Name:

Math/CS 231 Exam 1 Solutions 10/09/09

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1. Determine whether each of the following is a **statement**, a **predicate**, or neither. If possible, determine whether each of the **statements** is True or False.

   (a) $3 < \pi < 4$ Statement, True

   (b) Do not pass go. Neither

   (c) It was hot on Monday. Statement

   (d) $\forall x \in \mathbf{R}, x + 1 \geq x$ Statement, True

   (e) iPods are white. Statement, False

   (f) $2^n \geq 100$ Predicate

   (g) Are you hungry? Neither

2. Short answers.

(a) There are two ways to convert a predicate $P(x)$ into a statement. What are they?

Given a value to x or use universal or existential quantifiers on x.

(b) Let P denote the set of all people, let $Student(x)$ be the predicate "x is a student," and let $Redhair(x)$ be the predicate "x has red hair." Consider the statement:

$$\exists x \in P, Student(x) \rightarrow Redhair(x)$$

Does this statement say that there is a student who has red hair? If not, explain why, and rewrite the statement so that it does say that there is a student who has red hair.

No, this statement says that if a person is a student, then he or she has red hair. However, there is no guarantee that any person is a student, so it doesn'tguarantee existence. Correct statement:

$$\exists x \in P, Student(x) \wedge Redhair(x)$$

3. (a) Give the **converse and inverse** of the statement:

$$(\sim p) \vee q \to r$$

Converse:

$$r \to (\sim p) \vee q$$

Inverse:

$$\sim ((\sim p) \vee q) \to \sim r \equiv (p \wedge \sim q) \to \sim r$$

(b) Give the **contrapositive**, and (non-trivial!) **negation** of the statement:
If it rains or my ankle is still sore then I won't play Badminton.
Contrapositive:
If I play badminton, then my ankle is not still sore and it is not raining.
Negation:
Either it is raining or my ankle is still sore, and I will play badminton.

4. Using logical equivalences, prove the following:

(a) $(p \wedge (p \wedge q)) \wedge (\sim p \vee q) \equiv (p \wedge q) \wedge (\sim p \vee q)$
$\equiv ((p \wedge q) \wedge \sim p) \vee ((p \wedge q) \wedge q)$
$\equiv (c \wedge q) \vee (p \wedge q)$
$\equiv c \vee (p \wedge q)$
$\equiv p \wedge q$

5. Convert the following hexadecimal (base-16) number to binary: $7B9F$

$$
\begin{aligned}
7_{16} &= 0111_2 \\
B_{16} &= 1011_2 \\
9_{16} &= 1001_2 \\
F_{16} &= 1111_2 \\
7B9F_{16} &= [0111][1011][1001][1111]_2
\end{aligned}
$$

6. For the following table,

   (a) construct a Boolean expression having the table as its truth table

   (b) design a minimal circuit having the table as its input/output table

P	Q	R	S
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

(a) Identify all rows in the table with an output of 1 and construct boolean expressions that are true only for these rows:

P	Q	R	S	Boolean Expression
1	1	1	1	$P \wedge Q \wedge R$ or (PQR)
1	0	1	1	$P \wedge \sim Q \wedge R$ or $(P\overline{Q}R)$
0	0	1	1	$\sim P \wedge \sim Q \wedge R$ or $(\overline{PQ}R)$

"Or" these expressions together for the final solution:
$(P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$ or $(P\overline{Q}R) + (\overline{P}QR) + (\overline{PQ}R)$

(b)

$$
\begin{aligned}
& (P \wedge Q \wedge R) \vee (P \wedge \sim Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R) \\
\equiv\ & ((P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)) \wedge R \\
\equiv\ & ((P \wedge (Q \vee \sim Q)) \vee (\sim P \wedge \sim Q)) \wedge R \\
\equiv\ & (P \vee (\sim P \wedge \sim Q)) \wedge R \\
\equiv\ & ((P \vee \sim P) \wedge (P \vee \sim Q)) \wedge R \\
\equiv\ & (P \vee \sim Q) \wedge R
\end{aligned}
$$

Circuit not drawn, it's much harder to do this digitally! Talk to me if you are not sure how the circuit diagram corresponding to $(P \vee \sim Q) \wedge R$ should look like.

7. Rewrite the following statements formally (using quantifiers and variables). Assume the domain is all real numbers **R**, and use the following predicate definitions:

- Integer(x): x is an integer
- Irrational(x): x is irrational
- Unique(x): x is unique
- Prime(x): x is prime

(a) No irrational number is an integer.

$$\forall x \in \mathbf{R}, Irrational(x) \rightarrow \sim Integer(x)$$

(b) There is a unique integer x whose multiplicative inverse $\frac{1}{x}$ is also an integer.

$$\exists x \in \mathbf{R}, Unique(x) \wedge Integer(x) \wedge Integer(\frac{1}{x})$$

(c) There is no largest prime.

$$\forall x \in \mathbf{R}, Prime(x) \rightarrow \exists y \in \mathbf{R}, Prime(y) \wedge y > x$$

8. Write the non-trivial **negation** of the following statements:

   (a) For any real number x, if $x(x+1) > 0$, then either $x > 0$ or $x < -1$.
   Negation: There is a real number x such that $x(x+1) > 0$ and $x \leq 0$ and $x \geq -1$.

   (b) There exists integer k such that for all odd integers n, $n = 2k+1$.
   Negation: For all integers k there exists an odd integer n such that $n \neq 2k+1$.

   (c) An integer is even if and only if it is divisible by 2.
   Negation: There exists an integer that is either even and not divisible by 2, or is odd and divisible by 2.