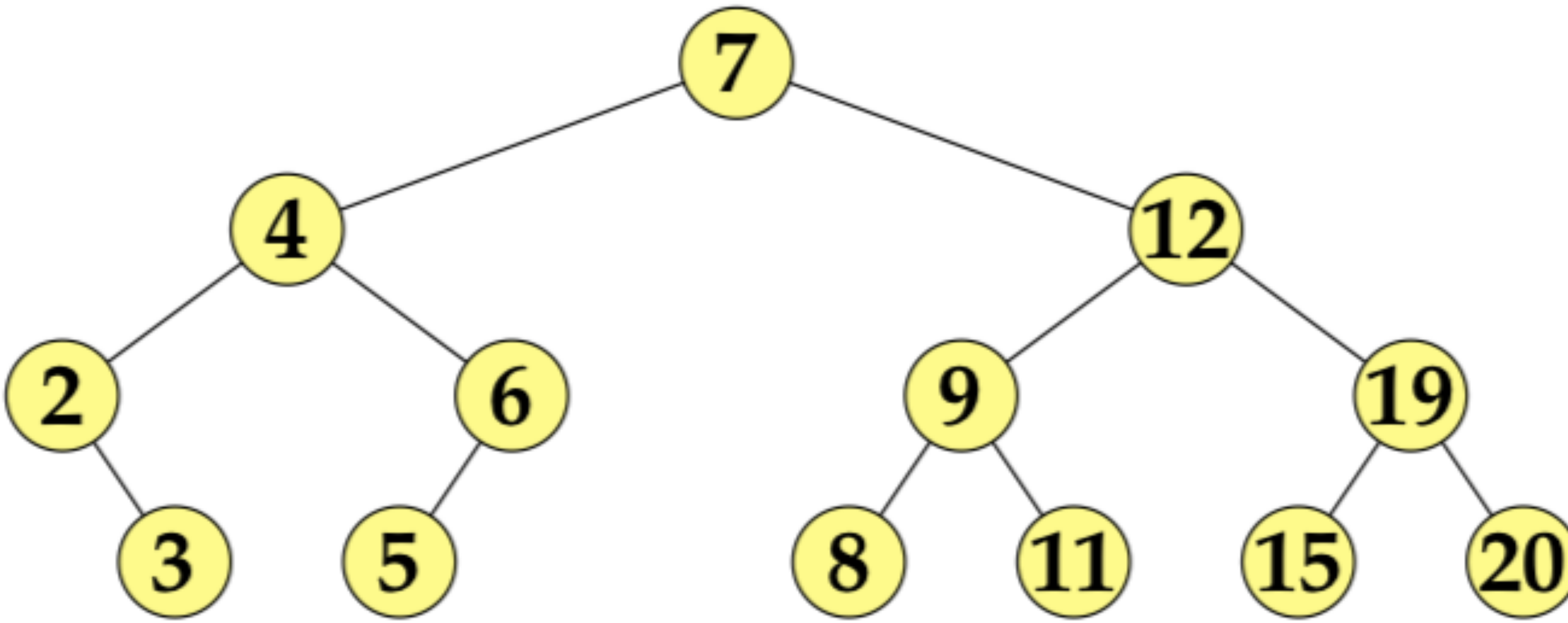
Trees

Traversals / Removal / Balancing

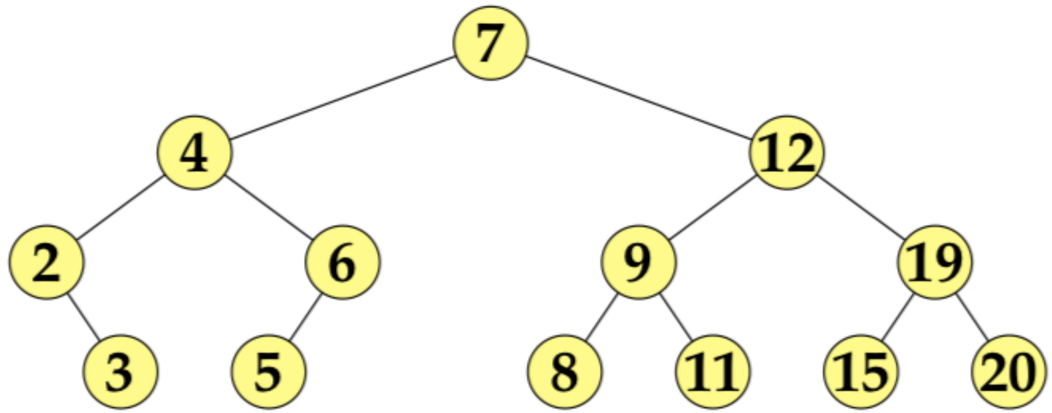
Traversals / Printing



Postorder traversal

```
public void printPostOrder() {
    printPostOrderUtil(root, 0);
    System.out.println();
}

private void printPostOrderUtil(Node treePart, int depth) {
    if (treePart==null) return;
    System.out.print("["+treePart.payload+","+depth+"]");
    printPostOrderUtil(treePart.left, depth+1);
    printPostOrderUtil(treePart.right, depth+1);
}
```

Breadth First traversal



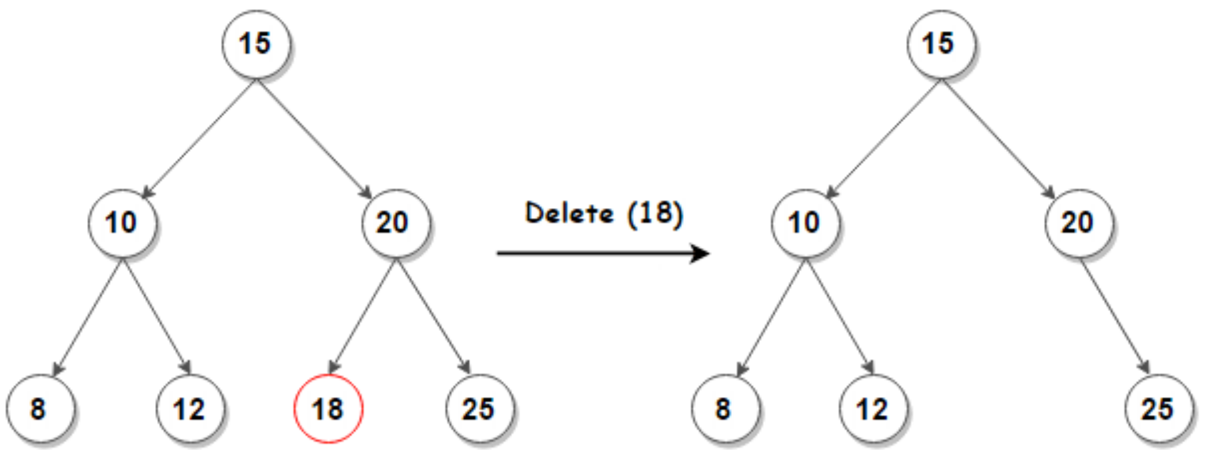
0 [7]
1 [4 12]
2 [2 6 9 19]
3 [3 5 8 11 15 20]

Remove

- `boolean remove(E element);`
- returns true if element existed and was removed and false otherwise
- Cases
  - element not in tree
  - element is a leaf
  - element has one child
  - element has two children

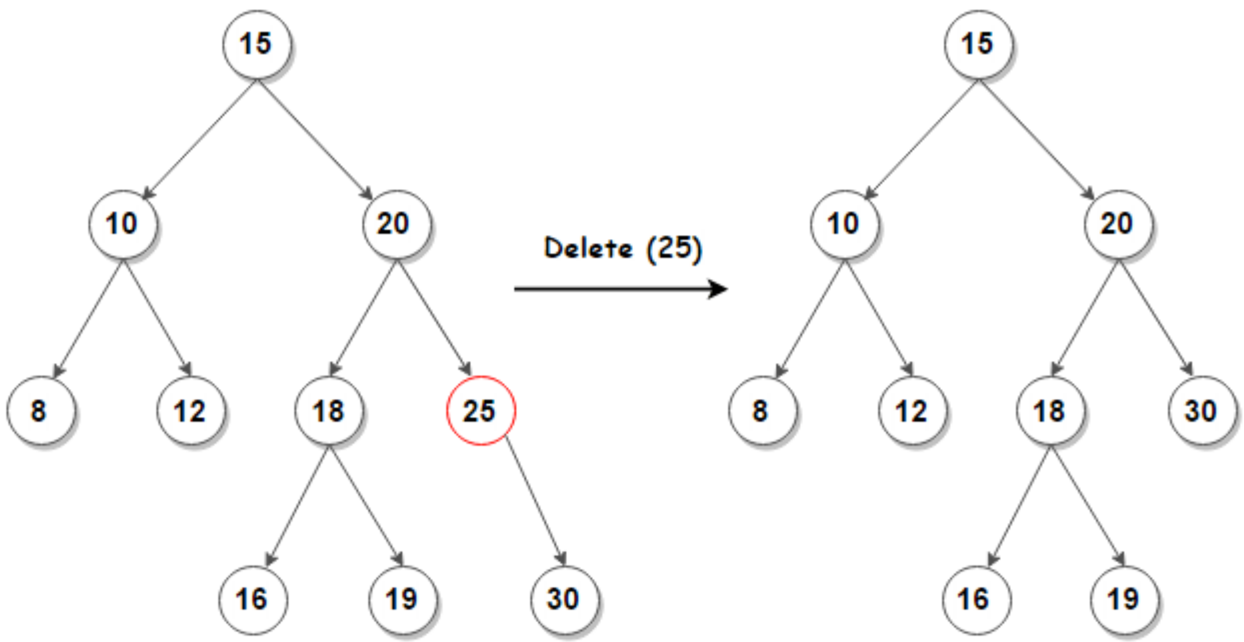
Leaf

- Just delete



One child

- Replace with child – skip over like in linked list

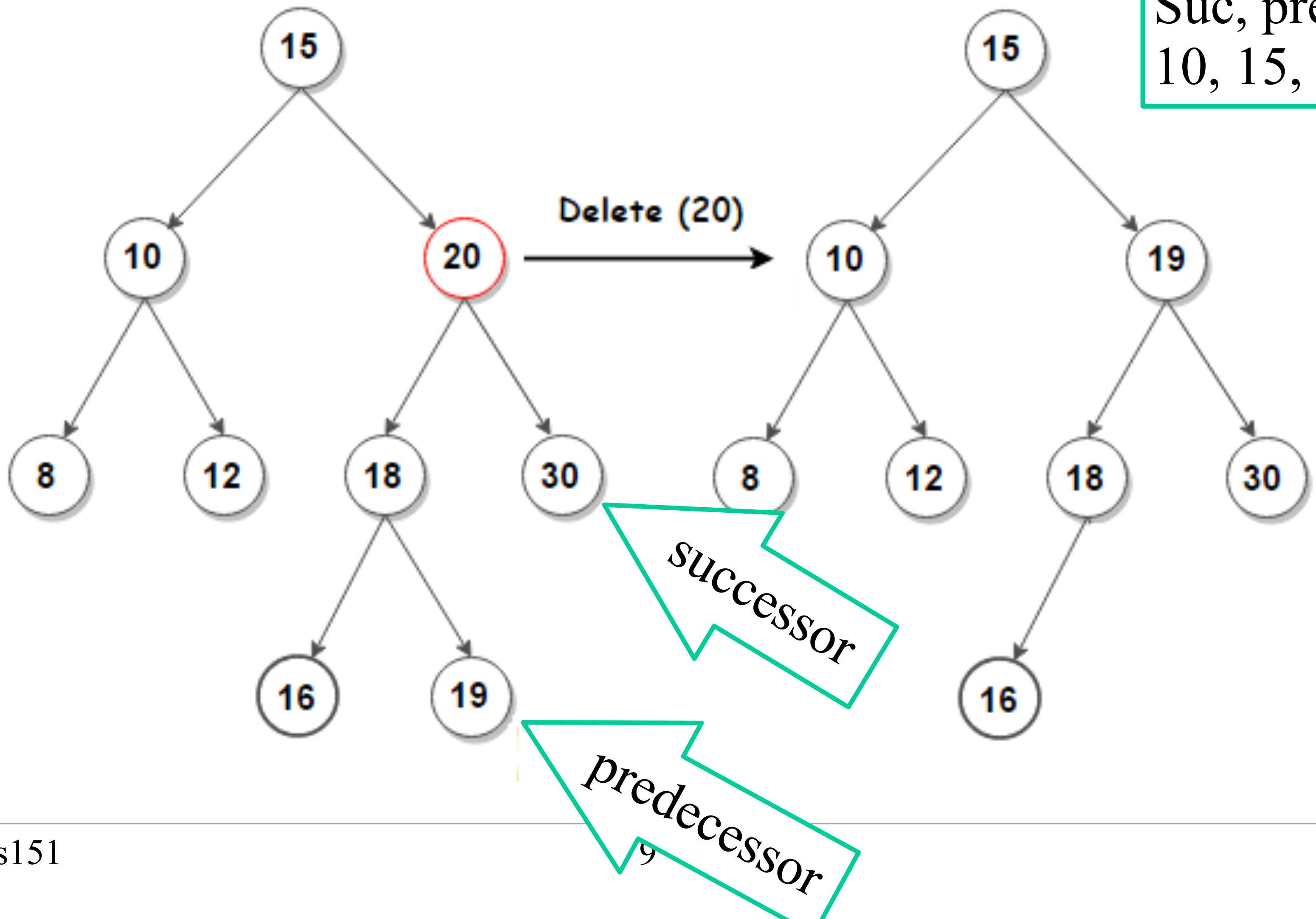


Two Children

- Replace with in-order predecessor or in-order successor
- in-order predecessor
  - rightmost child in left subtree
  - max-value child in left subtree
- in-order successor
  - leftmost child in right subtree
  - min-value child in right subtree

2 child replacement

Suc, pred for 10, 15, 19?



remove pseudocode

```
boolean remove(element)
  return removeUtil(element, root, null);

boolean removeUtil(element, node, parent)
  if (node==null) return false;
  if (node.payload>element)
    removeUtil(element, node.left, node);
  else if (node.payload<element)
    removeUtil(element, node.right, node);
  else
```

remove pseudocode 2

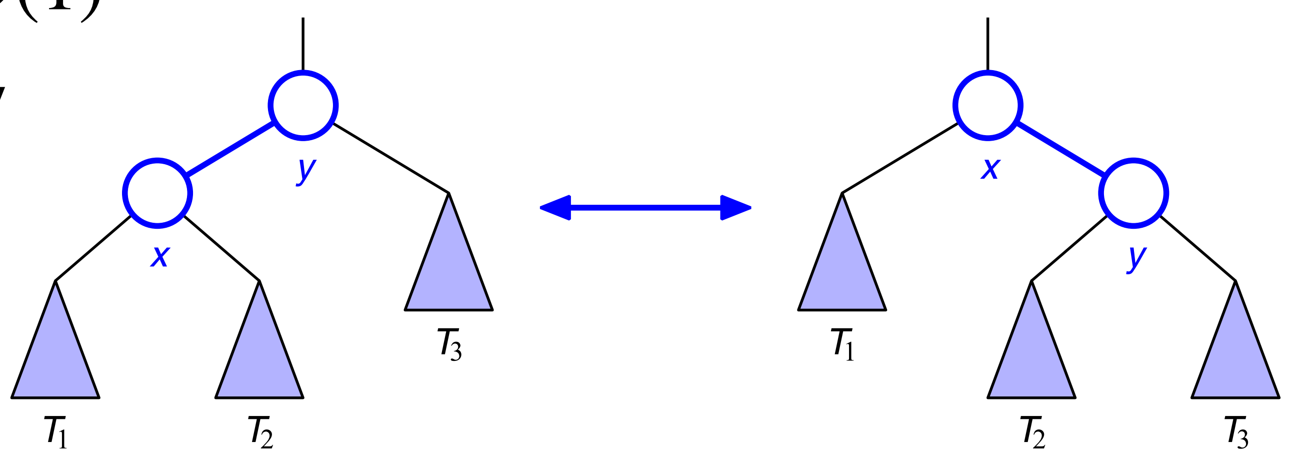
```
// found the node to delete
if (node.right==null && node.left==null)
  // at a leaf
  parent.remove(node)
  return true
if (node.right==null)
  // one descendent on left
  attach node.left to parent
  return true;
if (node.left==null)
  // one descendent on right
  attach node.right to parent
  return true;
```

remove pseudocode 3

```
// two children
successorNode = inorderSucessor(node.right)
node.payload=successorNode.payload
removeUtil(successorNode.payload, node.right
node);
return true;
```

Balanced Search Trees

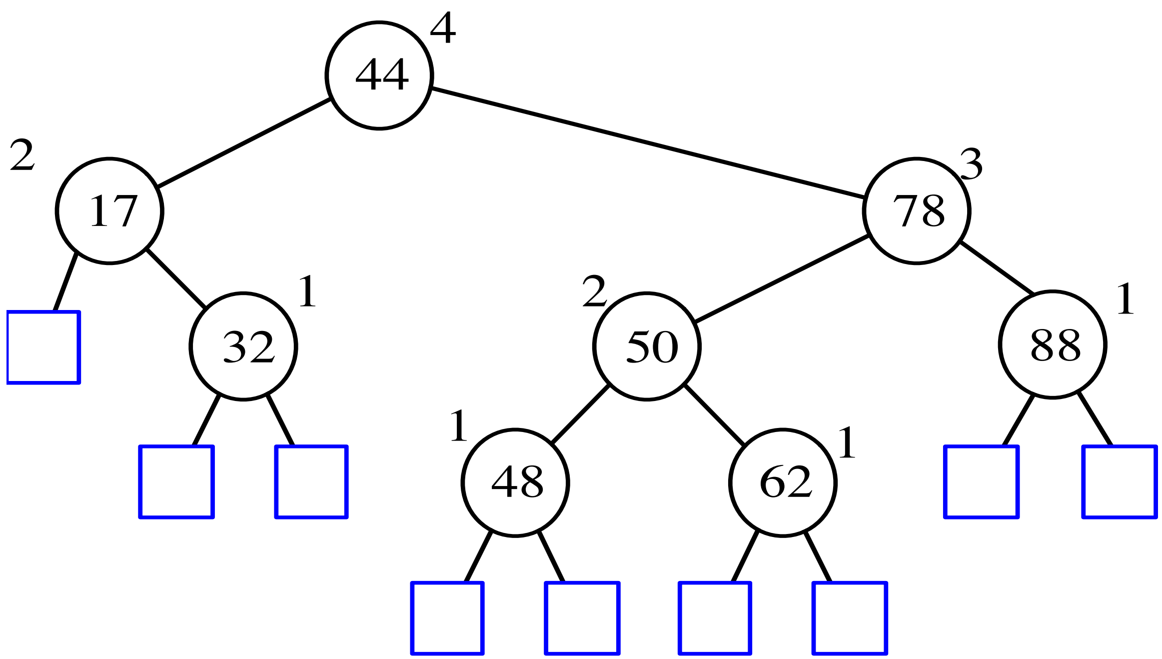
- A variety of algorithms augment a standard BST with occasional operations to reshape, reduce height and maintain balance.
- General approach: Rotation — moves a child to be above its parent,
  - ideally $O(1)$
  - certainly O(lgn)



AVL Trees

- Height-balance property
  - For every internal node, the avlHeight of the two children differ by at most 1
    - avlHeight = max distance from null endpoint
- Any binary tree satisfying the height-balance property is an AVL tree
- A height-balanced tree has height O(lg n)
  - max height is provably 1.44*lg(n)

AVL Tree Example



Insertion

- Maintain with each node the avlHeight.
- On insertion, first recur down through tree to insert.
- Then as you unwind recursion, update the avlHeight of each node.
- If height changes, check the height of other child
  - if not in balance then fix

Insertion code to maintain height

```
private class Node {
    Comparable<E> element;
    int avlHight;
    Node right;
    Node left;

    public Node(Comparable<E> e) {
        avlHight = 1;
        element=e;
        right=null;
        left=null;
    }
}
```

More insertion (pseudo)code

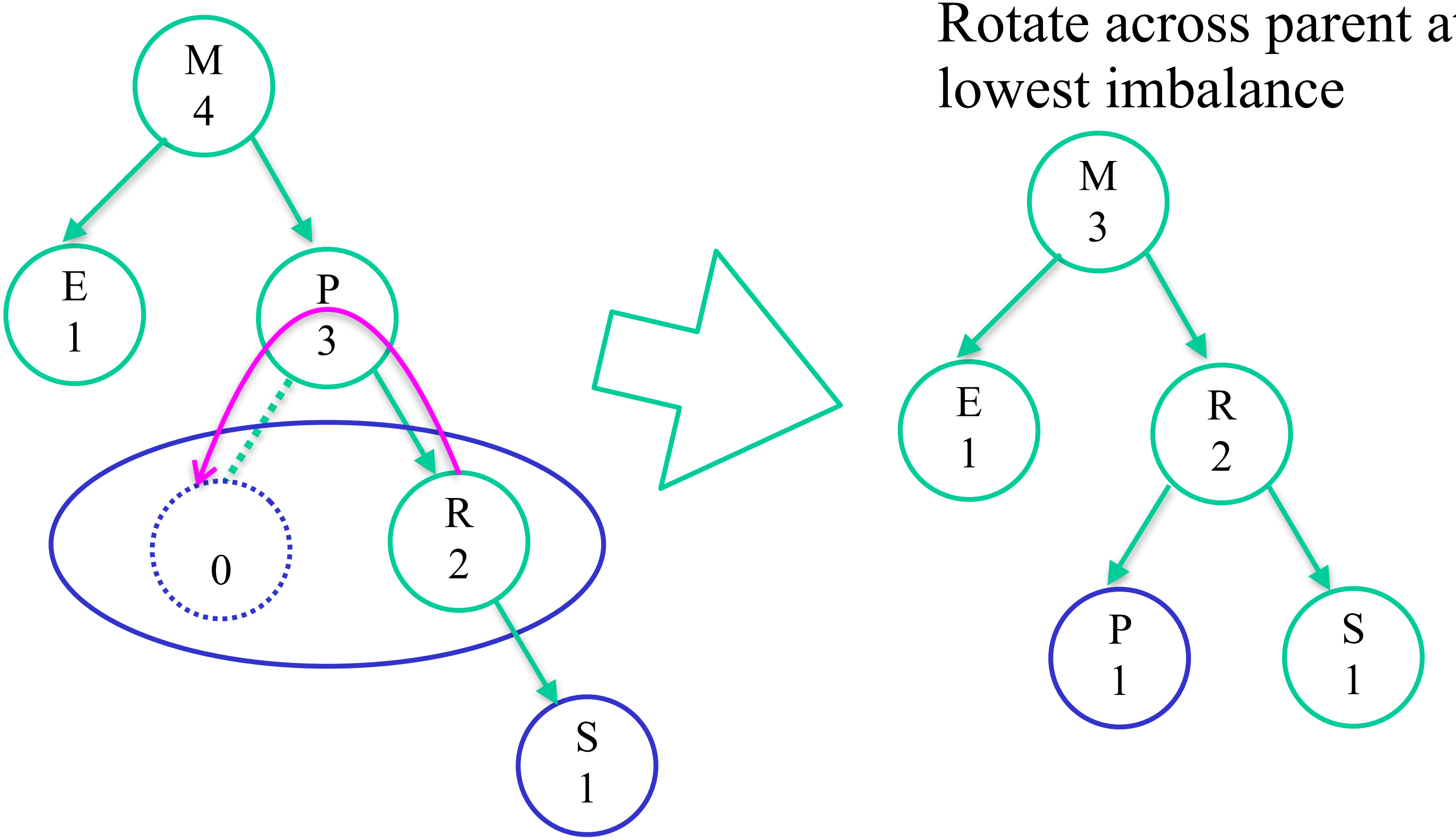
```
int insertUtil(node, element):

    if element==node.payload
        return -1;

    avlD=2; //!!
    if node.payload > element:
      if node.left==null
        node.left=new Node(payload)
      else
        avlD = 1+insertUtil(node.left,element);
    else
        // same but for right

    node.avlHieght = greater of avlD and
                     node.avlHeight

    return node.avlHeight
```

Fixing height imbalances Rotation!!

- Two types of rotation
- Single
  - left subtree of left node causes imbalance
  - right subtree of right node causes imbalance
- Double
  - right subtree of left node causes imbalance
  - left subtree of right node causes imbalance
    - The first rotation of a double puts the tree into position for a single rotation!

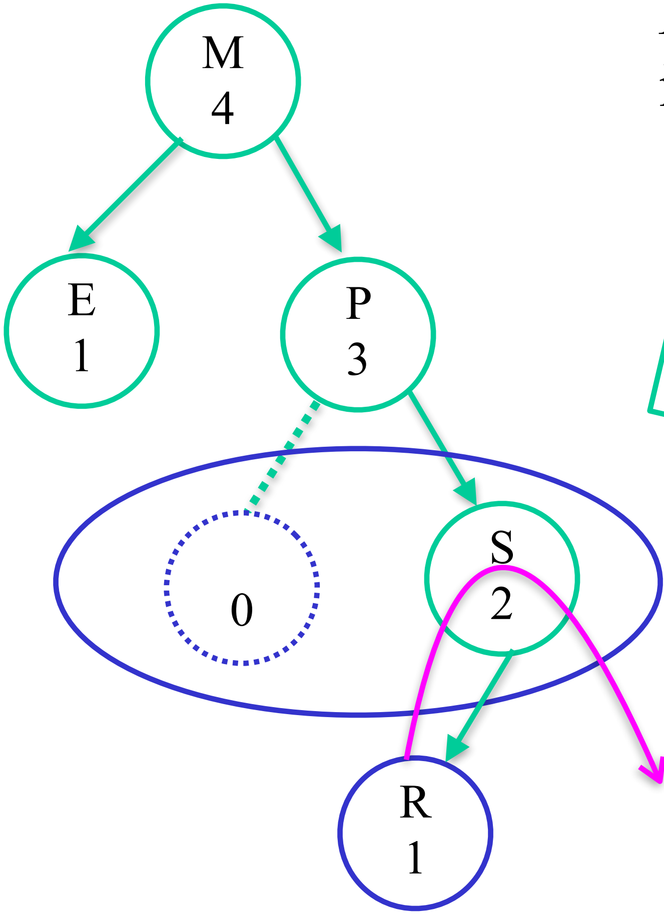
Single Rotation

Rotate across parent at the lowest imbalance

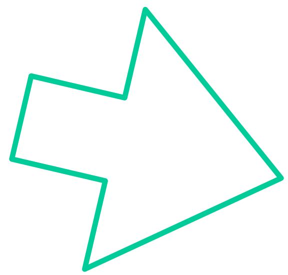


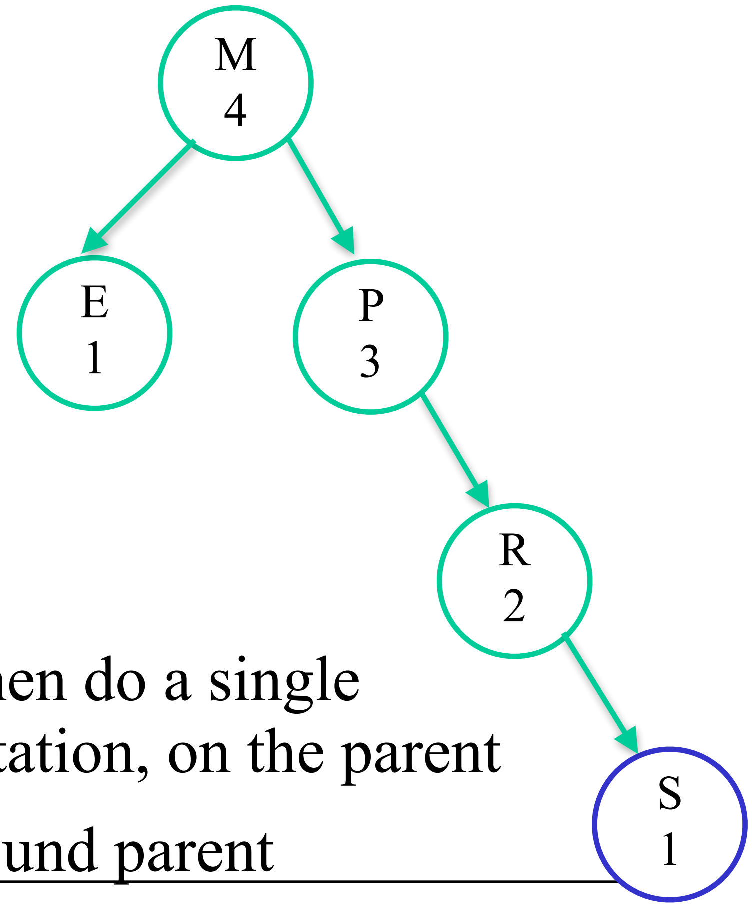
right-right => counter-clockwise rotation left-left => cw rotation

Double Rotation



First rotate across the child imbalance, This shifts from R-L to R-R





Then do a single rotation, on the parent

r,l => cw around child, then ccw around parent

Lab Part 1

- Given the following data show the tree after each operation, while keeping the tree balanced using AVL
- for deletions, always delete using inorder predecessor.
- iXXX == insert XXX into tree
- dXXX == delete XXX from tree

- i1024, i512, i256, i128, i64, i32, i16, i750, i875, d128, d32

- solutions:
  - https://cs.brynmawr.edu/cs151/L22/balance1.jpg
  - https://cs.brynmawr.edu/cs151/L22/balance2.jpg

Lab Part 2

Show the tree after each insertion / deletion
Before and after AVL rotation,
Send photo to gtowell151@cs.brynmawr.edu

insert	100
insert	200
insert	300
insert	400
insert	500
insert	600
insert	700
insert	800
insert	900
insert	750
insert	1000
insert	850
delete	400
delete	300
delete	200
delete	700
delete	500