1. Determine whether each of the following is a **statement**, a **predicate**, or neither. Give true values to **statements** when possible.

   (a) $\sqrt{2} > 1$ Statement, True

   (b) $\sin^2 x + \cos^2 x = 1$ Predicate

   (c) When is the square of a number greater than one? Neither

   (d) Get some rest. Neither

   (e) a is an even number. Predicate

   (f) $\forall x \in \mathbb{R}, (x^2 > 3 \rightarrow x > 1)$ Statement, False

2. Give the **converse, inverse, contrapositive**, and (non-trivial!) **negation** of the following statements.

   (a) $\sim p \to r$
   Converse: $r \to \sim p$
   Inverse: $p \to \sim r$
   Contrapositive: $\sim r \to p$
   Negation: $(\sim p) \wedge \sim r$

3. Simplify the following statement, citing laws ued at every step:

$$\sim ((p \vee q) \rightarrow (p \wedge q))$$

$\sim ((p \vee q) \rightarrow (p \wedge q))$	Original
$\equiv \sim ((p \wedge q) \vee \sim (p \vee q))$	Implication
$\equiv \sim (p \wedge q) \wedge \sim\sim (p \vee q)$	DeMorgan's
$\equiv \sim (p \wedge q) \wedge (p \vee q)$	Double negative
$\equiv p \oplus q$	Definition of $\oplus$

4. Convert the following binary number to hexadecimal (base-16): 1101001110011010_2

   $1101_2 \quad 0011_2 \quad 1001_2 \quad 1010_2$

   $D_{16} \quad\; 3_{16} \quad\; 9_{16} \quad\; A_{16}$

   Answer: $D39A_{16}$

5. Convert the following base-5 number to decimal: 1234_5
   $1234_5 = 1 * 5^3 + 2 * 5^2 + 3 * 5^1 + 4 * 5^0 = 125 + 50 + 15 + 4 = 194_{10}$

6. Find the decimal equivalent of the following 16-bit 2's complement number: 1001110000000101_2

1001	1100	0000	0101		original
0110	0011	1111	1010		1's complement
0110	0011	1111	1011		+1
6	3	F	B		hex
6×16^3	$+3 \times 16^2$	$+15 \times 16$	$+11$	$= 25595$	decimal

Answer: -25595

7. For the following table,

(a) construct a Boolean expression having the table as its truth table

(b) design a **minimal** circuit having the table as its input/output table

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

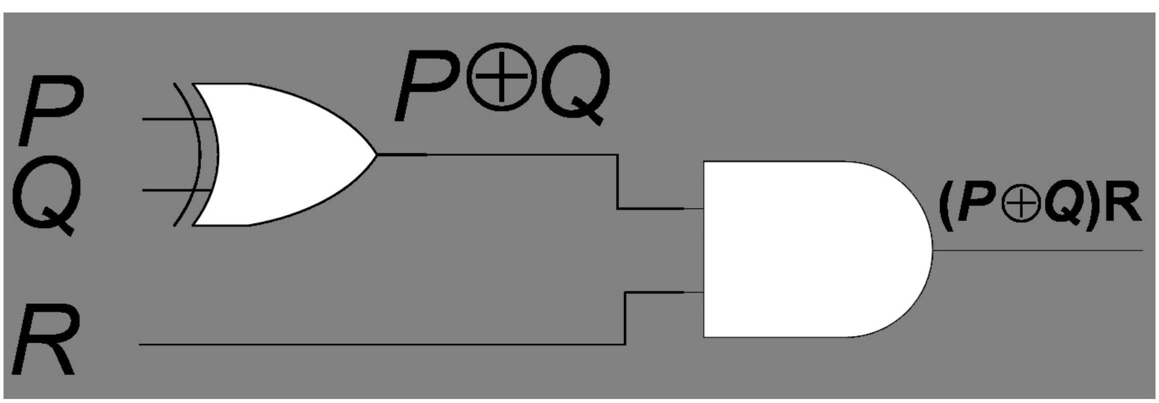
(a) Identify all rows in the table with an output of 1 and construct boolean expressions that are true only for these rows:

P	Q	R	S	Boolean Expression
1	0	1	1	$P\wedge \sim Q \wedge R$ or $(P\overline{Q}R)$
0	1	1	1	$\sim P \wedge Q \wedge R$ or $(\overline{P}QR)$

"Or" these expressions together for the final solution:
$(P\wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R)$ or $(P\overline{Q}R) + (\overline{P}QR)$

(b)

$$
\begin{aligned}
& (P\wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R) \\
\equiv\ & ((P\wedge \sim Q) \vee (\sim P \wedge Q)) \wedge R \\
\equiv\ & (((P\wedge \sim Q) \vee \sim P) \wedge ((P\wedge \sim Q) \vee Q)) \wedge R \\
\equiv\ & (((P\vee \sim P) \wedge (\sim Q\vee \sim P)) \wedge ((P \vee Q) \wedge (\sim Q \vee Q))) \wedge R \\
\equiv\ & ((\sim Q\vee \sim P) \wedge (P \vee Q)) \wedge R \\
\equiv\ & (\sim (Q \wedge P) \wedge (P \vee Q)) \wedge R \\
\equiv\ & (P \oplus Q) \wedge R
\end{aligned}
$$



8. Rewrite the following statements formally (by defining predicates and using quantifiers and variables, in other words, no English in the final statement). You may assume the domain is $\mathbb{R}$, and use the following predicate definitions only:

   - $Integer(x)$: x is an integer
   - $Even(x)$: x is even (an even integer)

   (a) Every integer is either even or odd.

   $$\forall x, Integer(x) \rightarrow Even(x) \vee \sim Even(x)$$

   (b) The sum of two odd integers is an even integer.

   $$\forall x \forall y, Integer(x) \wedge \sim Even(x) \wedge Integer(y) \wedge \sim Even(y) \rightarrow Even(x+y)$$

   (c) There is an integer whose multiplicative inverse is also an integer.

   $$\exists x, Integer(x) \wedge Integer(\frac{1}{x})$$

9. Write the non-trivial **negation** of the following statements:

   (a) There exist a real number x such that for all real numbers y, $xy > y$.
   For all real numbers x, there exists a real number y such that $xy \leq y$.

   (b) $\forall n \in \mathbb{Z}, (n \text{ even} \rightarrow n+1 \text{ odd})$.
   $\exists n \in \mathbb{Z}, n \text{ even} \wedge n+1 \text{ even}$

10. Consider the following statements:

    (a) All hummingbirds are richly colored.
    (b) No large birds live on honey.
    (c) Birds that do not live on honey are dull in color.
    (d) Hummingbirds are small.

10.1. Express each of the the statements using quantifiers and predicates (no English). Assume the domain is all birds. Clearly define the predicates you are using.

- $H(x) : x$ is a hummingbird.
- $R(x) : x$ is richly colored.
- $L(x) : x$ is large.
- $LH(x) : x$ lives on honey.

(a) $\forall x, H(x) \to R(x)$
(b) $\forall x, \sim L(x) \vee \sim LH(x)$
(c) $\forall x, \sim LH(x) \to \sim R(x)$
(d) $\forall x, H(x) \to \sim L(x)$

10.2. Can you conclude (d), i.e. "Hummingbirds are small", from the first three? Prove this using logical equivalences and rules of inference. Clearly label which rule you are using on each step.

1	$\forall x, H(x) \to R(x)$	hypothesis (a)
2	$\forall x, \sim LH(x) \to \sim R(x)$	hypothesis (c)
3	$\forall x, R(x) \to LH(x)$	Contrapositive from 2
4	$\forall x, H(x) \to LH(x)$	Universal transitivity from 1 and 3
5	$\forall x, \sim L(x) \vee \sim LH(x)$	hypothesis (b)
6	$\forall x, \sim LH(x) \vee \sim L(x)$	Communicative from 5
7	$\forall x, LH(x) \to \sim L(x)$	Definition of implication from 6
8	$\forall x, H(x) \to \sim L(x)$	Universal transitivity from 4 and 7