbers, exactly $n - 1$ multiplications are needed to compute the product of these n numbers, no matter how parentheses are inserted into their product.
7. For each of the following, give a *recursive* definition. Remember to indicate the initial terms or base:
   (a) $a_n = \sum_{i=0}^{n} i$
   (b) The sequence that generates the terms 3, 6, 12, 24, 48, 96, 192, ...
   (c) The set of non-negative even numbers
   (d) The set of all even numbers
8. Find explicit formulae for the following recursively defined sequences, and prove correctness using induction.
   (a) $a_k = k - a_{k-1}$, $\forall k \geq 1$, $a_0 = 0$.
   (b) $a_k = 2a_{k-2}$, $\forall k \geq 2$, $a_0 = 1, a_1 = 2$.
9. Prove the correctness of the following algorithm:

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[Pre-condition: i=1 and sum=0]
while(i<=100)
  sum := sum + i
  i := i + 1
end while
[Post-condition: sum = 1 + 2 + ... + 100]
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   State your loop invariant clearly.
10. Given the following recursive definition of a set S:
    - Basis: $\lambda \in S$
    - Recursive: $x \in S \rightarrow axa \in S$

    Prove using structural induction, that $\forall s \in S, |s|$ is even.