1. Determine whether each of the following is a **statement**, a **predicate**, or neither. If possible, determine whether each of the **statements** is True or False.

   (a) $\sqrt{2} > 1$ Statement, True

   (b) $\sin^2 x + \cos^2 x = 1$ Predicate

   (c) When is the square of a number greater than one? Neither

   (d) Get some rest. Neither

   (e) a is an even number. Predicate

   (f) $\forall x \in \mathbf{R}, (x^2 > 3 \rightarrow x > 1)$ Statement, False

2. Give the **converse, inverse, contrapositive**, and (non-trivial!) **negation** of the following statements.

   (a) $\sim p \rightarrow r$
   Converse: $r \rightarrow \sim p$
   Inverse: $p \rightarrow \sim r$
   Contrapositive: $\sim r \rightarrow p$
   Negation: $(\sim p) \wedge \sim r$

3. Find an equivalent statement that contains only $\vee$ or $\wedge$:

$$\sim ((p \vee q) \rightarrow (p \wedge q))$$

$$\begin{aligned}
\sim ((p \vee q) &\rightarrow (p \wedge q)) \\
&\equiv \sim ((p \wedge q) \vee \sim (p \vee q)) \\
&\equiv \sim (p \wedge q) \wedge (p \vee q) \\
&\equiv (\sim p \vee \sim q) \wedge (p \vee q) \\
&\equiv ((p \vee q) \wedge (\sim p)) \vee ((p \vee q) \wedge (\sim q)) \\
&\equiv (p \wedge \sim p) \vee (\sim p \wedge q) \vee (\sim q \wedge p) \vee (\sim q \wedge q) \\
&\equiv (c \vee (\sim p \wedge q)) \vee (\sim q \wedge p \vee c) \\
&\equiv (\sim p \wedge q) \vee (\sim q \wedge p)
\end{aligned}$$

4. Convert the following binary number to hexadecimal (base-16): 1101001110011010
   [1101][0011][1001][1010] In binary, this converts to $D39A_{16}$

5. Convert the following base-5 number to decimal: 1234
   $1234_5 = 1 * 5^3 + 2 * 5^2 + 3 * 5^1 + 4 * 5^0 = 125 + 50 + 15 + 4 = 194_{10}$

6. For the following table,

   (a) construct a Boolean expression having the table as its truth table

   (b) design a circuit having the table as its input/output table

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

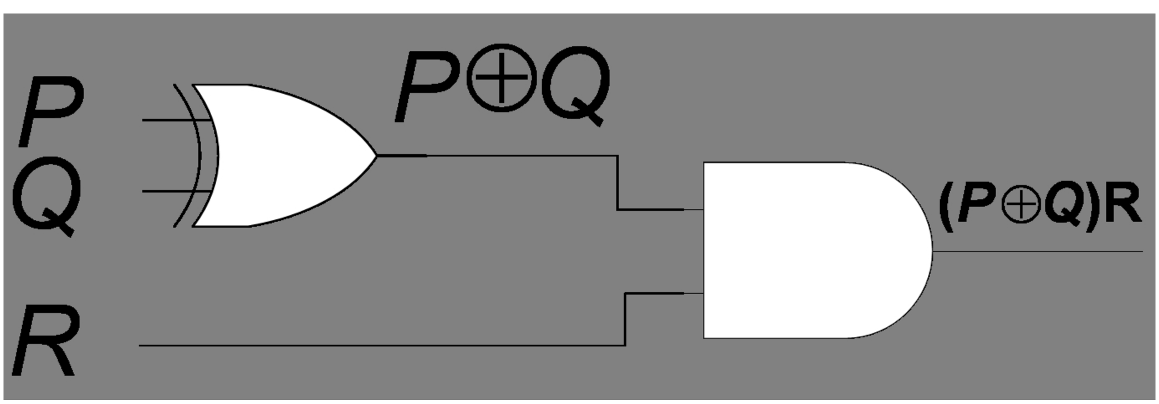
(a) Identify all rows in the table with an output of 1 and construct boolean expressions that are true only for these rows:

P	Q	R	S	Boolean Expression
1	0	1	1	$P\wedge \sim Q \wedge R$ or $(P\overline{Q}R)$
0	1	1	1	$\sim P \wedge Q \wedge R$ or $(\overline{P}QR)$

"Or" these expressions together for the final solution:
$(P\wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R)$ or $(P\overline{Q}R) + (\overline{P}QR)$

(b)

$$
\begin{aligned}
& (P\wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R) \\
\equiv\ & ((P\wedge \sim Q) \vee (\sim P \wedge Q)) \wedge R \\
\equiv\ & (((P\wedge \sim Q) \vee \sim P) \wedge ((P\wedge \sim Q) \vee Q)) \wedge R \\
\equiv\ & (((P\vee \sim P) \wedge (\sim Q \vee \sim P)) \wedge ((P \vee Q) \wedge (\sim Q \vee Q))) \wedge R \\
\equiv\ & ((\sim Q \vee \sim P) \wedge (P \vee Q)) \wedge R \\
\equiv\ & (\sim (Q \wedge P) \wedge (P \vee Q)) \wedge R \\
\equiv\ & (P \oplus Q) \wedge R
\end{aligned}
$$



7. Rewrite the following statements formally (using quantifiers and variables):

   (a) Some integers are not necessarily the product of two prime numbers.
   Let P(x) = x is prime
   $\exists x \in \mathbb{Z}, \forall a, b \in \mathbb{Z}, P(a) \wedge P(b) \rightarrow x \neq a \times b$

8. Write the non-trivial **negation** of the following statements:

   (a) There exist a real number x such that for all real numbers y, $xy > y$.
   For all real numbers x, there exists a real number y such that $xy \leq y$.

   (b) $\forall n \in \mathbf{Z}, (n \text{ even} \rightarrow n + 1 \text{ odd})$.
   $\exists n \in \mathbf{Z}, n \text{ even} \wedge n + 1 \text{ even}$