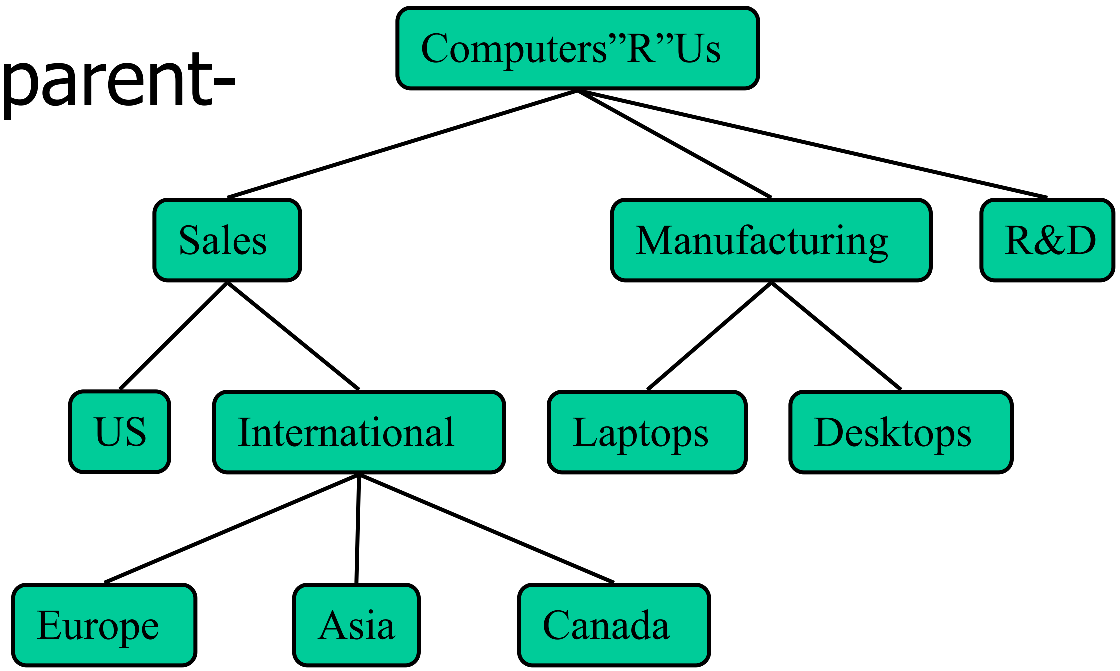
Trees

Tree

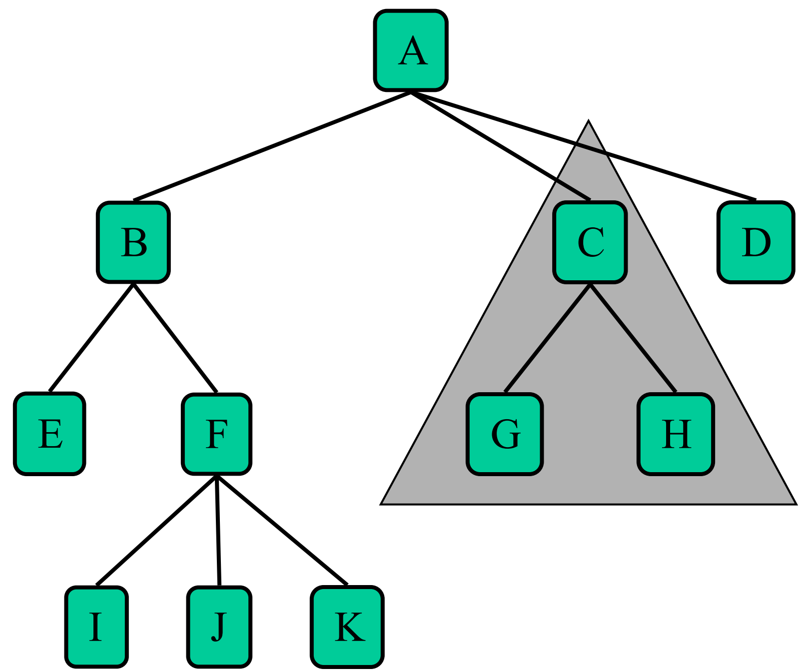
- A tree is an abstract model of a hierarchical structure
- Nodes have a parent-child relation
- No loops
- One Path



Terminology
Same as for heaps

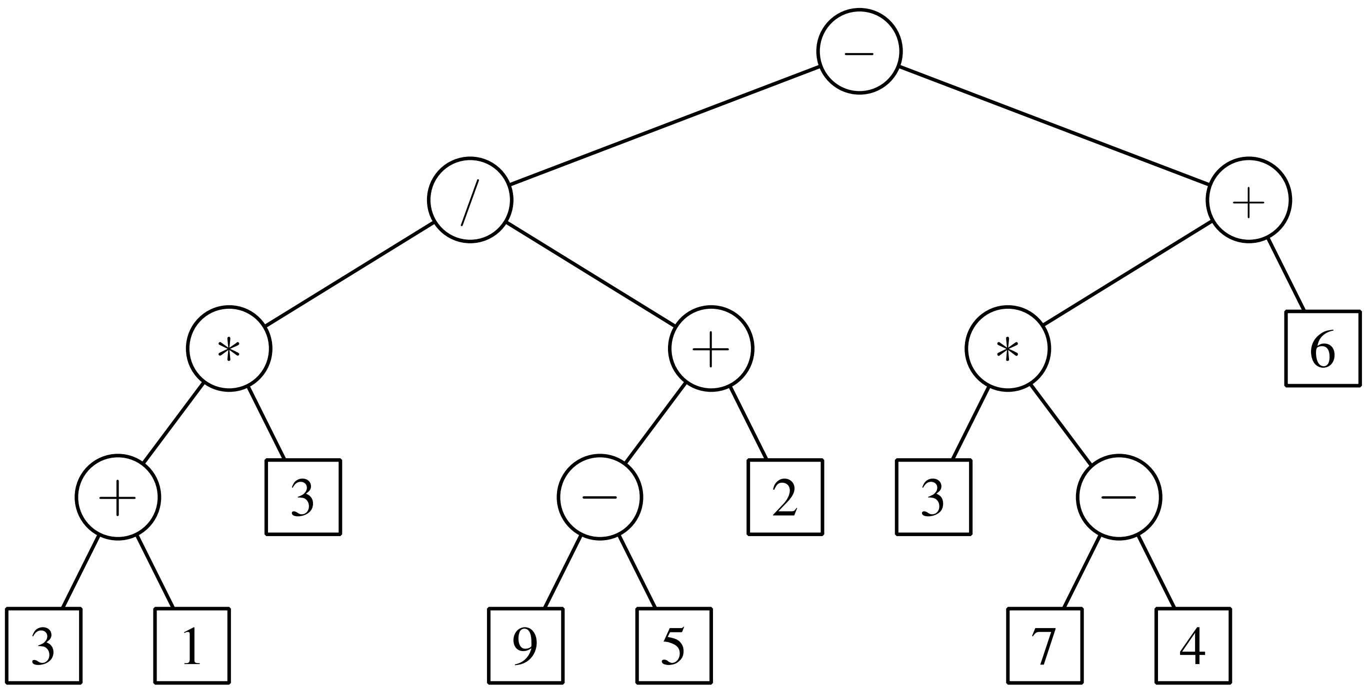
- root: no parent – A
  - There is only one root
- external node/leaf: no children – E, I, J, K, G, H, D
- internal node: node with at least one child - A, B, C, F
- ancestor/descendent
- depth - # of ancestors
- Height - max depth

- Subtree: tree consisting of a node and its descendants



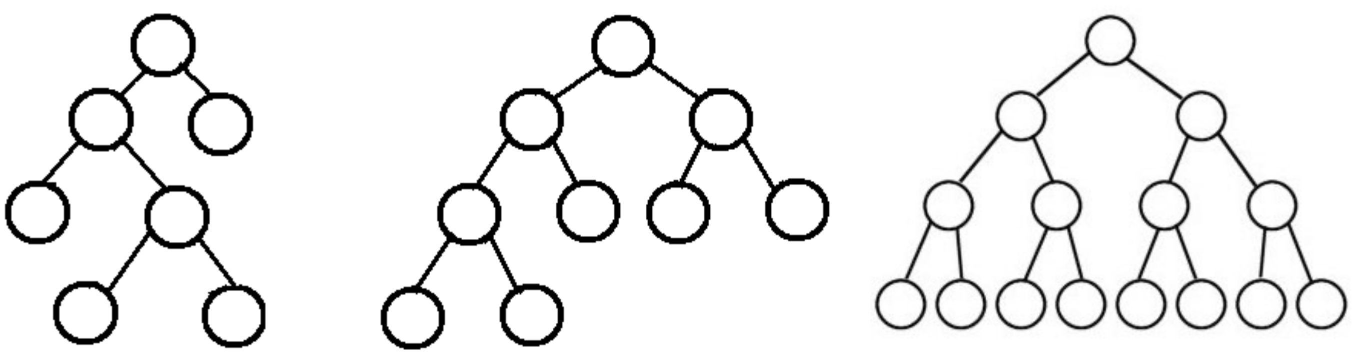
Binary Tree

- An tree with every node having at most two children – left and right



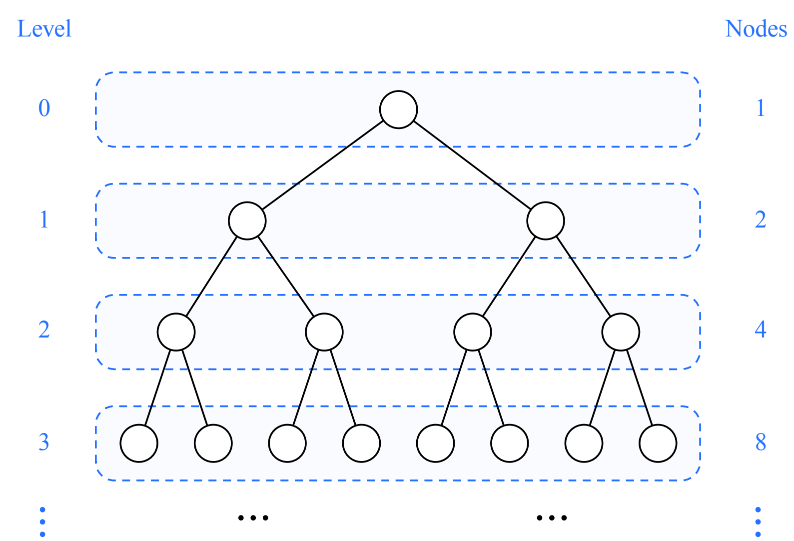
Type of Binary Trees

- A binary tree is **complete** if every level (except possibly the last) is filled
  - A (binary) heap is a complete binary tree
  - A Complete binary tree has height = $\log_2(n)$



Binary Tree Properties

- Let n denote the number of nodes and h the height of a binary tree
  - $h + 1 \leq n \leq 2^{h+1} - 1$
  - $\log(n + 1) - 1 \leq h \leq n - 1$
- Height of a binary tree is usually (you hope) $O(logn)$ of the max number of nodes
  - worst case ??

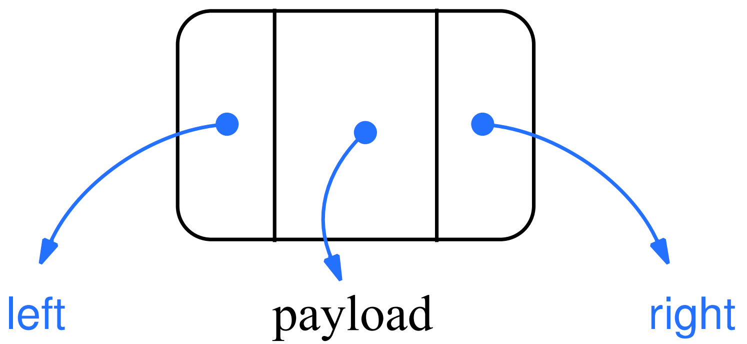


Interface

```
public interface TreeInterface<B>
{
    int size();
    int height();
    boolean isEmpty();
    boolean contains(B element);
    void insert(B element);
    B remove(B element);
    String printNaturalOrder();
}
```

Implementation

```
protected class Node<F extends Comparable<F>> {
       F payload;
       Node<F> right;
       Node<F> left;

        public Node(F e) {
            payload = e;
            right = null;
            left = null;
        }

        public String toString() {
            return payload.toString();
        }
}
```



This looks a lot like a doubly linked list!!
So, is a doubly linked list a tree?

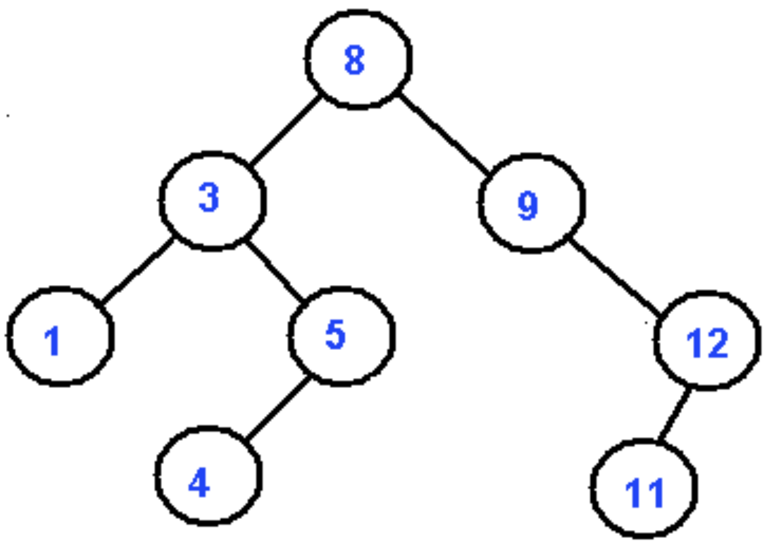
Class

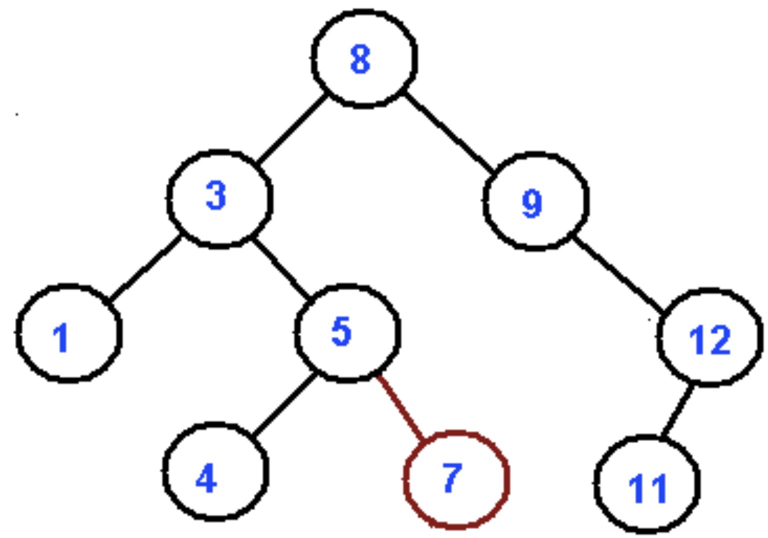
```
public class LinkedBinaryTree<E extends Comparable<E>>
implements TreeInterface<E> {

   protected Node . . .

   protected int size;
   protected Node<E> root;
```

Class name violates Encapsulation!

Insertion

- smaller to the left, bigger to the right





Following this pattern creates a “Binary Search Tree”

size() without size

- Size (number of nodes) of tree is
  - size of right subtree plus
  - size of left subtree plus
  - 1

Its recursive!!!

```
public int size() {
      return sizeAltUtil(root);
 }

private int sizeAltUtil(Node<E> treepart) {
      if (treepart == null)
          return 0;
      return sizeAltUtil(treepart.right) +
             sizeAltUtil(treepart.right) +
             1;
 }
```

Height / maxDepth

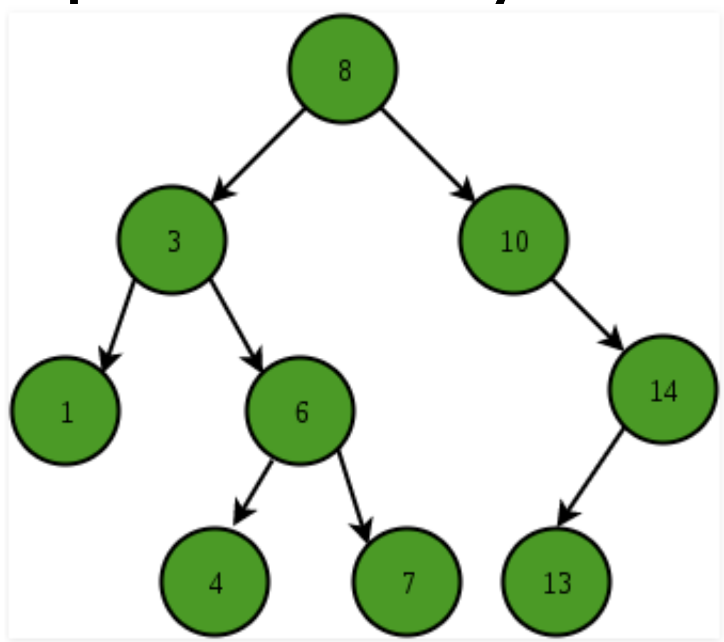
Again, using a recursive helper method

```
@Override
public int height()
{
   return maxDepthUtil(root, 1);
}

int maxDepthUtil(Node n, int depth) {
   …}
```

live write

contains

- returns true if found in the tree, false otherwise
- Assumes / requires Binary search tree



Contains Algorithm

- compare with root of **current subtree**
  - root is empty – return false
  - root == element – return true
  - root < element – recurse on right child
  - root > element - recurse on left child

- Comparisons are assumed to be done using Comparable interface (ie, the compareTo method)
  - <E extends Comparable<E>>

Pseudo Code

```
findRec(root, key):
  if root == null:
    return false
  if root.key == key:
    return true
  if root.key > key:
    return findRec(root.left, key)
  else
    return findRec(root.right, key)
```

Contains Code

- Write using a recursive helper method

```
public boolean contains(E element) {
        if (root==null) return false;
        return containsUtil(root, element)!=null;
    }
private Node containsUtil(Node treepart, E toBeFound) {
  … }
```

live write

Unordered Contains

- Suppose that you did not know relation among children (you do NOT have a binary search tree)
  - So thing being looked for could be either left or right
  - How would you change containsUtil function
    - Would a tree be a useful structure in this case?

insert

- `void insert(E element);`
- new node is always inserted as a leaf
- inserts to
  - left subtree if element is smaller than subtree root
  - right subtree if larger
- Pre-case: if root=null then root=new Node

```
public void insert(E element) {
        if (root==null) {
            root=new Node<E>(element);
            size = 1;
        } else
            insertUtil(root, element);
    }
```

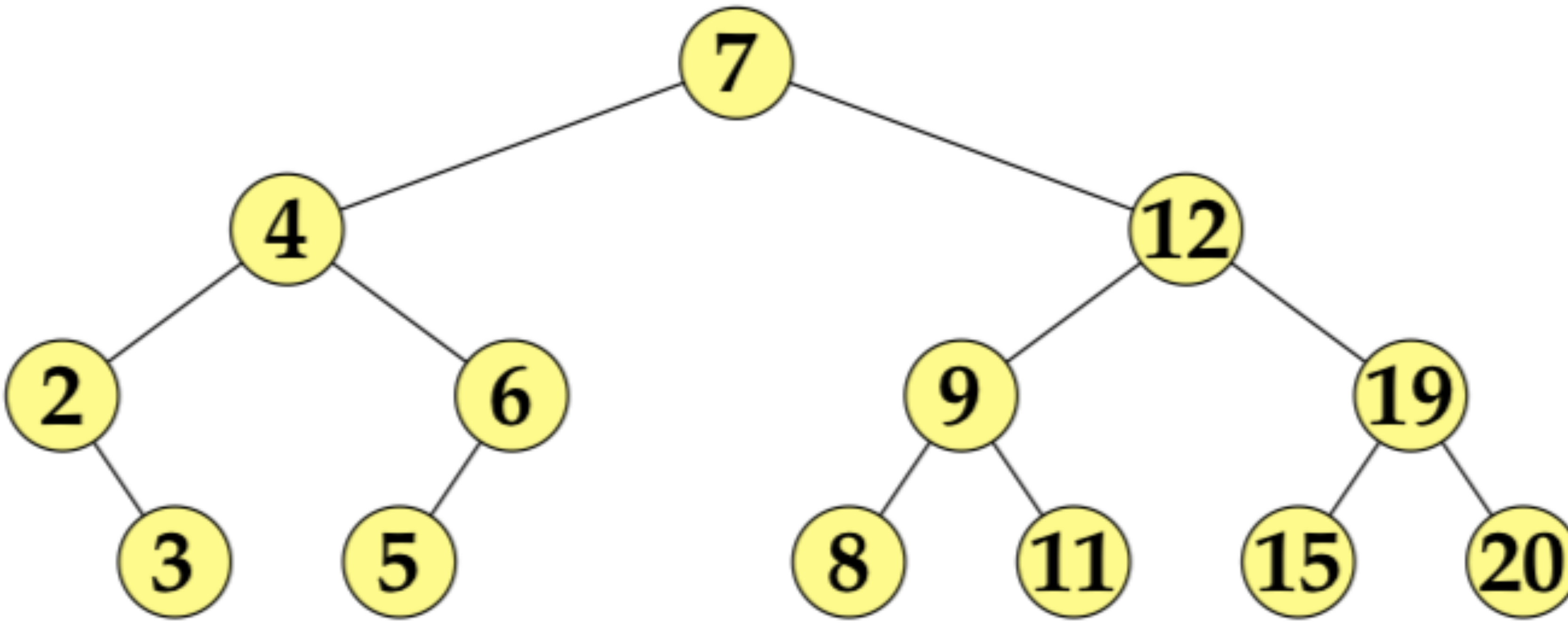
Groups

- Draw binary search trees for data received from left to righto
  - 4, 5, 6, 49, 43, 31, 19, 10, 11, 8, 17
  - 17, 31, 8, 19, 43, 11, 5, 49, 10, 6, 4

- Write insertUtil

```
private void insertUtil(Node treepart, E toBeAdded) {
 … }
```

Traversals / Printing



Postorder traversal

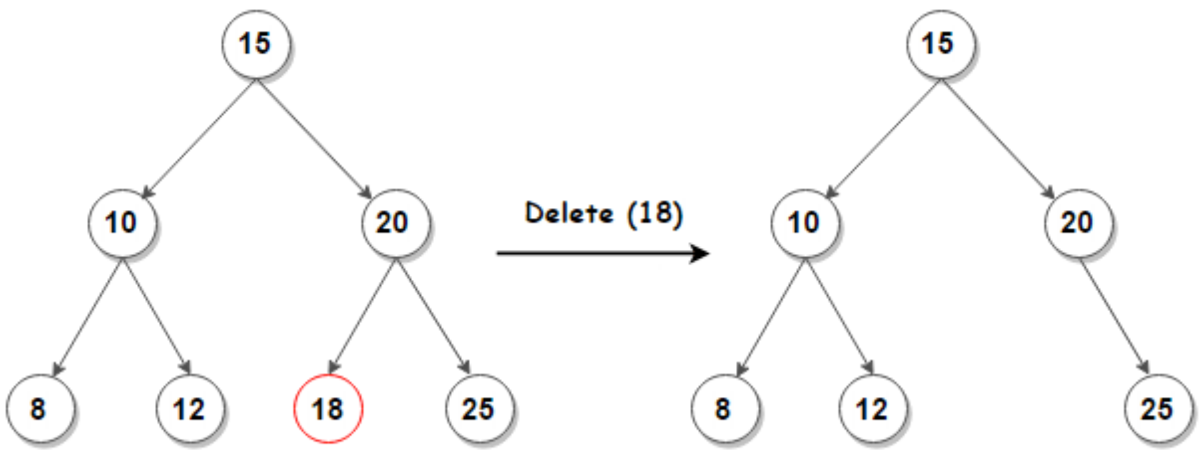
```
public void printPostOrder() {
    iPrintPostOrder(root, 0);
    System.out.println();
}

private void iPrintPostOrder(Node treePart, int depth) {
    if (treePart==null) return;
    iPrintPostOrder(treePart.left, depth+1);
    iPrintPostOrder(treePart.right, depth+1);
    System.out.print("["+treePart.payload+","+depth+"]");
}
```

Remove

- `boolean remove(E element);`
- returns true if element existed and was removed and false otherwise
- Cases
  - element not in tree
  - element is a leaf
  - element has one child
  - element has two children

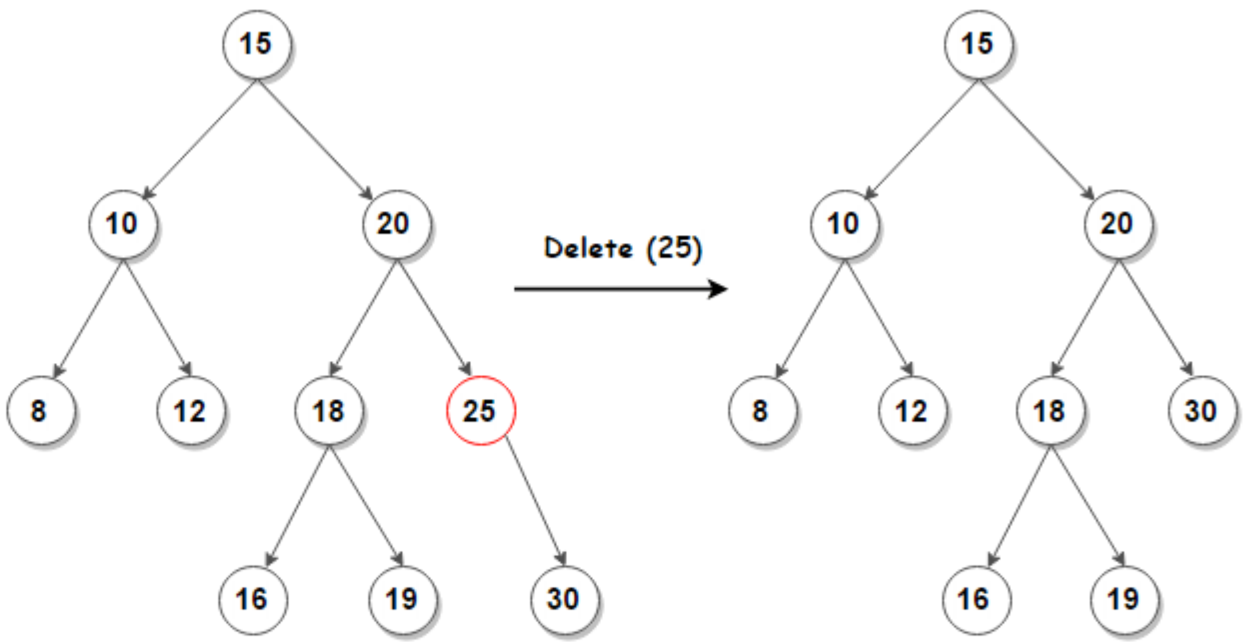
Leaf

- Just delete



One child

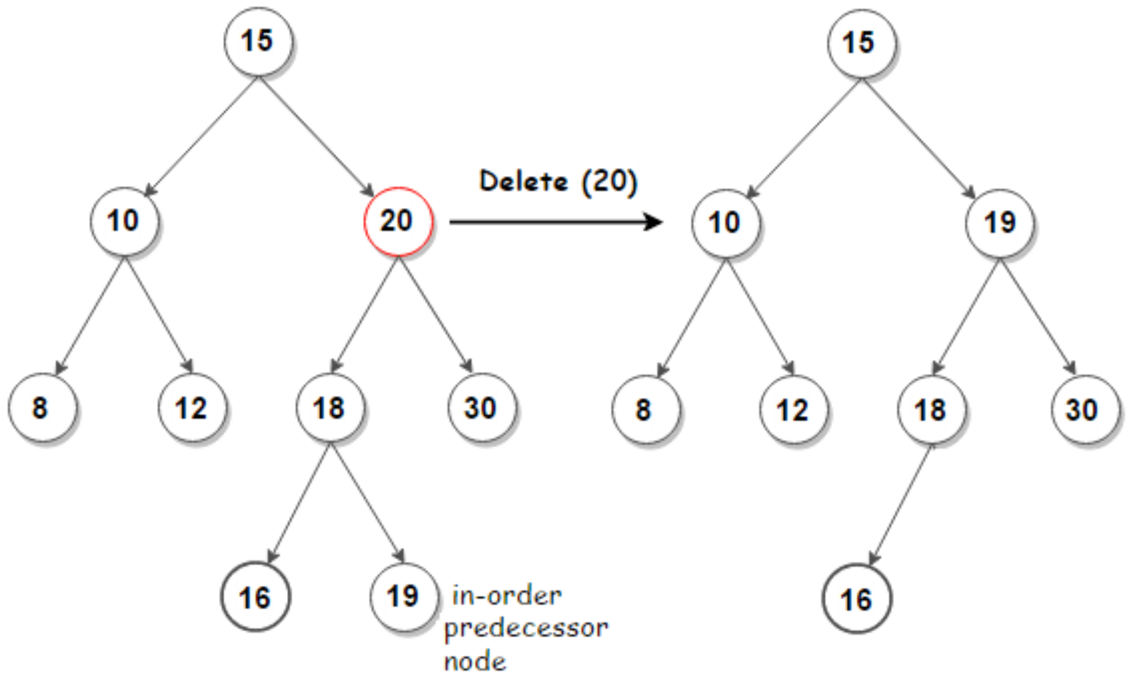
- Replace with child – skip over like in linked list



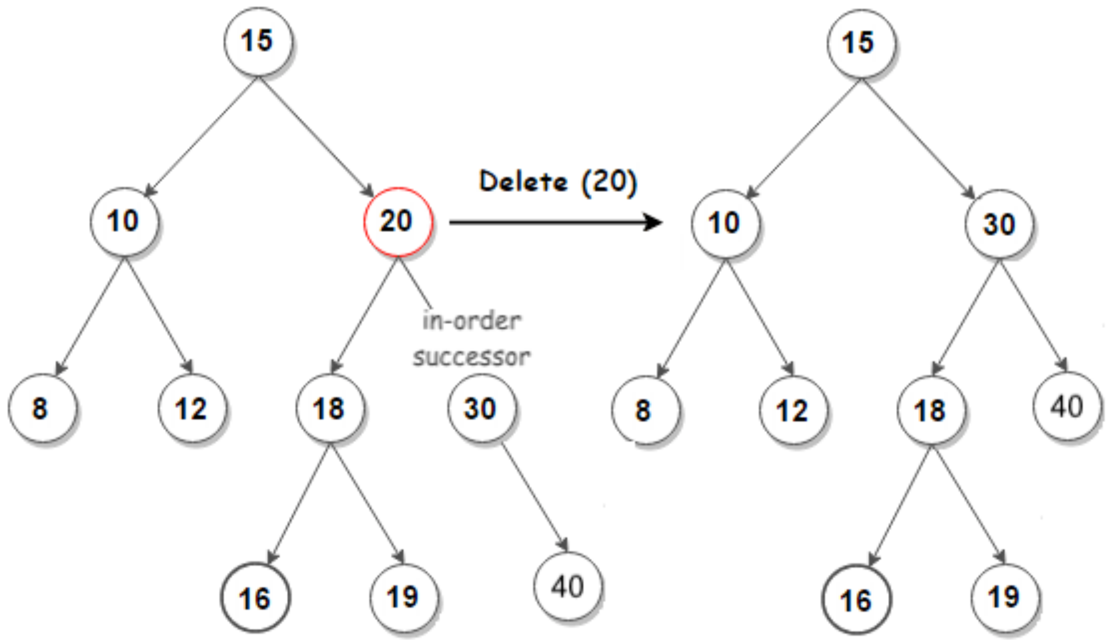
Two Children

- Replace with in-order predecessor or in-order successor
- in-order predecessor
  - rightmost child in left subtree
  - max-value child in left subtree
- in-order successor
  - leftmost child in right subtree
  - min-value child in right subtree

Replace with Predecessor



Replace with Successor



Practice

- Given the data:
- 6, 19, 10, 5, 43, 31, 11, 8, 4, 17, 49, 36

- Draw the binary tree
- Write the preorder traversal of your tree
- Write the postorder traversal of your tree
- What the height of the tree?
- If the data were re-arranged, what is the shorted possible tree?